# Dynamic Analysis Of Piezoelectric Transducers Using Finite Element Method

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**Abstract**

*Piezoelectric transducers have been extensively applied over the last decade to diverse areas such as band-pass filters and high–energy ultrasonic devices. They are typically utilized for generating an acoustic pulse proportional to the input external loading and detecting the same and converting it in to electric signal. Other application of piezoelectric are in loudspeaker, microphone, phonograph, wrist watches, oscillator, resonator, transducer etc. Piezoelectric crystals are widely used in aerospace industry, automobile industry and geological instruments as sensors and actuators. In the current engineering world automation is an integral part of the machines, processes, automobiles etc. In automation we need sensors and actuators. Piezoelectric material is widely used for the same purpose. Various type of analysis can be done by Finite Element Modelling of Piezoelectric Transducer e.g. Calculation of Eigen values, mode shapes, electrical output on application of strain and vice versa.*

## 1. Introduction

The word *piezoelectricity* literally means 'pressure electricity'; the prefix *piezo* is derived from the Greek word *piezein* which means "to press". Piezoelectricity is the property of a crystal by which electric polarization is produced by mechanical strain in crystal and conversely production of strain on application of voltage. The electric polarization produced is proportional to the applied strain and changing direction with it. Piezoelectricity is different from electrostriction, another effect which causes a solid dielectric to change shape on application of a voltage. In piezoelectricity, a reversal of voltage reverses the sign of the resulting strain whereas for electrostriction the strain is an even function of the applied voltage.

Piezoelectric transducers have been extensively applied over the last decade to diverse area such as band-pass filters and high–energy ultrasonic devices. They are typically utilized for generating an acoustic pulse proportional to the input external loading and detecting the same and converting it in to electric signal. Different other application of piezoelectric are in loudspeaker, microphone, phonograph, wrist watches, oscillator, producing very selective filter circuit, resonator, transducer etc. Piezoelectric crystals are widely used in aerospace industry, automobile industry and geological instruments as sensors and actuators.

A lot of work has been done in the field of finite element modelling of piezoelectric transducer on different type of uses. *A.V. Belokon et al* [1] proposed new scheme which develops the technique for the finite element analysis to take account of attenuation in piezoelectric media. *K.Y.Sze, Y.S.pan* [4] employed hybrid variational principles for finite element formulation of piezoelectric transducer. *F.Cote et al* [6] validates both theoretically and experimentally the implementation of a multilayered three-dimensional model based on the analogy between thermal strains and piezoelectric strains.

Application of piezoelectric crystals can be divided in to two categories e.g. sensors and actuators. In the following work, main emphasis is given to the analysis of piezoelectric crystals as sensors. An analysis of transducers have also been given for dynamic loading, which includes the analysis of simple cantilever beam made of piezoelectric crystal, analysis of shell made of piezoelectric material with ends of brass, analysis of cantilever beam with piezoelectric strip as transducer, and analysis of proving ring with piezoelectric strips as sensors. The above analysis has been done for dynamic loading considering the sinusoidal load as dynamic load. Other than the above work, a separate work is presented to understand the dependency of piezoelectric response on working temperature.

## 2. Theoretical concept about Piezoelectric Properties

"Piezoelectricity is electric polarization produced by mechanical strain in crystals belonging to certain classes, the polarization being proportional to the strain and changing direction with it." The production of an electric polarization by mechanically inducing a strain in a crystal is called the direct piezoelectric effect. The converse effect, whereby a mechanical strain is produced in a crystal by a polarizing electric field, also exists.

All the crystalline materials are anisotropic and do not have the same properties in all the directions as do the isotropic material. Crystals can be divided into 32 classes on the basis of symmetry they possess and, of these 32 classes , only 20 possess the property of piezoelectricity and 12 do not because these 12 classes of crystals have the centre of symmetry where as a crystal possessing the centre of symmetry can not be piezoelectric because no combination of uniform stresses will produce a separation of the centres of gravity of +ve and –ve charges and produce an induced dipole moment which is necessary for the polarization by stresses. Most unsymmetrical type of system is the triclinic system, all three of whose crystallographic axis makes oblique angles, and the length of the unit cells on the three-axis is unequal. Such a material will have 21 elastic constants, 18 piezoelectric constants, 6 dielectric constants. A most symmetric crystal (cubic crystal) has 3 elastic constants, 1 piezoelectric and 1 dielectric constants. Thus on the basis of symmetry the piezoelectric crystals are classified in different classes.

According to the linear theory of piezoelectricity, the constitutive equations that couples the deformation and electric field in the piezoelectric plate are given by-

$$\tau_{ij} = c_{ijkl} S_{kl} - e_{kij} E_k$$

$$D_i = e_{ikl} S_{kl} + \varepsilon_{ik} E_k$$

Where- $c_{ijkl}$ = stiffness coefficient
$e_{kij}$ = piezoelectric constants
$\varepsilon_{ik}$ = dielectric constants

In order to determine the solution of piezoelectric vibration problem, we will have to know array of material coefficients for the particular symmetry of the material. Here we are considering the compressed matrix notation that turns out to be more useful then the extended tensor notation. When discussing symmetry. The matrix notation consist of replacing *ij* and *kl* by *p* or *q* where *i,j,k* and *l* take the values *1,2* and *3*, and *p,q* take the values *1,2,3,4,5* and *6.*

So $$c_{ijkl} = c_{pq}$$
$$e_{ikl} = e_{ip}$$
$$\tau_{ij} \equiv T_{ij} = T_p \equiv \tau_p$$

.

