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Dynamic Analysis of Laminated Composite Plate Structure with Square Cut-Out under Hygrothermal Load

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***Abstract*— Effect of temperature and moisture on the dynamic properties of epoxy-based laminated composite plates with a square cut-out are investigated. Eight-noded isoparametric plate elements with 6 degrees of freedom per node have been implemented in the present computations. First order transverse shear deformation based on Yang-Norris-Stavsky theory [1] is used along with rotary inertia of the material. The analysis considers material properties of the laminae at elevated temperature and moisture concentrations. Residual stresses due to hygrothermal environment are taken into account. A set of new results with various cut-out ratios and by varying thickness of the plates are presented.**

Keywords—Free Vibration, Laminates, Hygrothermal Load, Square cutout

I. INTRODUCTION

Epoxy-based laminated composite plates with cut-outs are inevitable in naval, aeronautical, mechanical and civil structures. Presence of cut-out changes the stiffness of the structure. Also stress concentration occurs near the opening of the plate. When these structures are subjected to high temperature or moisture, the strength and stiffness of structural composites are strongly affected. The matrix is more susceptible to the elevated temperature and moisture concentrations than the fibre, hence, the deformation is observed to be more in the transverse direction. The rise in hygrothermal stress reduces the elastic modulus of the material and induces initial internal stresses, which may affect the stability as well as the safety of the structures. Thus, knowledge of the dynamic behavior of a structure over a range of temperature and moisture concentrations is essential for the design of structures in a hygrothermal environment.

Whitney and Ashton [2] considered the effect of environment on the free vibration of laminated composite plates by using Ritz method for symmetric laminates and equilibrium equation of motion for unsymmetric laminates. Dhanaraj and Palaninathan [3] showed that the natural frequency decreases linearly with the increase in temperature. Shen and Springer [4, 5] showed that the physical change due to temperature and moisture effects in the resin results in decreasing the tensile properties and reducing the transverse and shear modulus. Sai Ram and P.K. Sinha [6] studied the moisture and temperature effect in free vibration of symmetric and antisymmetric laminates accounting transverse shear deformation with simply supported and clamped boundary condition. Lee *et al*. [7] have predicted the natural frequencies of composite rectangular plates with cutouts, neglecting shear deformations and rotary inertia. Ashwini kumar and Srivastava [8] studied the free vibration of laminated composites with delaminations around a central cutout. Reddy [9] used the quadratic isoparametric element to study the free vibration of antisymmetric angle-ply laminated plates including transverse shear deformation and rotary inertia based on YNS theory [1]. Ali *et al*. [10] presented a simplified method for the dynamic analysis of plates with cutouts, based on Rayleigh's principle. A FEM formulation using HSDT is used in the analysis. Marsh *et al*. [11] studied the moisture absorption and diffusion into epoxy and epoxy-glass composites. Bahrami and Nosier [12] presented the interlaminar hygrothermal normal and shear stress distributions in long laminated composite plates.

In the present investigation results are presented for symmetric laminates subjected to uniform distribution of temperature and moisture throughout its volume in absence of any external loading. Four layered graphite/epoxy laminates with simply supported and all side clamped boundary conditions are considered. Lamina material properties at elevated temperature and moisture conditions [6] are considered. Results showing change in natural frequencies of laminates at elevated temperature and moisture concentrations for various fibre angle orientations, boundary conditions, plate thicknesses and various cut-out ratios are presented. Since the evaluation of shear correction factor from the exact theory of elasticity is difficult in the present case, a commonly used value of 5/6 is assumed [9].

