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Dynamic and Stiffness Modeling of New 3DOF PKM for High Speed Machining Application

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***Abstract*—many applications in the field of production automation, such as machining, assembly and material loading require machines that are capable of high speed, accelerationand rigidity.This paper mainly addresses the issue of dynamic and stiffness formulation of a three prismatic-revolute-spherical PKM (3-PRS PKM) using screw theoryand virtual work approach. First the dynamic formulation is performed and later the stiffness equation is derived using virtual work principle. In order to build up the stiffness and dynamics model, kinematics, Jacobian, Hessian and Finite Element Analysis are also performed as the basis.The study of robot dynamics is necessary for its mechanical design and synthesis, providing the information on the force that must be resisted by joints, links and actuators while the study of stiffness is highly demanded to predict the rigidity performance of the PKM. So that, the inverse dynamics is developed with capable of calculating the force along the driving link direction. Also, Analytical stiffness model, a function of Jacobian matrix and components stiffness matrix, is obtained first using the principle of virtual work. Stiffness model is also a six by six dimensional matrix and can provide the information of actuation and constraint stiffness simultaneously.**

Keywords: PKMs, Hessian, 3PRS, Dynamics, stiffness analysis, FEA

I. INTRODUCTION

A parallel platform manipulator (PPM) is a device whose end-effector is attached to the ground via multiple serial chains that provides closed kinematic loops in the system for better load handling capacity and stiffness. It is these qualities that make it applicable in a wide range of applications ranging from flight simulators to high speed milling machines. However, it is these closed kinematic loops that increase the complexity of their analysis to a great extent(Y.G Li).

An important advantage of parallel platform manipulator is that its superior structural rigidity renders it as the better choice over serial chain manipulator (SCM) for moving heavy loads and high precision machining tasks. The advantages include very high accuracy, better stiffness ratio, more payload capacityand better inertial distribution among others(24-41).

Y.G Li, T.Huang and H.T .Liu[1] "a general approach for formulating dynamics of lower mobility parallel manipulators. Moreover, Joshi, S., Liu, H., Chetwynd, D.G.,Li,Z., [2]"formulated the generalized jacobian analysis of lower mobility manipulators, Dressler, I., Robertson. And Johansson, R.[3] "has designed the accuracy of kinematic and dynamic models of a Gantry-Tau parallel kinematic robot, Li Y G, Song Y M, Feng Z Y, et al.[8] has derived the Inverse dynamics of 3-RPS parallel mechanism by Newton-Euler formulation in Chinese and Liu, H., Huang, T.,Chetwynd, D.G.[11-20], An Approach for Acceleration Analysis of Lower Mobility Parallel Manipulators.

Since the industrial revolution, there has been an ever increasing to improve product quality and reduce manufacturing cost parallel kinematic machines are applied on various area such as, For high speed and high precision machining center, for machining purpose, air plane simulators and For pick and place, assembly and carrying load purpose in higher industries like aircraft, electronics and automotive factories and robotic assistance surgery. For the successful implementation of parallel manipulators in different engineering and technological areas studying the motion and rigidity has no any equivalent substitution. As it is already discussed before the dynamics and stiffness analysis will be studied in this paper as follows.

II. CAD MODEL KINEMATIC DESCRIPTION

The architecture of 3PRS PKMs is showing on fig.1 which is composed of moving platform, a fixed base the three supporting limbs with identical configurations. Each limb connects the fixed base to the moving platform by prismatic, revolute and spherical joint respectively and the prismatic joint is the active joint which is actuated by the linear actuator servo motor.

The considered machine is 3-DOF PKM, which can be showed mathematically by using mobility criterion.

$M = 6(n - j - 1) + \sum_{i=1}^{j} f_i$, where M is the DOF, n is number of links in the system, j is number of joint and f_i is the number of DOF of the i^{th} joints. For this manipulator n=8,j=9,f_i=3 for spherical joint and f_i=1 for prismatic and revolute joints. Therefore the above equation will give as the system has 3 DOF.

$$M = 6(8-9-1)+3(1+1+3) = 3\text{DOF} \qquad (1)$$

The vectors and reference frames are described also in the figure for the sake of analysis, as shown in the figure 2 a fixed Cartesian reference coordinate frame P (x, y, z) is attached at the center point O of the fixed triangle base platform $\Delta C_1C_2C_3$. And the moving coordinate system O'(u, v, w) is attached on the moving platform a point O' Which is a center of $\Delta Q_1Q_2Q_3$. For simplicity without losing generality, let x-axis be aligned toward OC_3 and the u-axis pointing along the direction of $\overline{O'Q_3}$.

From the above expression R_1,R_2,R_3 are the vector of fixed base platform from the center point O to C_i and r_1,r_2, $r_3$are the vectors of the moving platform from the center point O' to Q_i.

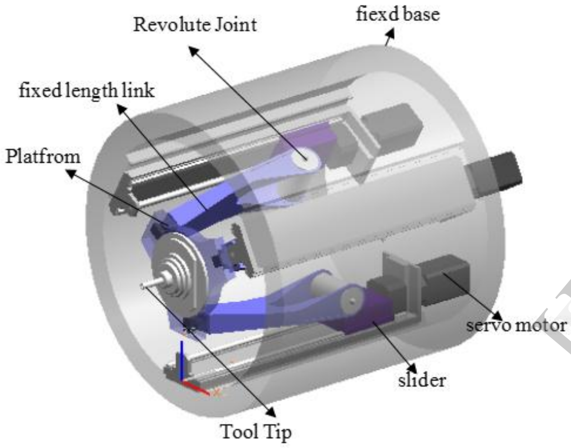

figure 1: 3D CAD model of the machine

III. VELOCITY ANALYSIS

In this section the velocity analysis of 3PRS have done by applying the principle of screw theory. There are five joint screws associated with each limb. The first joint is the only actuated joint and the remaining joints are passive. The instantaneous twist of the moving platform can be expressed as a linear combination of five screws.

$$\$p = \dot{h}_{1,i}\hat{\$}_{1,i} + \dot{\theta}_{2,i}\hat{\$}_{2,i} + \dot{\theta}_{3,i}\hat{\$}_{3,i} + \dot{\theta}_{4,i}\hat{\$}_{4,i} + \dot{\theta}_{5,i}\hat{\$}_{5,i} \qquad (8)$$