## 3. Piezoelectric Analysis

The piezoelectric effect is the coupling of stress and electrical field in a material: an electrical field causes the material to strain, and vice versa. ANSYS/Standard has the capability to perform fully coupled piezoelectric analysis. The elements that are used in this case contain both displacement degrees of freedom and the electric potential as nodal variables.

### 3.1 Equilibrium and Flux Conservation

The piezoelectric effect is governed by coupled mechanical equilibrium and electric flux conservation equations.

The mechanical equilibrium equation is

$$\int_V \sigma : \delta\varepsilon dV = \int_S t.\delta u dS + \int_V f.\delta u dV$$

where

$\boldsymbol{\sigma}$ is the "true" (Cauchy) stress at a point currently at **x**;

**t** is the traction across a point of the surface of the body;

***f*** is the body force per unit volume in the body

**δu** is an arbitrary, continuous vector field (the virtual velocity field).

The electrical flux conservation equation is

$$\int_V q.\delta E dV = \int_S q_S.\delta\varphi dS + \int_V q_V \delta\varphi dV$$

where **q** is the electric flux vector;

***qs*** is the electric flux per unit area entering the body at a point on its surface;

***qV*** is the electric flux entering the body per unit volume;

$\delta\varphi$ is an arbitrary, continuous, scalar field (the virtual potential).

The electric flux vector q is known as the electrical displacement, and the potential gradient E is known as the electrical field.

Constitutive Behavior: Material Coupling

Currently the assumption of linear materials is utilized. The basic equations for a piezoelectric linear medium are defined in the following. Mainly three alternative forms of the constitutive equations are presented as *e*-form, *d*-form, and *g*-form.

In ANSYS the constitutive equations in the *e*-form are used:

$$\sigma_{ij} = D^{E}_{ijkl} \varepsilon_{kl} - e^{\varphi}_{mkl} E_m$$

and

$$qi = e^{\varphi}_{ijl} \varepsilon_{jk} + D^{\varphi(\varepsilon)}_{ij} E_j$$

These are expressed in terms of the piezoelectric stress coefficient matrix $e^{\varphi}_{mkl}$. However, ANSYS also allows the input of piezoelectric constants in terms of the piezoelectric strain coefficient matrix $d^{\varphi}_{mkl}$.

The constitutive equations in the $g$-form can also be expressed as

$$\sigma_{ij} = D^{q}_{ijkl}\varepsilon_{kl} - D^{q}_{ijkl} g^{\varphi}_{mkl} q_m$$

and

$$q_i = D^{\varphi(\sigma)}_{im} g^{\varphi}_{mjk}\sigma_{jk} + D^{\varphi(\sigma)}_{ij} E_j$$

These equations can be convenient in interpreting and verifying the results of piezoelectric analyses.

**3.2Kinematics**

For the piezoelectric elements both displacements and electric potentials exist at the nodal locations. The displacements and electrical potentials are approximated within the element as

$$u = N^N u^N$$

and

$$\varphi = \mathbf{N}^N \varphi^N$$

where $\mathbf{N}^N$ is the array of interpolating functions and $\mathbf{u}^N$ and $\boldsymbol{\varphi}^N$ are nodal quantities. The body forces and charges as well as the surface forces and charges are interpolated in a similar manner.

The strains and electrical potential gradients are given as

$$\varepsilon = B^N_u u^N$$

and

$$E = -B^N_{\varphi}\varphi^N$$

where $B^N_u$ and $B^N_{\varphi}$ are the spatial derivatives of $\mathbf{N}^N$ .In geometrically nonlinear analyses these spatial derivatives are defined in the current configuration.

**3.3 System Equations**

With these approximate fields and the constitutive properties given above, in conjunction with the equilibrium and conservation equations, the following system of equations is derived in terms of nodal quantities:

$$M^{MN}\ddot{u}^N + K^{MN}_{uu} u^N + K^{MN}_{\varphi u}\varphi^N = P^M$$

and

$$K^{MN}_{\varphi u} u^N - K^{MN}_{\varphi\varphi}\varphi^N = -Q^M$$

where

$$M^{MN} = \int_V \rho N^M . N^N dV$$

is the mass matrix (no inertia terms exist for the electrical flux conservation equation),

$\rho$ is the mass density,

$K^{MN}_{uu} = \int_V B^M_u : D_m : B^N_u dV$ is the displacement stiffness matrix,

$K^{MN}_{\varphi\varphi} = \int_V B^M_{\varphi} : D\varphi : B^N_{\varphi} dV$ is the dielectric "stiffness" matrix,

$K^{MN}_{\varphi u} = \int_V B^M_{\varphi} . e : B^N_u dV$ is the piezoelectric coupling matrix,

$PM = \int_V N^M . P_v dV + \int_S N^M . P_s dS + P^M_c$ is the mechanical force vector,

and

$Q^M = \int_V N^M . Q_v dV + \int_S N^M . Q_s dS + Q^M_c$ is the electrical charge vector.

In these expressions the constitutive properties are specified in a matrix form where $\mathbf{D}_m$ is the mechanical relationship, $\mathbf{D}_{\varphi}$ is the electrical relationship, and e is the piezoelectrical relationship.