II. MATHEMATICAL FORMULATION

A. Governing Equations for a Laminate

A laminated composite plate of uniform thickness, t consisting of a number of thin laminae each of which may

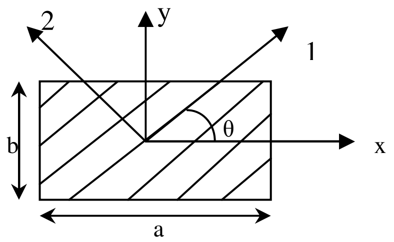

Fig. 1: *Arbitrarily oriented lamina, Axis 3 coincides with z axis*

be arbitrarily oriented at an angle θ (Fig. 1) with reference to the x-axis of the co-ordinate system is considered. The constitutive equations for the plate, when it is subjected to uniform temperature and moisture concentration is given by,

$$\{F\}=[D]\{\varepsilon\}-\{F^N\}, \qquad (1)$$

where,

$\{F\}$ = Mechanical force resultant

$= \{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y\}^T$,

$\{F^N\}$ = Hygrothermal force resultant

$=\{N_x^N, N_y^N, N_{xy}^N, M_x^N, M_y^N, M_{xy}^N, Q_x^N, Q_y^N, 0, 0\}^T$,

$\{\varepsilon\}$ = Strain vector= $\{\bar{\varepsilon}_x, \bar{\varepsilon}_y, \bar{\gamma}_{xy}, K_x, K_y, K_{xy}, \phi_x, \phi_y\}^T$

$$[D]=\begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{44} & A_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{45} & A_{55} \end{bmatrix} \qquad (2)$$

where, the hygrothermal force and moment resultants are

$$\{N^N_x \;\; N^N_y \;\; N^N_{xy}\}^T=\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\left[Q'_{ij}\right]_k [e]_k dz,$$

$$(i, j = 1,\ 2 \text{ and } 6)$$

and (3)

$$\{M^N_x \;\; M^N_y \;\; M^N_{xy}\}^T= \sum_{k=1}^{n}\int_{z_{k-1}}^{z_k} z\left[Q'_{ij}\right]_k [e]_k dz,$$

$$(i, j = 1,\ 2 \text{ and } 6).$$

where $\{e\}_k = \{e_x \quad e_y \quad e_{xy}\}^T$ = Hygrothermal strain = $[R]\{\beta_1, \beta_2\}_k^T\ (C\text{-}C_0) + [R]\{\alpha_1, \alpha_2\}_k^T\ (T\text{-}T_0)$

in which $[R] = \begin{bmatrix} cos^2\theta & sin^2\theta \\ sin^2\theta & cos^2\theta \\ \sin 2\theta & -sin\ 2\theta \end{bmatrix}$,

(C and C_o) and (T and T_o) are the instantaneous and reference moisture concentrations and temperatures respectively.

The stiffness coefficients of the laminate are defined as

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\left[Q'_{ij}\right]_k\{1, z, z^2\}\ dz,$$

for *i, j =1,2,6*

and (4)

$$(A_{ij}) = \alpha\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\left[Q'_{ij}\right]_k dz,$$

for *i, j = 4,5*

α is a shear correction factor, taken as 5/6, to take account for the non uniform distribution of the transverse shear strain across the thickness of the laminate.

$\left[Q'_{ij}\right]_k$ is defined as,

$$\left[Q'_{ij}\right]_k = [R_1]^{-1}\left[Q_{ij}\right]_k[R_1]$$

for *i, j=1,2,6*

and

$$\left[Q'_{ij}\right]_k = [R_2]^{-1}\left[Q_{ij}\right]_k[R_2]$$

for *i, j=4,5*

where,

$$[R_1] = \begin{bmatrix} cos^2\theta & sin^2\theta & 2\sin\theta\cos\theta \\ sin^2\theta & cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & cos^2\theta - sin^2\theta \end{bmatrix},$$

$$[R_2] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix},$$

$$\left[Q_{ij}\right]_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$$

for *i, j=1,2,6*

$$\left[Q_{ij}\right]_k = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix},\quad i, j = 4,5$$

in which,

$$Q_{11} = E_1/(1-\nu_{12}\nu_{21}),$$
$$Q_{12} = \nu_{12}E_2/(1-\nu_{12}\nu_{21}),$$
$$Q_{22} = E_2/(1-\nu_{12}\nu_{21}),$$
$$Q_{44} = G_{13},\ Q_{55} = G_{23}.$$

Where, E_1 and E_2 are Young's moduli along the 1 and 2 axes, respectively and ν_{ij} is Poisson's ratio of transverse strain in the j direction to axial strain in i direction when stressed along i-direction only.