Where

$$\hat{\$}_{1,i} = \begin{bmatrix} s_1 \\ 0 \end{bmatrix}, \quad \hat{\$}_{2,i} = \begin{bmatrix} (r_i - l_i)\times s_2 \\ s_2 \end{bmatrix}, \quad \hat{\$}_{3,i} = \begin{bmatrix} r_i \times s_3 \\ s_3 \end{bmatrix}$$

$$\hat{\$}_{4,i} = \begin{bmatrix} r_i \times s_4 \\ s_4 \end{bmatrix}, \hat{\$}_{5,i} = \begin{bmatrix} r_i \times s_5 \\ s_5 \end{bmatrix} \qquad (9)$$

Where $\hat{\$}_{i,j}$ is a unit vector along the joint J^{th} joint of the i^{th} limb. These five screws form five system for which a one system of reciprocal screw exists. This reciprocal screw lies on the intersection of the two planes. The first plane is perpendicular for the prismatic joint axis and the second plane is containing both the revolute and spherical joint. This reciprocal screw denoted as $\hat{\$}_{r1,i}$ is zero pitch screw passing through the center of spherical joint and parallel to $s_{2,i}$.

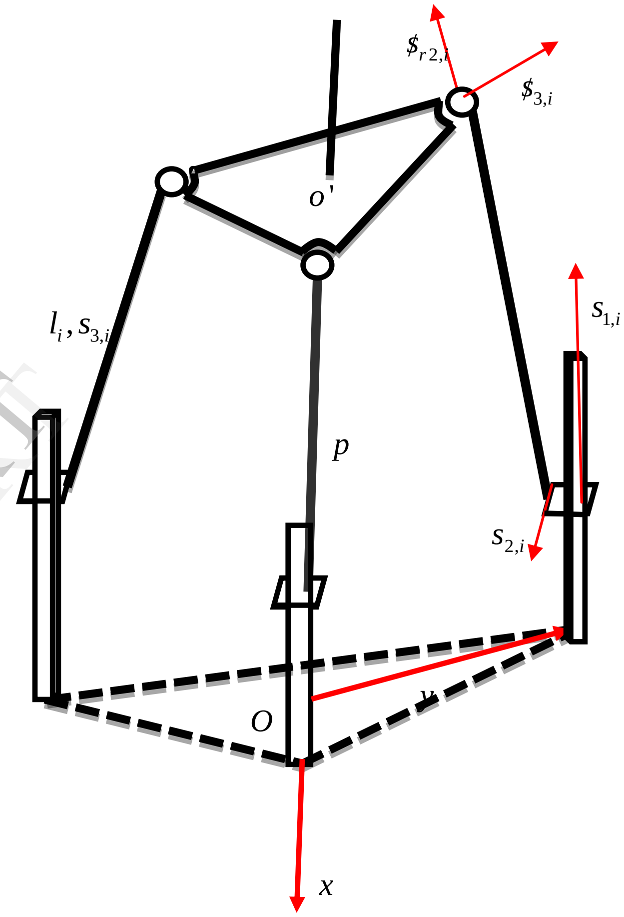

figure 2: 2D schematic diagram of the mechanism

$$\hat{\$}_{r1,i} = \begin{bmatrix} (r_i \times s_{2,i}) \\ s_{2,i} \end{bmatrix} \qquad (10)$$

By taking the inner product of both sides of the instantaneous with the constraint wrench we found the constraint Jacobian (J_C) .

$$J_C = \begin{bmatrix} s_{2,1}{}^T & (r_1 \times s_{2,1})^T \\ s_{2,2}{}^T & (r_2 \times s_{2,2})^T \\ s_{2,3}{}^T & (r_3 \times s_{2,3})^T \end{bmatrix} \qquad (11)$$

This matrix represents the constraint imposed by the revolute joint. An additional basis screw which is reciprocal to the passive joint of the i[th] limb can be identified as zero pitch screw passing through the center of spherical joint. This reciprocal screw represents wrench of actuation and it is normal to the previous system. This can be expressed by

$$\hat{\$}_{r2,i} = \begin{bmatrix} r_i \times s_{3.i} \\ s_{3.i} \end{bmatrix} \tag{12}$$

Take the orthogonal product of this reciprocal wrench for both sides of the twist screw. Then we found J_x.

$$\hat{\$}p \otimes \hat{\$}_{r2,i} = \left(\hat{\$}_{1,i} \otimes \hat{\$}_{r2,i}\right)\dot{h}_i \tag{13}$$

$$J_x = \begin{bmatrix} s_{3,1}{}^T & (r_1 \times s_{3,1})^T \\ s_{3,1}{}^T & (r_2 \times s_{3,1})^T \\ s_{3,1}{}^T & (r_3 \times s_{3,1})^T \end{bmatrix} \tag{14}$$

Equation 13 can be rewrite three times, once for each limb and it yields.

$J_x \$p = J_q \dot{q}$ Where the inverse Jacobian is J_q is:-

$$J_q = \begin{bmatrix} s_{3,1}{}^T s_{1,1} & 0 & 0 \\ 0 & s_{3,2}{}^T s_{1,2} & 0 \\ 0 & 0 & s_{3,3}{}^T s_{1,3} \end{bmatrix} \tag{15}$$

The actuation Jacobian, is responsible to relating the Cartesian velocity with joint rate $J_{\dot{q}}$, is a function of J_x and J_a.

$$J = \begin{bmatrix} J_a \\ J_c \end{bmatrix} \quad \text{Where,} \quad J_a = \frac{J_x}{J_q} \tag{16}$$

Equation 16 is a generalized Jacobian that relates the velocity of joint rate to the velocity of the moving platform.:

IV. ACCELERATION ANALYSIS

For the acceleration analysis of this 3PRS parallel manipulator we used a new approach [11] named Hessian matrix to achieve an explicit and compact form equation.

$$Ja,i = [sta,_{1,i} \quad sta,_{2,i}, sta,_{3,i} \quad sta,_{4,i} \quad sta,_{5,i}] \tag{17}$$

$$J_{c,i} = stc_{1,i} = \begin{bmatrix} (b_i \times ls_{3,i}) \times n_{1,i} \\ n_{1,i} \end{bmatrix} \quad \text{for i=1,2,3} \tag{18}$$

Where $J_{c,i}$ is constraint Jacobian for a limb.