**3.4 Specifying Piezoelectric Material Properties**

A piezoelectric material responds to an electric potential gradient by straining, while stress causes an electric potential gradient in the material. This coupling between electric potential gradient and strain is the material's piezoelectric property. The material will also have a dielectric property so that an electrical charge exists when the material has a potential gradient. Piezoelectric material behavior is discussed in the mechanical properties of the material must be modeled by linear elasticity. The mechanical behavior can be defined by

$\sigma_{ij} = D^{E}_{ijkl}\varepsilon_{kl} - e^{\varphi}_{mij} E_m$ in terms of the piezoelectric stress coefficient matrix, $e^{\varphi}_{mij}$,

and,

$\sigma_{ij} = D^{E}_{ijkl}\left(\varepsilon_{kl} - d^{\varphi}_{mkl} E_m\right)$ in terms of the piezoelectric strain coefficient matrix, $d^{\varphi}_{mkl}$. The electrical behavior is defined by

$$q_i = e^{\varphi}_{ijk}\varepsilon_{jk} + D^{\varphi(\sigma)}_{ij} E_j$$

Where

$\sigma_{ij}$ is the mechanical stress tensor;

$\varepsilon_{ij}$ is the strain tensor;

$q_i$ is the electric "displacement" vector;

$D^{E}_{ijkl}$ is the material's elastic stiffness matrix defined at zero electrical potential gradient (short circuit condition);

$e^{\varphi}_{ijk}$ is the material's piezoelectric stress coefficient matrix,

$d^{\varphi}_{mkl}$ is the material's piezoelectric strain coefficient matrix.

$\varphi$ is the electrical potential;

$D^{\varphi(\sigma)}_{ij}$ is the material's dielectric property,

$E_i$ is the electrical potential gradient vector, $-\partial\varphi / \partial x_i$.

### 3.5 Electrical Output of Piezoelectric

In the analysis we are more interested in the electrical output. The electrical response of a piezoelectric material is assumed to be made up of piezoelectric and dielectric effects

$$qi = e^{\varphi}_{ijl}\varepsilon_{jk} + D^{\varphi(\varepsilon)}_{ij}E_j$$

where

φ- the electrical potential,

$qi$ - the component of the electric flux vector (also known as the electric displacement) in the *i*th material direction,

$e^{\varphi}_{ijk}$ - the piezoelectric stress coupling,

$\varepsilon_{ij}$ - a small-strain component,

$D^{\varphi}_{ij}$ - the material's dielectric matrix for a fully constrained material, and

$\boldsymbol{E_i}$ - the gradient of the electrical potential along the *i*th material direction, $-\partial\varphi/\partial x_i$ .

### 3.6 Specifying Piezoelectric Material Properties

The piezoelectric material properties can be defined by giving the stress coefficients, $e^{\varphi}_{mij}$ (this is the default), or by giving the strain coefficients, $d^{\varphi}_{mkl}$ . In either case, 18 components must be given in the following order (substitute *d* for *e* for strain coefficients):

$e^{\varphi}_{111}$ $e^{\varphi}_{122}$ $e^{\varphi}_{133}$ $e^{\varphi}_{112}$ $e^{\varphi}_{113}$ $e^{\varphi}_{123}$
$e^{\varphi}_{211}$ $e^{\varphi}_{222}$ $e^{\varphi}_{233}$ $e^{\varphi}_{212}$ $e^{\varphi}_{213}$ $e^{\varphi}_{223}$
$e^{\varphi}_{311}$ $e^{\varphi}_{322}$ $e^{\varphi}_{333}$ $e^{\varphi}_{312}$ $e^{\varphi}_{313}$ $e^{\varphi}_{323}$

The first index on these coefficients refers to the component of electric displacement (sometimes called the electric flux), while the last pair of indices refers to the component of mechanical stress or strain.

Thus, the piezoelectric components causing electrical displacement in the 1-direction are all given first, then those causing electrical displacement in the 2-direction, and then those causing electrical displacement in the 3-direction. (Some references list these coupling terms in a different order.)

The dielectric matrix can be isotropic, orthotropic, or fully anisotropic. For non-isotropic dielectric materials a local orientation for the material directions must be specified

The dielectric matrix $D^{\varphi(\sigma)}_{ij}$ can be fully isotropic, so that

$$D^{\varphi(\varepsilon)}_{ij} = D^{\varphi(\varepsilon)}\delta_{ij}$$

You specify the single value $D^{\varphi(\varepsilon)}$ for the dielectric constant. $D^{\varphi(\varepsilon)}$ must be determined for a constrained material. Isotropic behavior is the default.

For orthotropic behavior you must specify three values in the dielectric matrix( $e^{\varphi}_{11}$, $e^{\varphi}_{22}$ and $e^{\varphi}_{33}$). For fully anisotropic behavior you must specify six values in the dielectric matrix ($e^{\varphi}_{12}$, $e^{\varphi}_{22}$, $e^{\varphi}_{22}$, $e^{\varphi}_{13}$, $e^{\varphi}_{23}$ and $e^{\varphi}_{33}$).

## 4. Modelling and Analysis

**4.1** *Finite element modelling of cantilever beam made of piezoelectric material PZT- 4 and determination of eigenvalues and corresponding mode shapes.*

To start with the piezoelectric transducer, first take the simplest case i.e. a cantilever beam having piezoelectric material. To understand the nature of piezoelectric as a single material this condition has been taken. To start the analysis we divide the real structures in three parts , beam, plate and shell. Model of the first analysis has been prepared as shown in Fig 4.1.

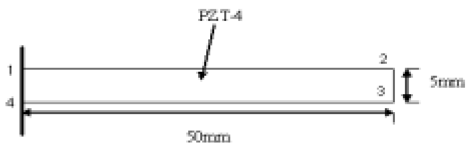



Fig 4.1. *Model of simple beam made of piezoelectric (PZT-4)*

In the above problem the material used is PZT-4. The main purpose of the analysis is to learn about the proper modeling of the piezoelectric material structures. Generally for a simple material we are having only 6 degree of freedoms but for the material like piezoelectric we have one more degree of freedom which is directly related to the electrical properties of the piezoelectric material. In stead of the eigen values and corresponding mode shapes , analysis of electric potential at points 1,2,3,4( fig4.1.1 ) is also done.