The non-linear strains can be expressed as,

$$\varepsilon_{xnl} = [\bar{u}^2{}_{,x} + \bar{v}^2{}_{,x} + \bar{w}^2{}_{,x} + 2z(\bar{u}_{,x}\theta_{y,x} - \bar{v}_{,x}\theta_{x,x}) + z^2(\theta^2{}_{y,x} + \theta^2{}_{x,x})]/2,$$

$$\varepsilon_{ynl} = [\bar{u}^2{}_{,y} + \bar{v}^2{}_{,y} + \bar{w}^2{}_{,y} + 2z(\bar{u}_{,y}\theta_{y,y} - \bar{v}_{,y}\theta_{x,y}) + z^2(\theta^2{}_{x,y} + \theta^2{}_{y,y})]/2,$$

$$\gamma_{xynl} = [\bar{u}_{,x}\bar{u}_{,y} + \bar{v}_{,x}\bar{v}_{,y} + \bar{w}_{,x}\bar{w}_{,y} + z(\bar{u}_{,y}\theta_{y,x} + \bar{u}_{,x}\theta_{y,y}) - z(\bar{v}_{,y}\theta_{x,x} + v_{,x}\theta_{x,y}) + z^2(\theta_{y,x}\theta_{y,y} + \theta_{x,x}\theta_{x,y})],$$

$$\gamma_{xznl} = [\bar{u}_{,x}\theta_y + \bar{v}_{,x}\theta_x + z(\theta_y\theta_{y,x} + \theta_x\theta_{x,x})],$$

$$\gamma_{yznl} = [\bar{u}_{,y}\theta_y + \bar{v}_{,y}\theta_x + z(\theta_y\theta_{y,y} + \theta_x\theta_{x,y}) \qquad (5)$$

where, *u, v* and *w* are *x, y* and *z* component of structural displacement respectively and θ_x and θ_y are total rotations of a plate element along x and y –directions.

B. *Finite Element Formulation*

Eight-noded isoparametric plate elements with 6 degrees of freedom per node have been implemented in the present computations. The stiffness matrix, the initial stress stiffness matrix, the mass matrix and the nodal load vectors of the element are derived by using the principle of minimum potential energy [6]. The element displacements are expressed in terms of their nodal values by using the element shape functions ($N_i(\xi,\eta)$)and are given by

$$u_0 = \sum_{i=1}^{8} N_i(\xi,\eta) u_{o_i},$$

$$v_0 = \sum_{i=1}^{8} N_i(\xi,\eta) v_{o_i},$$

$$w = \sum_{i=1}^{8} N_i(\xi,\eta) w_i, \qquad (6)$$

$$\theta_x = \sum_{i=1}^{8} N_i(\xi,\eta) \theta_{x_i},$$

$$\theta_y = \sum_{i=1}^{8} N_i(\xi,\eta) \theta_{y_i}$$

in which u_{oi}, v_{oi}, w_i, θ_{xi}, θ_{yi} are the displacements at a node i.

1) *Element Stiffness Matrix*

The linear strain matrix {ε} is obtained by expression

$$\{\varepsilon\} = [B]\{\delta_e\} \qquad (7)$$

The element stiffness matrix is given by

$$[K_e] = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} [B]^T [D][B] dx\, dy \qquad (8)$$

Where [B] is the linear strain displacement matrix and [D] is the elasticity matrix [6].