Here $s_{4,i} = \begin{bmatrix} \dfrac{s_{3,i} \times s_{5.i}}{\left\| s_{3,i} \times s_{5.i} \right\|} \end{bmatrix}$ and $s_{5,i} = -Rs_{2,i}$ for i=1,2,3

$$J_i = \begin{bmatrix} J_{a,i} & J_{c,i} \end{bmatrix}, \text{ Jacobian for each limb.} \tag{19}$$

The Hessian matrix can be found from the derivative of screws in the Lie bracket form. This yields

$$\boldsymbol{H}_{a,i} = \begin{bmatrix} \mathbf{0} & [\hat{\$}_{ta,1,i}\ \hat{\$}_{ta,2,i}] & [\hat{\$}_{ta,1,i}\ \hat{\$}_{ta,3,i}] & \cdots & [\hat{\$}_{ta,1,i}\ \hat{\$}_{ta,n_i,i}] \\ \mathbf{0} & \mathbf{0} & [\hat{\$}_{ta,2,i}\ \hat{\$}_{ta,3,i}] & \cdots & [\hat{\$}_{ta,2,i}\ \hat{\$}_{ta,n_i,i}] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & [\hat{\$}_{ta,n_i-1,i}\ \hat{\$}_{ta,n_i,i}] \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{20}$$

$$\boldsymbol{H}_{c,i} = \begin{bmatrix} \mathbf{0} & [\hat{\$}_{tc,1,i}\ \hat{\$}_{tc,2,i}] & [\hat{\$}_{tc,1,i}\ \hat{\$}_{tc,3,i}] & \cdots & [\hat{\$}_{tc,1,i}\ \hat{\$}_{tc,6-n_i,i}] \\ \mathbf{0} & \mathbf{0} & [\hat{\$}_{tc,2,i}\ \hat{\$}_{tc,3,i}] & \cdots & [\hat{\$}_{tc,2,i}\ \hat{\$}_{tc,6-n_i,i}] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & [\hat{\$}_{tc,5-n_i,i}\ \hat{\$}_{tc,6-n_i,i}] \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{21}$$

$$\boldsymbol{H}_{ac,i} = \begin{bmatrix} [\hat{\$}_{ta,1,i}\ \hat{\$}_{tc,1,i}] & \cdots & [\hat{\$}_{ta,1,i}\ \hat{\$}_{tc,6-n_i,i}] \\ \vdots & \vdots & \vdots \\ [\hat{\$}_{ta,n_i,i}\ \hat{\$}_{tc,1,i}] & \cdots & [\hat{\$}_{ta,n_i,i}\ \hat{\$}_{tc,6-n_i,i}] \end{bmatrix} \tag{22}$$

$A = J_i \ddot{q}_i + \dot{q}_i{}^T H_i \dot{q}_i$ This equation puts the screw derivative ($[* \quad *]$) and its coefficient side by side. i.e. $\dot{q}_i$ is coefficient and H_i is screw derivative in Lie bracket form. And the overall Hessian matrix can be written in the form of

$$H_i = \begin{bmatrix} H_{a,i} & H_{ac,i} \\ 0 & H_{c,i} \end{bmatrix} \tag{23}$$

The matrix in eq. (23) is generalized Hessian with $6 \times 6 \times 6$ matrix for each limb contain both the actuation and constraint joints and eq. (20, 21, 22) are the element of general Hessian matrix.

This generalized matrix will help us to solve the dynamics of the generalized lower DOF parallel manipulators in efficient, compact and robust way and it is believed solving the Hessian in this form will simplified the dynamics work in unbelievable status.

V. DYNAMICS FORMULATION

This work comes with applying screw mathematics in order to utilize the advantage of Robust behavior of virtual work for solving inverse dynamics of symmetrically configured PKM.

Gravity of the moving platform and the *p*th ($1 \le p \le n_i - 1$) link of the *i*th ($i = 1, 2, \cdots, l$) limb can be expressed as follow

$$\$_{wg} = \begin{bmatrix} m_C \boldsymbol{g} \\ \mathbf{0} \end{bmatrix}, \ \$_{wg,p,i} = \begin{bmatrix} m_{p,i} \boldsymbol{g} \\ \mathbf{0} \end{bmatrix} \tag{24}$$

where $\boldsymbol{g}$ is the vector of acceleration of the gravity, m_C and $m_{p,i}$ is the mass of the moving platform and the *p*th link of the *i*th limb respectively.

According to Euler's Theorem, the inertial forces of the moving platform can be expressed as

$\boldsymbol{\$}_{wI} = -\boldsymbol{m}_C \dot{\boldsymbol{\$}}_{Ct} - [\boldsymbol{\$}_{Ct} \times] \boldsymbol{m}_C \boldsymbol{\$}_{Ct}$ (25)

Where

$$\boldsymbol{m}_C = \begin{bmatrix} m_C \boldsymbol{E} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_C \end{bmatrix}, \ \dot{\boldsymbol{\$}}_{Ct} = \boldsymbol{A}_C + \boldsymbol{J}_{C\omega} \boldsymbol{\$}_t = \begin{bmatrix} \dot{\boldsymbol{v}}_C \\ \dot{\boldsymbol{\omega}} \end{bmatrix}$$

$$\boldsymbol{J}_{C\omega} = \boldsymbol{J}_{\omega} \boldsymbol{T}_0, \ \boldsymbol{J}_{\omega} = \begin{bmatrix} [\boldsymbol{\omega} \times] & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \ [\boldsymbol{\$}_{Ct} \times] = \begin{bmatrix} [\boldsymbol{v}_C \times] & \boldsymbol{0} \\ \boldsymbol{0} & [\boldsymbol{\omega} \times] \end{bmatrix}$$

where $\boldsymbol{I}_C = \boldsymbol{R} \boldsymbol{I}_{C0} \boldsymbol{R}^{\mathrm{T}}$ and $\boldsymbol{I}_{C0}$ denotes the inertial matrix of the moving platform about the mass centre described in the reference coordinate system and local coordinate system respectively, $\boldsymbol{R}$ is the rotation matrix of the local coordinate system with respect to the reference coordinate system.