***Element Type***

CPE4E – A 4 node bilinear plain strain piezoelectric quadrilateral.

*.Material*

PZT -4

*Boundary condition*

Surface 1-4 is rigidly fixed i.e. U1= U2=UR3=0

*Total no of elements*- **306**

*Total no of nodes*- **419**

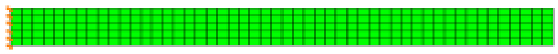


Fig 4.2 *Meshed Cantilever made of Piezoelectric PZT-4*

**Results and discussion**

Other then the eigen values and the mode shapes**, the** voltage output of the beam is also calculated at different points. This output is shown in table 4.1

***Electrical potential output***

| Frequency (cycle/time) | EPOT -1 | EPOT -2 | EPOT -3 | EPOT -4 |
|---|---|---|---|---|
| 1031.9 | -8.838 | -8.846 | -8.846 | -8.854 |
| 6259 | 5.056 | 5.003 | 5.003 | 4.949 |
| 16515 | -0.405 | -0.561 | -0.561 | -0.716 |
| 20726 | 35.7 | 38.68 | 38.68 | 35.7 |
| 29990 | -4.829 | -4.522 | -4.522 | -4.215 |
| 45594 | -22.48 | -22.98 | -22.98 | -23.48 |
| 62059 | 33.65 | 36.59 | 36.59 | 3.65 |

Table 4.1 *Electrical potential at point 1,2,3,4 with frequencies*.

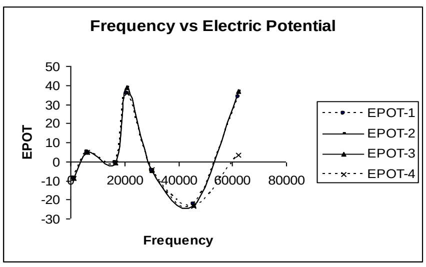



Fig 4.3 *Graphical representation of frequency vs. electric potential on the basis of data given inTable*4.1

**4.2 *Finite element modeling of cylindrical shell made of PZT-4 and having end cover of Brass and determination of eigenvalues and mode shapes***

The idea behind the solving such situation was to have a good understanding about the assembly of piezoelectric with the other materials. Also to understand the effect of the piezoelectric material while it is in the combination of other material.

The modeling of this type of shell is done by taking 2-D asymmetric element. The upper half of the diagram as shown in following figure is constructed. And then analysis is done

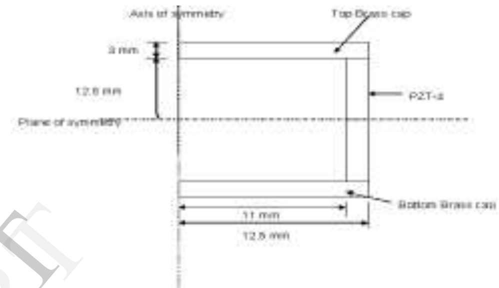



Fig- 4.4 *Model of piezoelectric shell with Brass end cap*

Considering the following;-

*Element type*

**Brass**

CAX4R: A 4 node bilinear axisymmetric quadrilateral

Total no of elements- 600

Total no of nodes- 656

**PZT-4**

CAX8RE: An 8 node biquadratic axisymmetric piezoelectric quadrilateral

Total no of elements- 600

Total no of nodes- 1941

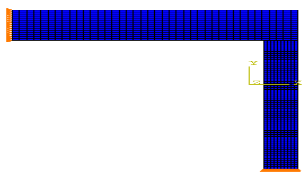


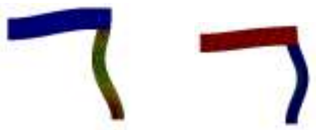
Fig: 4.5 *The mesh representing the shell element in axisymmetric Form*

**Boundary Condition**
At axis of symmetry no displacement in radial direction
At plane of symmetry no displacement in vertical direction
Result and discussion

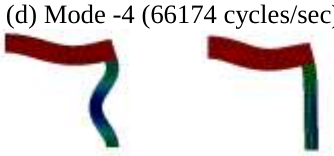
*Mode shapes*

(a)Mode-1(19347cycle/sec)

(b) Mode -2 (42564 cycles/sec)



(c) Mode -3 (60009 cycles/sec)
(d) Mode -4 (66174 cycles/sec)



(e)Mode -5 (90844 cycles/sec)

(f) Mode -6 (97955 cycles/sec)

Fig 4.6

*Mode shapes of the model for first 6 eigen values*

***Electric potential output***

In this problem the electrical potential is calculated at the plane of symmetry of the shell.

| Sr. N0. | Frequency (cycles/sec) | Electric potential |
|---|---|---|
| 1 | 19347 | 2.958 |
| 2 | 45564 | -7.174 |
| 3 | 60009 | 4.531 |
| 4 | 66174 | -2.606 |
| 5 | 90844 | -4.034 |
| 6 | 97955 | -9.35 |
| 7 | 118651 | -6.565 |

Table 4.2 *Electrical potential at plane of symmetry with frequencies.*

***4.3Finite element modelling of cantilever beam with piezoelectric transducer PZT-4 and determination of its response under sinusoidal loading***.

To solve this problem we consider a 2-D planer cantilever beam fixed at one end. In this cantilever beam we attach a piezoelectric element (PZT-4) as shown in figure with the TIE option The cantilever beam is kept at zero potential difference. A periodic load is applied on the right most corner with different frequencies.