2) *Element Geometric Stiffness Matrix*

Element stiffness matrix [6] is given by

$$[K_{Ge}^r] = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} [G]^T \{S^r\}[G] dx\, dy \qquad (9)$$

3) *Element Mass Matrix*

The elemental mass matrix [6] is defined as

$$[M_e] = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} [N]^T [\rho][N] dxdy \qquad (10)$$

4) *Element Load Vectors*

The element load vector [6] due to hygrothermal forces and moment is given by

$$\{P_e^N\} = \iint [B]^T \{F^N\} dxdy \qquad (11)$$

5) *Solution Process*

The stiffness matrix, the geometric stiffness matrix, the mass matrix and the load vectors of the element, given by equations (8-11), are calculated by expressing the integral in local natural co-ordinates, ξ and η of the element and then performing Gaussian integration. Generally, a 3 point Gauss quadrature is adopted to compute the bending stiffness of the elements, whereas a 2 point integration is applied to calculate the shear stiffness and mass matrix. The purpose of reduced integration is to reduce the shear stiffness of the element. Then the element matrices are expressed accordingly to global co-ordinates from local co-ordinates.

The initial displacements (δ^i) are then evaluated from equilibrium equation,

$$[K]\{\delta^i\} = \{P\} + \{P^N\} \qquad (12)$$

After obtaining the initial stress resultants {N_x^i, N_y^i, N_{xy}^i, M_x^i, M_y^i, M_{xy}^i, Q_x^i, Q_y^i} from "(1)", the natural frequencies of vibration of the plate are determined from the general equation of vibration,

$$|[K] + [K_\sigma] - \omega_n{}^2[M]| = 0$$

This generalized eigenvalue problem is solved by subspace iteration method.

III. NUMERICAL RESULTS AND DISCUSSION

The finite element formulation described in the earlier section has been used to generate numerical results to study the effects of cutout under hygrothermal environment of a laminated composite plate. Parametric studies have been provided for variation in cut-out ratio, stack up sequence, plate thicknesses, boundary conditions and various temperature and moisture concentrations.

A. *Mesh Convergence Study*

A mesh convergence study has been carried out to obtain minimum number of elements required to achieve the non-dimensional natural frequencies as accurately as possible optimizing the computation time. For this purpose, a square laminated composite plate having a square cutout (a/s=0.4) is taken and analyzed under simply supported (SSSS) and clamped (CCCC) boundary conditions. The side-to-thickness ratio (=s/t) is taken as 75. Three mesh sizes containing 144 (=9x9 mesh), 200 (=10x10 mesh) and 220 (=11x11 mesh) elements are considered for the convergence study. Details of a typical mesh having 200 elements are shown in Fig. 2.

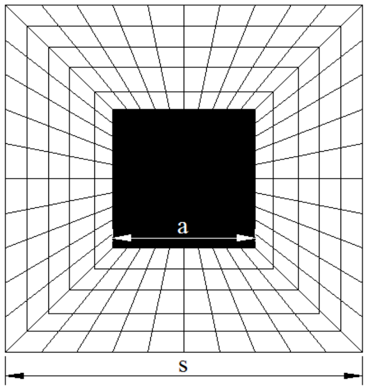

Fig. 2: *Discretization details for a square plate with square cutout*

Natural frequencies obtained for different meshes under different boundary conditions are tabulated in Table 1.

Table 1: *Non-Dimensional Frequency for Different Mesh Sizes*

MODE NO.	SSSS			CCCC		
	9 x 9	*10x10*	*11 x 11*	*9 x 9*	*10x10*	*11 x 11*
1	14.95	14.86	14.86	14.95	14.86	14.86
2	28.06	28.02	28.02	28.06	28.02	28.02
3	28.06	28.02	28.02	28.06	28.02	28.02

It can be observed that the fundamental frequency converges for a mesh containing 200 elements, or more. Hence, for subsequent studies, a mesh containing 200 (10x10 mesh) elements is taken.