Similarly, the inertial forces of the *p*th ($1 \le p \le n_i - 1$) link of the *i*th ($i = 1, 2, \cdots, l$) limb can be expressed as follows

$$\boldsymbol{\$}_{wI,p,i} = -\boldsymbol{m}_{p,i} \dot{\boldsymbol{\$}}_{Cta,p,i} - \left[\boldsymbol{\$}_{Cta,p,i} \times\right] \boldsymbol{m}_{p,i} \boldsymbol{\$}_{Cta,p,i} \qquad (26)$$

Where,

$$\boldsymbol{m}_{p,i} = \begin{bmatrix} m_{p,i} \boldsymbol{E} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{p,i} \end{bmatrix}, \ \dot{\boldsymbol{\$}}_{Cta,p,i} = \boldsymbol{A}_{C,p,i} + \boldsymbol{J}_{C\omega,p,i} \boldsymbol{\$}_t = \begin{bmatrix} \dot{\boldsymbol{v}}_{C,p,i} \\ \dot{\boldsymbol{\omega}}_{p,i} \end{bmatrix}$$

$$\boldsymbol{J}_{C\omega,p,i} = \boldsymbol{J}_{\omega,p,i} \boldsymbol{T}_{p,i} \boldsymbol{J}_{p,i} \boldsymbol{J}_{a,p,i}^{L}, \ \boldsymbol{J}_{\omega,p,i} = \begin{bmatrix} \left[\boldsymbol{\omega}_{p,i} \times\right] & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}$$

$$\left[\boldsymbol{\$}_{Cta,p,i} \times\right] = \begin{bmatrix} \left[\boldsymbol{v}_{C,p,i} \times\right] & \boldsymbol{0} \\ \boldsymbol{0} & \left[\boldsymbol{\omega}_{p,i} \times\right] \end{bmatrix}$$

where $\boldsymbol{I}_{p,i} = \boldsymbol{R}_{p,i} \boldsymbol{I}_{p,i,0} \boldsymbol{R}_{p,i}^{\mathrm{T}}$ and $\boldsymbol{I}_{p,i,0}$ denotes the inertial matrix of the link about the mass centre described in the reference coordinate system and local coordinate system respectively, $\boldsymbol{R}_{p,i}$ is the rotation matrix of the local coordinate system with respect to the reference coordinate system.

Let $\boldsymbol{\$}_{we} = \left(\boldsymbol{f}^{\mathrm{T}}, \boldsymbol{t}^{\mathrm{T}}\right)^{\mathrm{T}}$ be the load wrench applied on the reference point of the moving platform, where $\boldsymbol{f}$ and $\boldsymbol{t}$ is the vector of force and moment respectively. By using of the principle of virtual work, we can obtain

$$\delta w + \delta \boldsymbol{q}^{\mathrm{T}} \boldsymbol{\tau} + \delta \boldsymbol{\$}_t^{\mathrm{T}} \boldsymbol{\$}_{we} = 0 \quad (26)$$

where

$$\delta w = \delta \boldsymbol{\$}_{Ct}^{\mathrm{T}} \left(\boldsymbol{\$}_{wI} + \boldsymbol{\$}_{wg}\right) + \sum_{i=1}^{l} \sum_{p=1}^{n_i - 1} \delta \boldsymbol{\$}_{Cta,p,i}^{\mathrm{T}} \left(\boldsymbol{\$}_{wI,p,i} + \boldsymbol{\$}_{wg,p,i}\right)$$

$$\boldsymbol{\tau} = \begin{bmatrix} \boldsymbol{\tau}_a^{\mathrm{T}} & \boldsymbol{\tau}_c^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$

where $\boldsymbol{\tau}_a$ is the *f*-dimensional vector composed of the driving force coefficients, and $\boldsymbol{\tau}_c$ is the vector composed of the constraint force coefficients.

The equation of velocity of joint, twist of the moving platform, twist of the p[th] link of the i[th] limb can be rewritten in the variational form as follows

$$\delta \boldsymbol{\$}_{Ct} = \boldsymbol{T}_0 \delta \boldsymbol{\$}_t, \ \delta \boldsymbol{q} = \boldsymbol{J} \delta \boldsymbol{\$}_t, \ \delta \boldsymbol{\$}_{Cta,p,i} = \boldsymbol{J}_{C,p,i} \delta \boldsymbol{\$}_t \qquad (27)$$

Substituting Eqs(24), (25), (26) and (27) into Eq(27), the dynamics model can be constructed

$$\boldsymbol{\tau} = \boldsymbol{D}(\boldsymbol{r}) \dot{\boldsymbol{\$}}_t + \boldsymbol{H}(\boldsymbol{r}, \boldsymbol{\$}_t) \boldsymbol{\$}_t + \boldsymbol{G}(\boldsymbol{r}) + \boldsymbol{E}(\boldsymbol{r}) \qquad (28)$$

VI. STIFFNESS MODELING

Under the assumption that the platform and the machine frame are rigid, when the platform is subjected to the external wrench $\tau = [F^T, M^T]$ on the reference point p, where F and M are the external force and torque applied to the platform, the deformation of the limbs will causes the platform to experience a twist $\Delta x = [\Delta r^T, \Delta \alpha^T]$ in terms of the translational and rotational deformations along/about the axes of frame. Then, applying the virtual work principle to the platform gives

$$\Delta x^T \tau = \Delta \rho^T f \qquad (29)$$

Where $\Delta \rho$ and f represents the set of deflections and reaction force magnitude

$$\Delta \rho = J \Delta x$$

This equation $\Delta x^T \tau = \Delta \rho^T f$ can be re write

$\Delta x^T \tau - \Delta \rho^T f = 0$ Substitute $\Delta \rho$ from the above equation

$$(\tau^T - J f^T) \Delta x = 0$$

$$\tau^T - J f^T = 0$$

Taking the transpose yields: $\tau = J^T f$

Where $f = [f_a^T, f_c^T]$ is the internal wrench vector of limbs, where f_a and f_c are the generalized force of the PRS limbs and the PR limb related to the twist $\Delta \rho = [\Delta \rho_a^T, \Delta \rho_c^T]$, f_a is a force which is parallel to the screw axis while the f_c is parallel to the revolute axis.