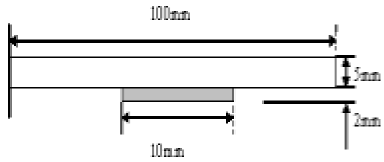



Fig 4.7 *Model of beam with piezoelectric element*.

***Material***
Piezoelectric Element - PZT-4
***Element***
**Beam**
CPS4R – A 4 node bilinear plane stress quadrilateral.
Total no of elements- 322
Total no of nodes – 504
**Transducer**
CPS4E - A 4 node bilinear plane stress piezoelectric quadrilateral.
Total no of elements- 80
Total no of nodes-105
**Boundary condition**
Rigidly fixed i.e. U1= U2=UR3=0

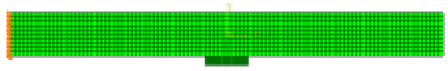


Fig: 4.8 Meshed structures *of Beam and Transducer*

## Result and Discussion

Following is the table of the electrical output with respect to the natural frequency of the system .

| Frequency | EPOT | Frequency | EPOT |
|---|---|---|---|
| 671.1 | 19.01 | 6040 | 22.36 |
| 1342 | 69.18 | 6711 | 7.918 |
| 2013 | 17.22 | 7383 | 9.147 |
| 2685 | 18.74 | 8054 | 1.076 |
| 3356 | 20.73 | 8725 | 0.432 |
| 4698 | 76.94 | 9396 | 10.22 |
| 5369 | 3.262 | | |

Table 4.3. *Frequency Vs Electric Potential in Piezoelectric Element*.

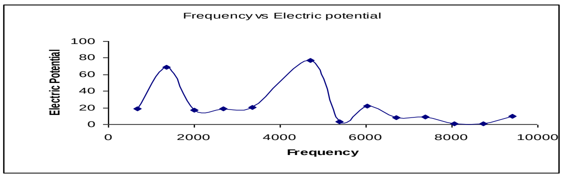



Fig: 4.9 *Frequency Vs output voltage*

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**4.4** *Finite element modelling of proving ring with piezoelectric layers as a sensor and dynamic analysis under sinusoidal loading with the variation in frequency with constant load and with the variation of load under constant frequency*.

Proving ring is a device which is used to calibrate the instruments which apply the static loading eg. Universal Testing Machine etc. The following model is presented with an expectation that the same ring can be used to calibrate the instruments involving the dynamic loading if we use the piezoelectric strips as a sensor.

The modeling of proning ring has been done as follows. It contains five parts, one is the ring made of stainless steel and rest of the four parts are made of piezoelectric material PZT-4. The steel ring is the main body and the rest of the four parts are made as integral part of the main ring with TIE option. The construction is as thefollowing diagram.

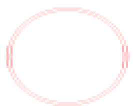


Fig 4.10 *Model of Proving Ring with Piezoelectric Strip*

Now each part will be described individually. The first part is the ring made of stainless steel as shown in the following figure. The steel ring is modeled as followed-

Inner Diameter – 175mm
Outer Diameter – 185mm
No. of elements – 70
No. of nodes - 350
Element type – CPS8R

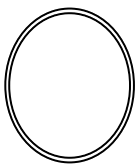


Fig 4.11 *Ring made of Stainless steel*

Other four part are made of piezoelectric material PZT-4.All the four strips are having thickness of 1mm. All the strips are modeled as follows

No. of elements – 40
No. of nodes - 203
Element type – CPS8RE

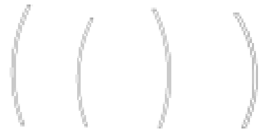


Fig 4.12 *Piezoelectric Strips*

### 4.4.1 Static Analysis

After the modeling of the transducer , the first part of the analysis is static analysis. In the part of static analysis , the analysis is done by applying the static load on the top point in the downward direction. The range of load applied is 20N to 500 N. The result of the static analysis is shown in the Table 4.5



| S. No. | Load | Max. Strain | Plastic Strain |
|---|---|---|---|
| 1 | 20 | $6.937 \times 10^{-12}$ | 0 |
| 2 | 40 | $1.137 \times 10^{-11}$ | 0 |
| 3 | 60 | $2.081 \times 10^{-11}$ | 0 |
| 4 | 80 | $2.775 \times 10^{-11}$ | 0 |
| 5 | 100 | $3.469 \times 10^{-11}$ | 0 |
| 6 | 150 | $5.203 \times 10^{-11}$ | 0 |
| 7 | 200 | $6.937 \times 10^{-11}$ | 0 |
| 8 | 250 | $8.672 \times 10^{-11}$ | 0 |
| 9 | 300 | $1.041 \times 10^{-10}$ | 0 |
| 10 | 350 | $1.214 \times 10^{-10}$ | 0 |
| 11 | 400 | $1.387 \times 10^{-10}$ | 0 |
| 12 | 450 | $1.561 \times 10^{-10}$ | 0 |
| 13 | 500 | $1.734 \times 10^{-10}$ | 0 |

Table 4.5 *Result of Static Analysis*

After going through the result of the above table, we see that the proving ring is safe for a load of 500N . Though the dynamic analysis is done for a small loads but static analysis shows that the ring is safe for the load of 500N. It may be safe for the higher load also but as per our current requirement the testing up to 500N is enough. Also we see that there is no plastic strain developed in the ring so our working limit is the elastic region only. So the conclusion of the static analysis is that we can use this ring for dynamic analysis to see the effect of various type of loading.