B. Validation Study

Two validation studies are made to test the accuracy of the computer program and the results are compared with those available in the literature. The cases are as follows:

i. Validation for laminated composite plate with square cutout
ii. Validation for laminated composite plate under hygro-thermal load

1) Laminated Composite Plate With A Square Cutout

A square laminated composite plate, with all edges simply supported, is considered [8]. The lay-up sequence is $[(\pm45°/0_2°)_3(90°/0_2°/90°)_2]_s$. Each lamina has material properties as follows: $E_1 = 13.00 \times 10^{10}$ N/m^2; $E_2 = 1.00\ 10^{10}$ N/m^2; $G_{12} = G_{31} = 0.5 \times 10^{10}$ N/m^2; $G_{23} = 0.33 \times 10^{10}$ N/m^2; $\nu_{12} = 0.35$; $\rho = 1500$kg/m^3 (available from Aeronautical Development Agency, Bangalore, India). The plate has side-to-thickness ratio (=s/t) of 75. First five Non-dimensionalized frequencies are obtained with a square (a/s = 0.4) cutout. The results are compared with those of Ashwini Kumar and R.P. Shrivastava [8] as shown in Table 2. From the table it is clear that the two results are in good agreement.

Table 2: *First Five Non-dimensional Frequency*

MODE	Ashwini kumar *et al.* [8]	Present
1	14.86	14.86
2	26.68	28.02
3	29.91	28.02
4	49.76	50.54
5	54.10	53.45

$$w_i = w_r * \left(\frac{s^2}{t}\right) * \left(\sqrt{\rho/E_2}\right)$$

where,

w_i =Non-dimensional frequency

w_r = Natural frequency in rad/sec

s = Length of the square plate

t = Thickness of the plate

ρ = Density of the plate

2) Laminated Composite Plate under Hygro-thermal Load

A simply supported square plate, with side to thickness ratio, s/t = 100, has been analyzed under increased temperature of 325K and moisture concentration = 0.1% and the first four non-dimensional frequencies are compared in Table 3, with those obtained by K. S. Sai Ram and P. K. Sinha [6]. The layup sequence of the laminate is taken as (0/90/90/0). Graphite-epoxy material with E_1=130 GPa, E_2=9.5GPa, G_{12}=6GPa, G_{13}= G_{12}, G_{23}= 0.5G_{12}, ν_{12}=0.3, β_1= 0.0, β_2= 0.44 and density = 1600kg/m^{3}are used. Material properties are assumed to be constant for this purpose, i.e., they do not vary with change in hygro-thermal conditions. From Table 3, it is evident that the program used, are giving quite accurate results.

Table 3: *First Four Non-dimensional Frequencies under Uniform Thermal and Hygral loading*

MODE NO.	Temperature=325K		Moisture Conc.=0.1%	
	Sai Ram et al.	*Present*	*Sai Ram et al.*	*Present*
1	8.088	8.096	9.429	9.438
2	19.196	19.297	20.679	20.783
3	39.324	39.489	40.068	40.236
4	45.431	46.165	46.752	47.505

C. *Case Studies*

1) *Study of Dynamic Behavior of Laminated Composite Plate for Varying cutout size*

Various sizes of square cut-outs are considered to examine the effect of cut-out size on the natural frequency of the laminate. The laminate (1.0m x 1.0m) has a varying side to thickness ratio (=s/t) of 25 and40. The layup sequence of the laminate is (0/90/90/0). The material properties for Graphite-epoxy material are given as follows: E_1=130 GPa, E_2= 9GPa, G_{12}=6GPa, G_{13}= G_{12}, G_{23}= 0.5G_{12}, ν_{12}=0.3, β_1= 0.0, β_2= 0.44 and density = 1600kg/m^3 are used. The cut out ratio (= a/s) is varied from 0.2 to 0.8 with increment of 0.2. External temperature is taken as 300K and the moisture concentration is taken as 0.5%. Two different types of boundary conditions, namely simply supported (SSSS) and clamped (CCCC), are taken in the analysis. Results obtained are shown in Table 4. To visualize the effect of cut out size a plot of the fundamental frequencies with various cut out sizes is presented in Fig 3.