Therefore the virtual work principle can be written

$$\Delta X^T \tau = \Delta \rho^T f \Leftrightarrow \Delta r^T F + \Delta \alpha^T = \Delta \rho_a^T f_a + \Delta \rho_c^T f_c$$

$$f_{ai} = [f_{a1}, f_{a2}, f_{a3}]^T,$$

$$\Delta \rho_a = [\Delta q_1, \Delta q_1, \Delta q_3]^T$$

$$f_{ci} = [f_{c1}, f_{c2}, f_{c3}]^T,$$

$$\Delta \rho_c = [\Delta c_1, \Delta c_2, \Delta c_3]^T$$

(30)

$f_a = k_a \Delta \rho_a$, $f_c = k_c \Delta \rho_c$ and

$$f = k_{ac}\Delta\rho$$

Where $k_{ac} = \begin{bmatrix} k_a & 0 \\ 0 & k_c \end{bmatrix}$

Here ka k_a and k_c are known as the component stiffness matrix of actuation and constraints respectively the formulation of their element

$$\tau = k\Delta x, k = J^T k_{ac} J \tag{31}$$

And the compliance model can be evaluated as

$$c = k^{-1}$$

A) **Formulation of k_{ac}**

As shown in the Figure 3 above the limb model has To formulate k_{ac} I group all the parts of an PRS limp in to four components: (1) the spherical joint (2) the limp body which is the fixed lengths link (3) R joint assembly (4) the lead screw assembly. as well as analytically convenient, they are sub systems that must realistically be subjected independently to design improvements.

k_a is given in a diagonal matrix i.e. $k_a = diag[k_{ai}]$ where $(i = 1,2,3)$ with k_{ai} being the axial stiffness coefficient at B_i along the S_{1i} axis of the ith limb referring to the fig 4. k_{ai} Can be modeled by four serially connected springs each representing the stiffness of one of the four components such

$$\frac{1}{k_{ai}} = \frac{1}{k_{lsa}} + \frac{1}{k_l} + \frac{1}{k_s} + \frac{1}{k_r} \tag{32}$$

Where k_{lsa}, k_l, k_s and k_r are the axial stiff nesses coefficients of the lead screw assembly ,the fixed length limb ,S joint and R joint assembly respectively.

Note that k_l , and k_r are constant and can be evaluated using finite element analysis. (FEA) by the ANSYS workbench which is very convenient to analysis a solid model like PKM.

The k_s varies with the configuration and should be evaluated as in the local frame since the spherical actuation is parallel to *w*, the coefficient stuffiness is calculated as follows

$$\frac{1}{k_s} = \frac{1}{k_{w1}} + \frac{1}{k_{w2}} + \frac{1}{k_{w3}} \tag{33}$$

The values of Eq (12) can be substituted from Table 1 below.

Table 2 The stiffness coefficient of the S joint

k_{u1}	k_{v1}	k_{w1}	k_{u2}	k_{v2}	k_{w2}	k_{u3}	k_{v3}	k_{w3}
31.4	33.3	436	4000	1810	2174	2119	270	588

k_{lsa} Is the lead screw assembly the combination of serially connected springs such as

$$\frac{1}{k_{lsa}} = \frac{1}{k_{ls}} + \frac{1}{k_n} + \frac{1}{k_{sb}} \tag{34}$$

Where k_{ls} , k_n and k_{sb} are the stiffness coefficients of lead screw.nut and support bearings respectively. k_{ls} is the lead screw which is the linear function of the limb length and can be defined as

$$k_{ls} = \frac{AE(L_{1i} + L_{2i})}{L_{1i}L_{2i}} \tag{35}$$

Where A, E stands for cross sectional area of the lead screw and yang's modular respectively

L_{1i} and L_{2i} are the distance b/n the nut and the supporting bearing located at both ends.

To find the constrained coefficient of stiffness matrix we can find as the same fashion of finding the actuation coefficient matrix. Similarly $k_c = diag[k_{ci}]$ $(i = 1,2,3)$ where k_{ci} is the bending stiffness coefficient at the platform along the S_{2i} axis of the i^{th} limb Then k_{ci} can be evaluated by taking reciprocal sum of the bending stiffness coefficient of the fixed length limb, S joint and R joint assembly respectively.

$$\frac{1}{k_{ci}} = \frac{1}{k_{cl}} + \frac{1}{k_{cs}} + \frac{1}{k_{cr}} \tag{36}$$

Again the k_{cs} can be evaluated by the configuration and should be evaluated as in the local frame since the spherical parallel to the constraint is to *u* the coefficient stuffiness is calculated as follows

$$\frac{1}{k_{cs}} = \frac{1}{k_{u1}} + \frac{1}{k_{u2}} + \frac{1}{k_{u3}} \tag{37}$$

The value of the stiffness will be substitute from table 4.1and easy to compute the value of k_{cs} .the k_{cl} and k_{cl} can be computed easy by FEA. Using the software ANSYS workbench by applying a 1KN force on the spherical joint which is parallel to the S_{2i}

B) **Formulation of overall stiffness matrix on tool tip**

Formulation of overall stiffness matrix applied on the center of end-effectors is calculated on Eq (36) which is

$$\tau = k\Delta x, \qquad k = J^T k_{ac} J$$

To find the overall coefficient of stuffiness matrix on tool tip it needs to transform the Eq(31) in to tool tip by the transformation matrix

$$T_c = \begin{bmatrix} E_3 & C[s_{1i}x] \\ 0_{3x3} & E_3 \end{bmatrix} \tag{38}$$

If let τ be imposed at the tool tip C and Δx be the corresponding small deflection twist the overall stiffness matrix about point C can easily be developed by replacing J in Eq.(31) with JT_C that is

$$K' = T_C{}^T J^T kJT_C$$

Where C is the distance from O' to C, $[s_{1i}]$ denotes the screw matrix of s_{1i} ,and E_3 denotes a unit matrix of order3.