### 4.4.2 Dynamic Analysis

Before going to the dynamic analysis of the proving ring with piezoelectric layers , the natural frequencies of the model has been calculated by using the step ***frequency*** and then ***model dynamics***. After the calculation of natural frequencies of the system , the analysis has been divided in two parts.

a) The analysis under the variation of load keeping the frequency constant.
b) The analysis under the constant load with the variation of frequency.

After the analysis the result are collected in graphical and numerical form and presented. Accordingly this analysis gives us the idea that a proving ring can also be used in calibration of equipments which apply dynamic loading.

**a) The analysis under the variation of load keeping the frequency constant.**

In this part of the analysis the analysis is done by keeping the frequency of loading constant at 100 cycles /second. The analysis time is 0.1 second. The output was calculated in the form of Electric flux at the outermost and inner most surfaces of the piezoelectric strips. The load is varied from 5N to 65 N in various steps. Following are the graphical results of the output with respect to the time.

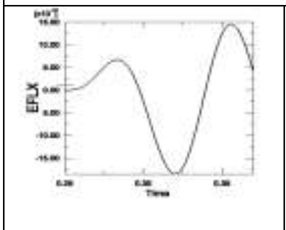
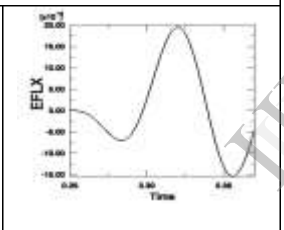
| Load 5N | Frequency 100 cycles/sec |
|---|---|
|  |  |
| **Inner Layer** | **Outer Layer** |

Fig 4.13 *Electric Flux Generated with time for ( 5N, 100 cycles/sec)*

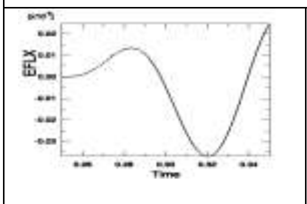
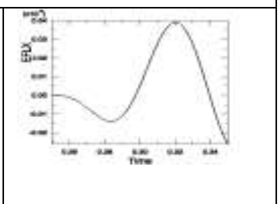
| Load 10N | Frequency 100 cycles/sec |
|---|---|
|  |  |
| **Inner Layer** | **Outer Layer** |

Fig:4.14 *Electric Flux Generated with time for ( 10N, 100 cycles/sec)*

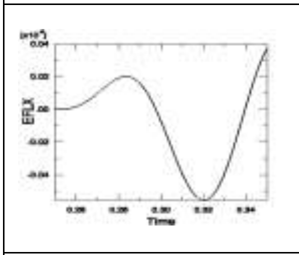
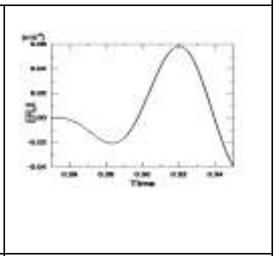
| Load 15N | Frequency 100 cycles/sec |
|---|---|
|  |  |
| **Inner Layer** | **Outer Layer** |

Fig 4.15 *Electric Flux Generated with time for ( 15N, 100 cycles/sec)*

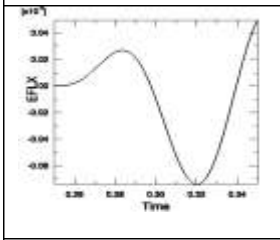
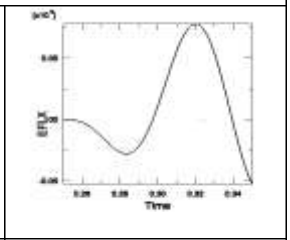
| Load 20N | Frequency 100 cycles/sec |
|---|---|
|  |  |
| **Inner Layer** | **Outer Layer** |

Fig 4.16 *Electric Flux Generated with time for ( 20N, 100 cycles/sec)*

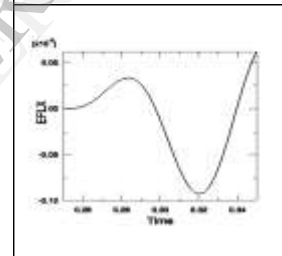
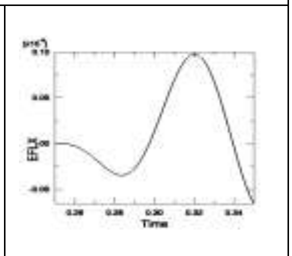
| Load 25N | Frequency 100 cycles/sec |
|---|---|
|  |  |
| **Inner Layer** | **Outer Layer** |

Fig 4.17 *Electric Flux Generated with time for ( 25N, 100 cycles/sec)*

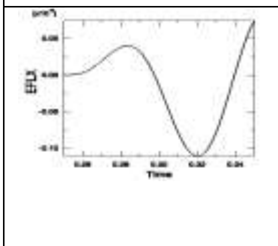
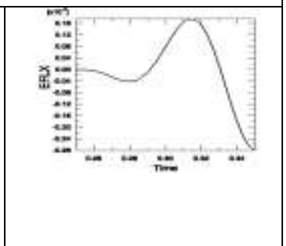
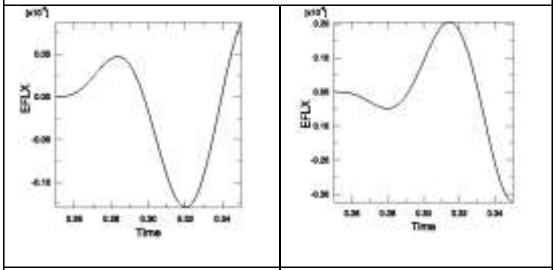
| Load 30N | Frequency 100 cycles/sec |
|---|---|
|  |  |
| **Inner Layer** | **Outer Layer** |