- It is observed that for smaller cut outs with s/t = 40, a/s = 0.2 to 0.4, the fundamental frequency decreases slightly, for simply supported edge condition. This indicates that stiffness of the plate decreases marginally when a smaller cut out is present. With the increase in cut out size, a/s =0.4 to 0.8, the fundamental frequency decreases so rapidly that instability occurs and no frequency could be found out. For s/t=25, the fundamental frequency increases with increase in cutout ratios. Here the decrease in mass is much compared to the decrease in stiffness. Hence there is an overall increase in frequency.

- A drastic increase in frequency is observed at a/s=0.8 for the plate clamped on all four edges. Thus for moderate and large cutouts the stiffness of the plate increases rapidly with the cut out area for clamped boundary condition.

- For second and third modes, the frequency decreases for increasing cutout ratio up to 0.6 for simply supported boundary conditions. After that, the frequency increases considerably. Similar phenomenon can be observed for clamped boundary condition. In this case (CCCC) the increase in natural frequency is much more compared to SSSS condition. It is also observed for the all the cases above, that the second and third modes are showing nearly similar frequencies. This is due to the similar mode shapes occurring owing to the structure symmetry.

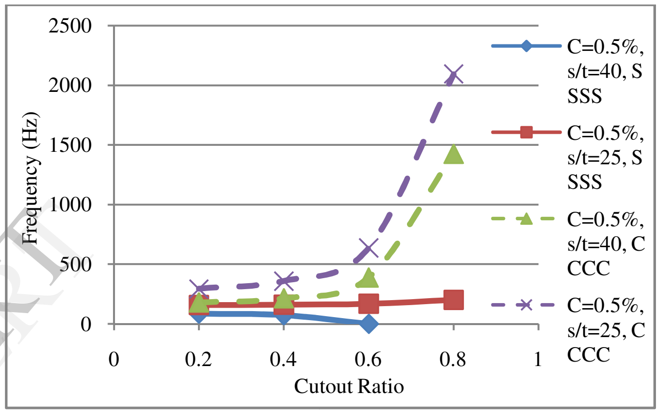

Fig. 3: *Graphical plot for fundamental frequency (Hz) vs. varying cutout size for different s/t ratios and boundary conditions*

Table 4: *First Three Natural Frequencies (Hz)for varying cutout size, boundary condition and s/t ratios*

		Mode No.	cutout ratio							
			Simply supported (SSSS)				**Clamped (CCCC)**			
			0.2	*0.4*	*0.6*	*0.8*	*0.2*	*0.4*	*0.6*	*0.8*
T = 300K, C = 0.5%	*s/t = 40*	1	85.90	74.30	-	-	180	214.4	389.1	1425.0
		2	227.80	208.80	-	-	342.9	324.4	440.6	1454.8
		3	227.90	209.50	-	-	343.0	325.0	444.3	1457.9
	s/t = 25	1	157.9	159.5	168.2	200.9	293.8	358.8	636.8	2093.4
		2	371.0	338.9	308.4	351.1	533.5	508.2	697.9	2114.4
		3	371.0	339.1	309.9	362.4	533.6	508.4	698.9	2116.5

2) *Study of Dynamic Behavior of Laminated Composite Plate Structure under Thermal Load*

Here the effect of increased thermal load on the dynamic behaviour of laminated composite plate (1.0m x 1.0m) with a square cutout (a/s=0.4), for different side to thickness ratio, has been analyzed. The layup sequence of the laminate is taken as (0/90/90/0). The temperature is varied from 300K to 400K. The material used in the analysis is graphite /epoxy with the following properties as given in Table 5.

Table 5: *Degraded Elastic Moduli for varying Temperature*

Elastic Moduli (GPa)	**Temp (K)**				
	300	*325*	*350*	*375*	*400*
E_1	130	130	130	130	130
E_2	9.5	8.5	8.0	7.5	7.0
G_{12}	6.0	6.0	5.5	5.0	4.75

Simply supported (SSSS) and clamped (CCCC) boundary conditions are taken in the analysis. The layup sequence is taken as (0/90/90/0).