$$c' = K'^{-1}$$

$$\Delta x' = c'\tau'$$

Where $\tau' = (F_x F_y F_z \frac{M}{r_C})^T$

$$\Delta x' = (\Delta u \Delta v \Delta w r_C \Delta\alpha_w)^T$$

r_C is the maximum radius of the cutting tool specified by the spindle head.

$$C' = K'^{-1}$$

$$K' = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{16}/r_c \\ & k_{22} & k_{23} & k_{26}/r_C \\ & & k_{33} & k_{36}/r_C \\ sym & & & k_{66}/r_C \end{bmatrix} \quad (39)$$

In order to evaluate the rigidity of a system we define the rigidity along/about three orthogonal axis of the frame $C-uvw$ by the diagonal corresponding element of C.

$$K_x = 1/C'(1.1) \quad K_y = 1/C'(2.,2) \quad K_z = 1/C'(3,3)$$

$$K_{rw} = r_C{}^2 / C'(6,6)$$

$$K_x = 1/C'(1.1)$$

$$K_y = 1/C'(2.,2) \quad K_z = 1/C'(3,3)$$

$$K_{rw} = r_C{}^2 / C'(6,6)$$

VII. STIFFNESS ANALYSIS

The stiffness of the 3PRS PKM is evaluated in the decomposing the machine in to limbs and apply a force on the spherical joint to find the actuated and constraint coefficient of stuffiness. With the aid of finite element analysis and numerical evaluated both actuated and constraint stiffness of the limb assembly is evaluated. The overall stiffness of the manipulator on the center of the plate form will be calculated as in Eq (31) indicated. Since in real sense the force/moment is applied on the too tip of the machine, it needs transform the stiffness matrix gained in Eq (31) to the tool tip by the transform matrix Eq (37) it gives a stiffness coefficient matrix on tool tip as shown in the Eq(39).then diagonal values in the stiffness matrix indicates the overall stiffness of the machine when force applied along *x*, *y* and *z* which explain in detail. In order to evaluate the rigidity of a system we define the rigidity along/about three orthogonal axis of the frame $C-uvw$ by the diagonal corresponding element of C.

$$K_x = 1/C'(1.1)$$

$$K_y = 1/C'(2.,2) \quad K_z = 1/C'(3,3)$$

$$K_{rw} = r_C{}^2 / C'(6,6)$$

5.2 Comparison with FEA Results

According to the above analysis, the detailed design was carried out and the stiffness of the virtual prototype was

	K_x $(N/\mu m)$	K_y $(N/\mu m)$	K_z $(N/\mu m)$	K_{rw} $(x10^6 Nm/ra$
Ana lytic	25.4	25.4	447.2	5.3
FE A	25.87	25.87	437.7	5.4

evaluated by ANSYS at four typical positions as shown from

Figure 3, Figure 4, Figure 5 and Figure 6 with 1KN force is applied at the tool tip along x, y and z and moment about z axis respectively. I can get the deformation easily from the FEA and the stiffness can get by $\frac{1000}{\Delta x}$. It can been seen from Table 3 that the estimated results of the mathematical models developed have a good match with those obtained by the FEA in terms of magnitude and distribution as well.

Table 4 Results obtained by the semi-analytical method and by FEA

The estimated linear stiffness along three orthogonal axis and the torsion stiffness about the *w*-axis of the $C-uvw$ frame, it can be seen that the stiffness distribution are tri-symmetrical in nature and the K_x and K_y are similar in magnitude.