Fig 4.18 *Electric Flux Generated with time for ( 30N, 100 cycles/sec)*

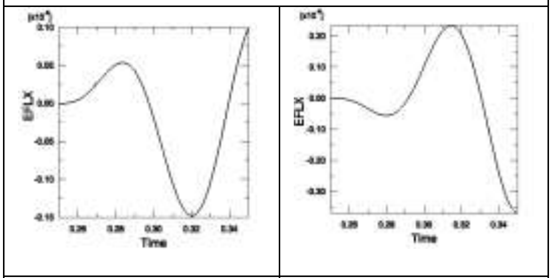
**Load 35N Frequency 100 cycles/sec**



| **Inner Layer** | **Outer Layer** |
|---|---|

Fig 4.19 *Electric Flux Generated with time for ( 35N, 100 cycles/sec)*

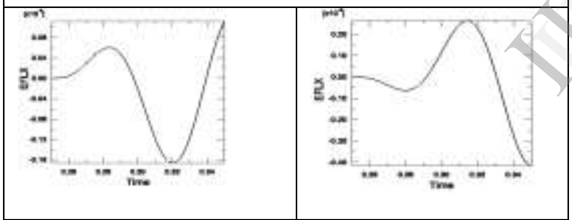
**Load 40N Frequency 100 cycles/sec**



| **Inner Layer** | **Outer Layer** |
|---|---|

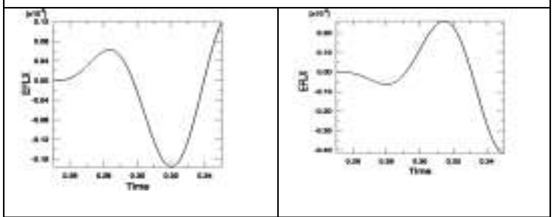
Fig 4.20 *Electric Flux Generated with time for ( 40N, 100 cycles/sec)*

**Load 45N Frequency 100 cycles/sec**



| **Inner Layer** | **Outer Layer** |
|---|---|

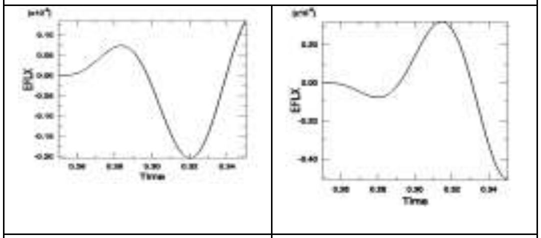
**Load 50N Frequency 100 cycles/sec**



| **Inner Layer** | **Outer Layer** |
|---|---|

Fig 4.21 *Electric Flux Generated with time for 45N, 100 cycles/sec, 50N, 100cycles/sec.*

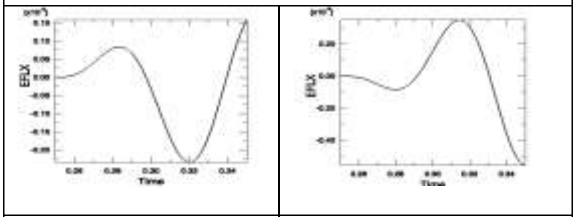
**Load 55N Frequency 100 cycles/sec**



| **Inner Layer** | **Outer Layer** |
|---|---|

Fig 4.22 *Electric Flux Generated with time for ( 55N, 100 cycles/sec)*

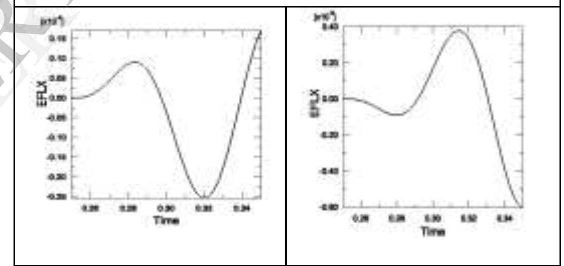
**Load 60N Frequency 100 cycles/sec**



| **Inner Layer** | **Outer Layer** |
|---|---|

Fig 4.23 *Electric Flux Generated with time for ( 60N, 100 cycles/sec)*

**Load 65N Frequency 100 cycles/sec**



| **Inner Layer** | **Outer Layer** |
|---|---|

Fig 4.24 *Electric Flux Generated with time for ( 65N, 100 cycles/sec)*

After seeing the above result , and analyzing minutely, we find that the output we are getting is in the form of sinusoidal curves, we also find that on increasing the load the output flux value is increasing but the response is almost the same wave form . It implies that on increasing the load the out put flux value is affected not the type of out put. The numerical data related to electric flux range with the variation in loading is given in the Table 4.6

| S. No. | Load (N) | Frequency (cycles/second) | Range of Electric Flux | |
|---|---|---|---|---|
| | | | Inner Layer | Outer Layer |
| 1 | 5 | 100 | -15 x $10^{-12}$ --- 15 x $10^{-12}$ | -15 x $10^{-12}$ --- 20 x $10^{-12}$ |
| 2 | 10 | 100 | -0.04x $10^{-9}$ --- 0.02 x$10^{-9}$ | -0.02x $10^{-9}$ --- 0.04 x$10^{-9}$ |
| 3 | 15 | 100 | -0.05x $10^{-9}$ --- 0.04 x$10^{-9}$ | -0.04x $10^{-9}$ --- 0.06 x$10^{-9}$ |
| 4 | 20 | 100 | -0.06x $10^{-9}$ --- 0.05 x$10^{-9}$ | -0.05x $10^{-9}$ --- 0.07 x$10^{-9}$ |
| 5 | 25 | 100 | -0.10x $10^{-9}$ --- 0.06 x$10^{-9}$ | -0.06x $10^{-9}$ --- 0.10 x$10^{-9}$ |
| 6 | 30 | 100 | -0.12x $10^{-9}$ --- 0.07 x$10^{-9}$ | -0.28x $10^{-9}$ --- 0.16 x$10^{-9}$ |
| 7 | 35 | 100 | -0.13x $10^{-9}$ --- 0.08 x$10^{-9}$ | -0.30x $10^{-9}$ --- 0.20 x$10^{-9}$ |
| 8 | 40 | 100 | -0.15x $10^{-9}$ --- 0.10 x$10^{-9}$ | -0.35x $10^{-9}$ --- 0.25 x$10^{-9}$ |
| 9 | 45 | 100 | -0.16x $10^{-9}$ --- 0.11 x$10^{-9}$ | -0.40x $10^{-9}$ --- 0.25 x$10^{-9}$ |
| 10 | 50 | 100 | -0.18x $10^{-9}$ --- 0.12 x$10^{-9}$ | -0.42x $10^{-9}$ --- 0.27 x$10^{-9}$ |
| 11 | 55 | 100 | -0.20x $10^{-9}$ --- 0.15 x$10^{-9}$ | -0.50x $10^{-9}$ --- 0.30 x$10^{-9}$ |
| 12 | 60 | 100 | -0.23x $10^{-9}$ --- 0.16 x$10^{-9}$ | -0.55x $10^{-9}$ --- 0.35 x$10^{-9}$ |
| 13 | 65 | 100 | -0.25x $10^{-9}$ --- 0.20 x$10^{-9}$ | -0.60x $10^{-9}$ --- 0.40 x$10^{-9}$ |

Table 4.6 : *Electric Flux Range for Load Variation at Constant Frequency*

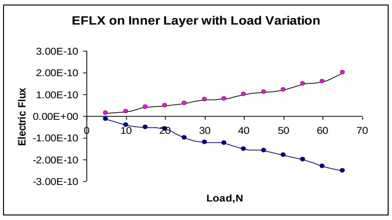



Fig 4.25 *Flux on Inner layer with load variation*

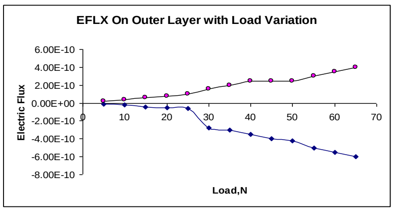



Fig 4.26 *Flux on Outer layer with load variation*

In the studies as described in earlier sections, the main emphasis is given to the sensor. Piezoelectric behave as sensors when it gives electrical response on the application of mechanical loading. On the basis of the result of the following-

In the section 4.1, the analysis of cantilever beam made of only piezoelectric material is done. The study shows that such a beam can be used as a transducer for the dynamic loading. Natural frequency of the system is also an important part so that we can avoid the loading frequency with the natural frequency.

In section 4.2, analysis of a shell made of PZT-4 with the brass cap is done. In which mode shape has been drawn and also the electrical response has been calculated. The same model has been tested for dynamic loading for a frequency of 250 cycles/sec and for a loading of 20N to 200 N. In which we find that electrical flux output is continuously increasing with increment of load. Thus we can say that such a transducer can be used as dynamic loading.

In section 4.3 analysis of a cantilever beam with a piezoelectric strip as a transducer is done. It is analyzed at a frequency of 250 cycles/ second and a load from 10N to 120N. We observe that with the increase in load the variation in electrical flux output is linear as shown in figure 4.22. Also we can understand that in stead of a complete beam of piezoelectric material, we can use a small strip of piezoelectric as transducer in big structures to analyze dynamic loading.

In section 4.4 the analysis of proving ring with piezoelectric strip by varying load and frequency is done. When we vary the load, we find that there is regular increment in electrical flux output while the frequency remains constant. and when the load is constant at 5N and frequency varied from 1000 cycles/sec to 15000cycles/sec then we find there is no change in the electrical flux output value even though the frequency changes. Hence by using such transducers we can get the magnitude as well as frequency of the dynamic load with proper calibration. The effect of temperature has been observed by which we can say that temperature has a good impact on the response of piezoelectric transducer. In the studies as mentioned above, the main emphasis is given to the sensor. Piezoelectric behave as sensors when it gives electrical response on the application of mechanical loading. After completing the above studies, we can now conclude that a piezoelectric sensor can be modeled successfully by finite element method even without any experimentation. After the detailed study of the work is done, we find that the electrical response of any sensor made of piezoelectric is dependent on the following factors –

- Loading condition i.e. what type of loading is there, loading frequency and magnitude of load.
- Temperature of the work environment.
- Natural frequency of the system.

- Curie temperature of the piezoelectric material.

Other then the above mentioned factors the electrical response also depend on the material properties, type of boundary condition and also the type of application.

***Future Prospect***

An important significance of the work presented in the previous sections is to analyze the piezoelectric transducers by finite element analysis and thus actual experiments can be avoided which is a time consuming and costly process. Piezoelectric transducers are widely used for the system where dynamic loading is present such as miniature accelerometer, high frequency accelerometer , load cell etc. all such systems can be modeled and analyzed by finite element analysis. Other then sensors, the reverse of it i.e. actuators can also be modeled analyzed by the finite element method.

We can also develop some systems on FEA packages and can understand the effect of various type of geometries , shape , size etc on the response of transducer. Also the temperature effect can be analyzed more elaborately .

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