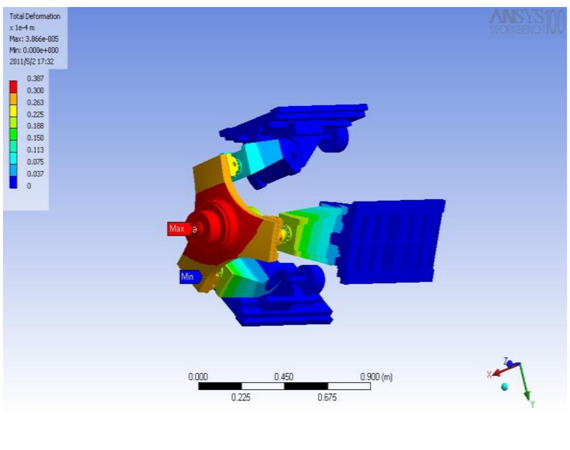

Figure 3 Deformation with 1KN force imposed at the spindle along x-axis

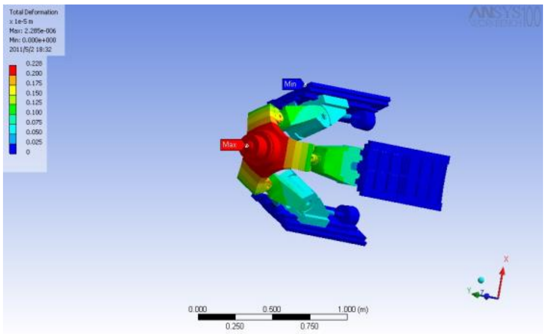

Figure 4 Deformation with 1KN force imposed at the spindle along z-axis

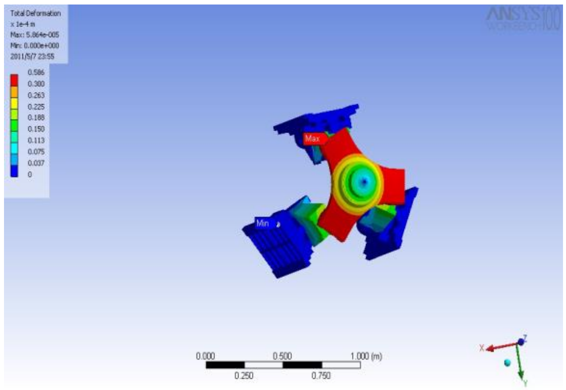

Figure 5 Deformation with 1KN force imposed at the spindle moment about z

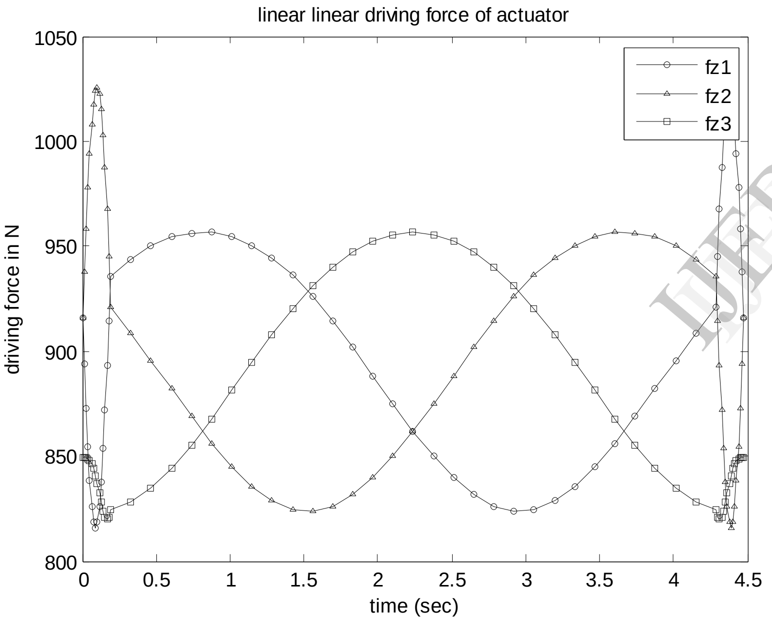

Figure 6. Linear actuating force

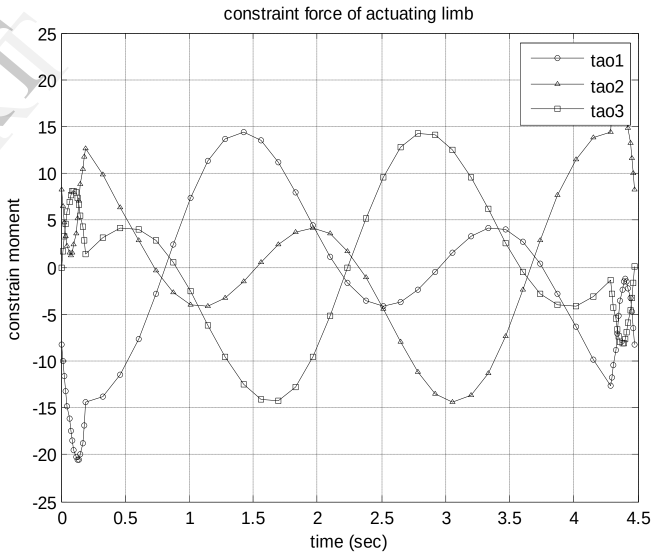

Figure 7. Constraint force of active limb

VIII. CONCLUSION

An approach for the dynamics and stiffness modeling 3PRS parallel mchanism is presented in this paper. With this approach,the analytical expressions of the mass centre velocity/acceleration of the link can be easily derived by using the operation of **Lie Algebra** and **reciprocal screw**. With the aid of the generalized Jaccobian and Hessian matrix, the new dynamics model can be applied to solve the generalized forces of actuation and constraint at the same time. The stiffness model of the lower mobility parallel manipulators can be formulated by two steps：

(1) formulation of the generalized Jacobian by simultaneously taking into account the deflections of the platform in both the free and fixed directions, and

(2) evaluation of the component stiffness matrices using the analytical approach, finite element analysis and/or their combinations, depending upon geometry complicity of the moving components. In concluding the robust dynamic equation is formulated and this is tested by developing simulation material and verify the result with numerical simulation shown in figure 7 and 8. Regarding the rigidity

analysis with the tool of ANSYS workbench we analysed the machine stiffness and compared the FEA result with the semi-analytical analysis and the estimated stiffness results have a good match with those obtained by the FEA. The result is in range of acceptable error as we can see it from fig.4,5 and 6 of ANSYS output compared with the FEA result in table 1.

REFERENCES

[1]. Zhang, F.; Zhang, D.; Yang, J., and Li, B., 2005, "Kinematics and Singularity Analysis of a 3-DOF Parallel Kinematic Machine.", IEEE. 29 July-1 Aug. 2005.

[2]. Joshi, S., Liu, H., Chetwynd, D.G.,Li,Z., "generalized jacobian analysis of lower mobility analysis of lower mobility manipulators," *IEEE transaction on robotics and automation, submitted* .

[3]. Dressler, I., Robertson. and Johansson, R.,2007, "Accuracy of kinematic and dynamic models of a Gantry-Tau parallel kinematic robot," *Proceedings of IEEE International Conference on Robotics and Automation*, pp.883- 888

[4]. Tsai, L.-W., 2000, "Solving the inverse dynamics of a Stewart-Gough manipulator by the principle of virtual work," *ASME Journal of Mechanical Design*, 122(3): pp. 3-9.[5] Li，Y.M. ，Xu，Q.S., 2005,

[5]. "Kinematic Analysis and Dynamic Control of a 3-PUU Parallel Manipulator for Cardiopulmonary Resuscitation" *International Conference on Advanced Robotics*, pp.344-351

[6]. Wisama Khalil and Sylvain Guegan, 2004, "Inverse and Direct Dynamic Modeling of Gough–Stewart Robots," *IEEE Transactions on Robotics*, 20(4):pp.754-762.

[7]. SUN Tao1, SONG YiMin1†, LI YongGang2 & LIU LinShan1 **"**Dimensional synthesis of a 3-DOF parallel manipulator based on dimensionally homogeneous Jacobian matrix"

[8]. Li Y G, Song Y M, Feng Z Y, et al. "Inverse dynamics of 3-PRS parallel mechanism by Newton-Euler formulation (in Chinese)". *Acta Aeron Astron Sin, 2007, 28:*

[9]. Carretero J A, Nahon M, Gosselin C M, et al. Kinematic analysis of a three-DOF parallel mechanism for telescope application. In: Proceedings of the 2007 ASME Design Automation Conference, Sacramento, California, 1997

[10]. Lee, K. M., Shah, D. K., 1988, "Dynamic analysis of a three-degrees-of-freedom in-parallel actuated manipulator," *IEEE Transactions on Robotics and Automation*, 1988, 4(3): 361-367

[11]. .Liu, H., Huang, T., Chetwynd, D. G., "An Approach for Acceleration Analysis of Lower Mobility Parallel Manipulators," *Proceedings of the ASME 2010 International Design Engineering Technical Conference & Computers and Information in Engineering Conference*, DETC2010-28020.

[12]. Tsai. Lung-Wen, "Robot analysis: the mechanics of serial and parallel manipulators," *J.WILEY & SONS, INC., 1999*

[13]. Rico, J. M., Duffy, J., 1996, "An application of screw algebra to the acceleration analysis of serial chains," *Mechanism and Machine Theory*, **31**(4): pp. 445-457.

[14]. Huang, Z., Zhao, Y. S., Zhao, T. S., 2006, "The Advanced Spatial Mechanism," Beijing: The High Education Press.

[15]. Meng-Shiun Tsai [a],*, Ting-Nung Shiau [a], Yi-Jeng Tsai [a], Tsann-Huei Chang, "Direct kinematic analysis of a 3-PRS parallel mechanism," *Mechanism and Machine Theory 38 (2003) 71–83*

[16]. G. Pond, J. Carretero, "Kinematic Analysis and Workspace Determination of the Inclined PRS Parallel Manipulator" proceeding of the 2004 ROMASY, Montreal, Quebec, Canada.

[17]. Zhu, S. J., Huang, Z., Guo, X. J., 2005, "Forward/reverse velocity and acceleration analyses for a class of lower-mobility parallel mechanisms," *Proceedings ASME Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, pp. 949-955.

[18]. Sameer A. Joshi, Lung-Wen Tsai*, "Jacobian Analysis of Limited-DOF Parallel manipulators"

[19]. Y.G. Li , H.T. Liu, X.M. Zhao , T. Huang ,*, Derek G. Chetwynd, "Design of a 3-DOF PKM module for large structural component machining" *Mechanism and Machine Theory 45 (2010) 941–954*

[20]. B.Dasgupta, P.Choudhury, "A general strategy based on Newton-Euler approach for the dynamic formulation of parallel manipulator"

[21]. Yangmin Li*, Qingsong Xu, "Kinematic analysis of a 3PRS parallel manipulator,"*robotics and computer-integrated manufacturing 23(2007)395-408*

[22]. Clavel R.DELTA, "a fast robot with parallel geometry" *proceeding of 18th international symposium on industrial robot, Lausanne; 1988.p.91- 100*

[23]. Lee KM, Arjunan S. "A three degree of freedom micromotion in parallel actuated manipulator" *IEEE Trans robot automat 1991 634-641*

[24]. Gosselin, C. M., "Stiffness mapping for parallel manipulators," IEEE Transactions on Robotics and Automation, vol. 6, no. 3, pp. 377–382, 1990.

[25]. Jian-Li "Design of 3-DOF Parallel Manipulators for Micro-Motion Applications" master's thesis in University of Ontario Institute of Technology 2010

[26]. Ting-Nung Shiau and Meng-Shiun Tsai "Research on Kinematic and Dynamic Characteristics of a 3-PRS Parallel Mechanism" Dissertation of Doctor of Philosophy in National Chung Cheng University June 2007

[27]. Wang, J.; Gosselin, C.M., 2004, "Kinematic Analysis and Design of Kinematically Redundant Parallel Mechanisms," ASME J. Mech. Des., 126(1), pp.109-118.

[28]. Mohamed, M.G; Gosselin, C.M., 2005, "Design and Analysis of Kinematically Redundant Parallel Manipulators with Configurable Platforms" IEEE Trans. Rob.,21(3), pp. 277-287.104

[29]. Zhang, D. and Wang, L., 2005, "Conceptual Development of an Enhanced Tripod Mechanism for Machine Tool", *Robotics and Computer Integrated Manufacturing* 21, no. 4-5 pp. 318-27.

[30]. Bi, Z. M. and S. Y. T. Lang. "Kinematic and Dynamic Models of a Tripod System with a Passive Leg." IEEE/ASME Transactions on Mechatronics 11, no. 1 (02, 2006): 108-111.

[31]. Chablat, D. and P. Wenger. "Architecture Optimization of a 3-DOF TranslationalParallel Mechanism for Machining Applications, the Orthoglide." *IEEE Transactions on Robotics and Automation* 19, no. 3 (06, 2003): 403-410

[32]. V.E. Gough and S.G White hall, 1962,"Univeral Tyre Test Machine" proceeding of 9th international congress, F.I.S.I.T.A.pp.117-137.

[33]. D. Stewart 1965,"Aplatform with six degree of freedom " in proceeding of institute of mechanical engineering, vol,180 No.5,pp371-386

[34]. K.Hunt, 1978, "kinematic geometry",clarendo press, oxford

[35]. Weck M., Staimer D., "Parallel kinematic machine tools---current states and future potentials," Ann. CIRP, vol. 51, no. 2, pp 617-683, 2002.

[36]. Neumann, K. E., 1988, Robot, US Patent 4732525.

[37]. Neumann, K. E., System and Method for Controlling a Robot, US Patent 6301525, 2001.

[38]. Tonshoff, H.K., Grendel, H. Kaak, R., "Structure and Characteristics of the Hybrid Manipulator George V," in: Boer, C.R. , Molinari-Tosatti, L. and Smith, K.S. (Eds.), Parallel Kinematic Machines, Springer-Verlag, London, pp. 365–376, 1999.

[39]. Huang, T., Li, M., and Li, Z. X., "A 5-DOF hybrid robot", Patent Cooperation Treaty (PCT), Int. Appl. PCT/CN2004/000479, 2004.

[40]. Huang, T., Li, M., Zhao, X. M., Mei, J. P., Chetwynd, D. G., Hu, S. J., 2005, Conceptual Design and Dimensional Synthesis for a 3-DOF Module of the TriVariant—a Novel 5-DOF Reconfigurable Hybrid Robot, *IEEE Trans. on Rob.*, 21(3), pp. 449-45

[41]. Gosselin, C. M., "Stiffness mapping for parallel manipulators," IEEE Transactions on Robotics and Automation, vol. 6, no. 3, pp. 377–382, 1990.