First three natural frequencies (Hz) are shown in Table 6. The fundamental frequency vs. temperature is plotted in Fig. 4.

- From the figure, it is observed that with increase in temperature, generally the structure turns soft thus reducing its stiffness.
- The decrease in natural frequency is nearly nonlinear for thinner plates (s/t=40).
- Clamped boundary condition imparts higher stiffnesses to the structure than simply supported conditions.
- First three mode shapes for clamped case are plotted in Fig. 5.
- From the figure it is seen that owing to the similarity in mode shapes for the second and third modes, the second and third modes are showing nearly same frequency of vibration.

Table 6: *First Three Natural Frequency (Hz) for varying Temperature*

	Mode No.	**Temp (K)**									
		SSSS					**CCCC**				
		300	*325*	*350*	*375*	*400*	*300*	*325*	*350*	*375*	*400*
s/t = 40	1	116.4	107.0	96.1	85.2	75.7	241.5	232.6	223.8	215.4	208.4
	2	230.5	223.8	215.5	207.6	201.7	343.7	336.2	327.2	318.4	311.9
	3	230.5	223.8	215.7	207.9	202.2	343.7	336.2	327.4	318.8	312.4
s/t = 25	1	182.6	176.3	168.0	160.1	154.1	375.8	367.9	359.3	350.9	344.4
	2	353.3	347.7	338.6	329.5	323.4	521.7	514.5	503.3	491.8	484.0
	3	353.3	347.8	338.7	329.6	323.5	521.7	514.5	503.3	491.9	484.1

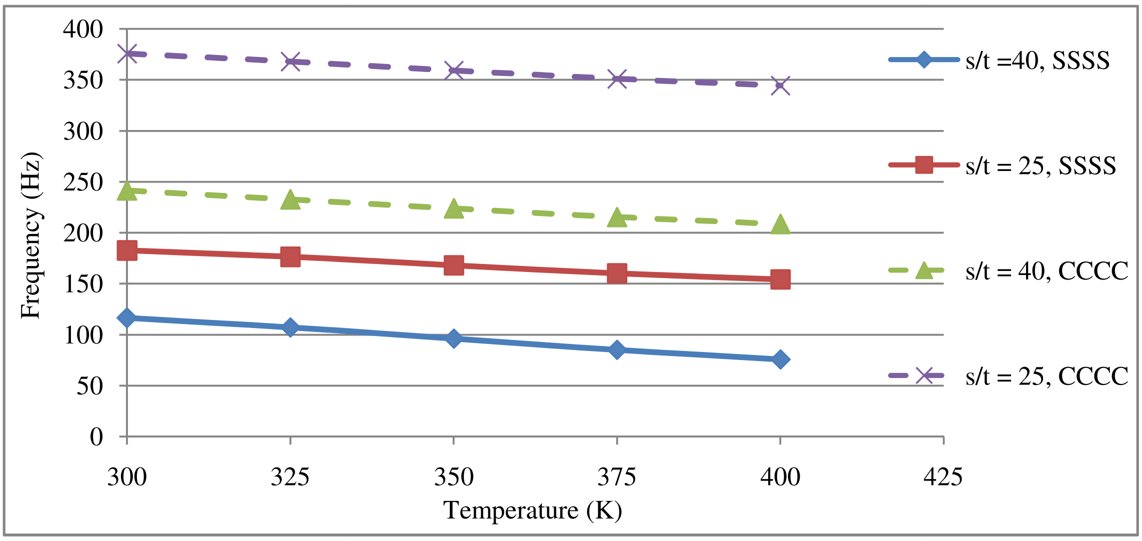

Fig. 4: *Graphical plot for fundamental frequency vs. varying temperature for various boundary conditions and side to thickness ratio*

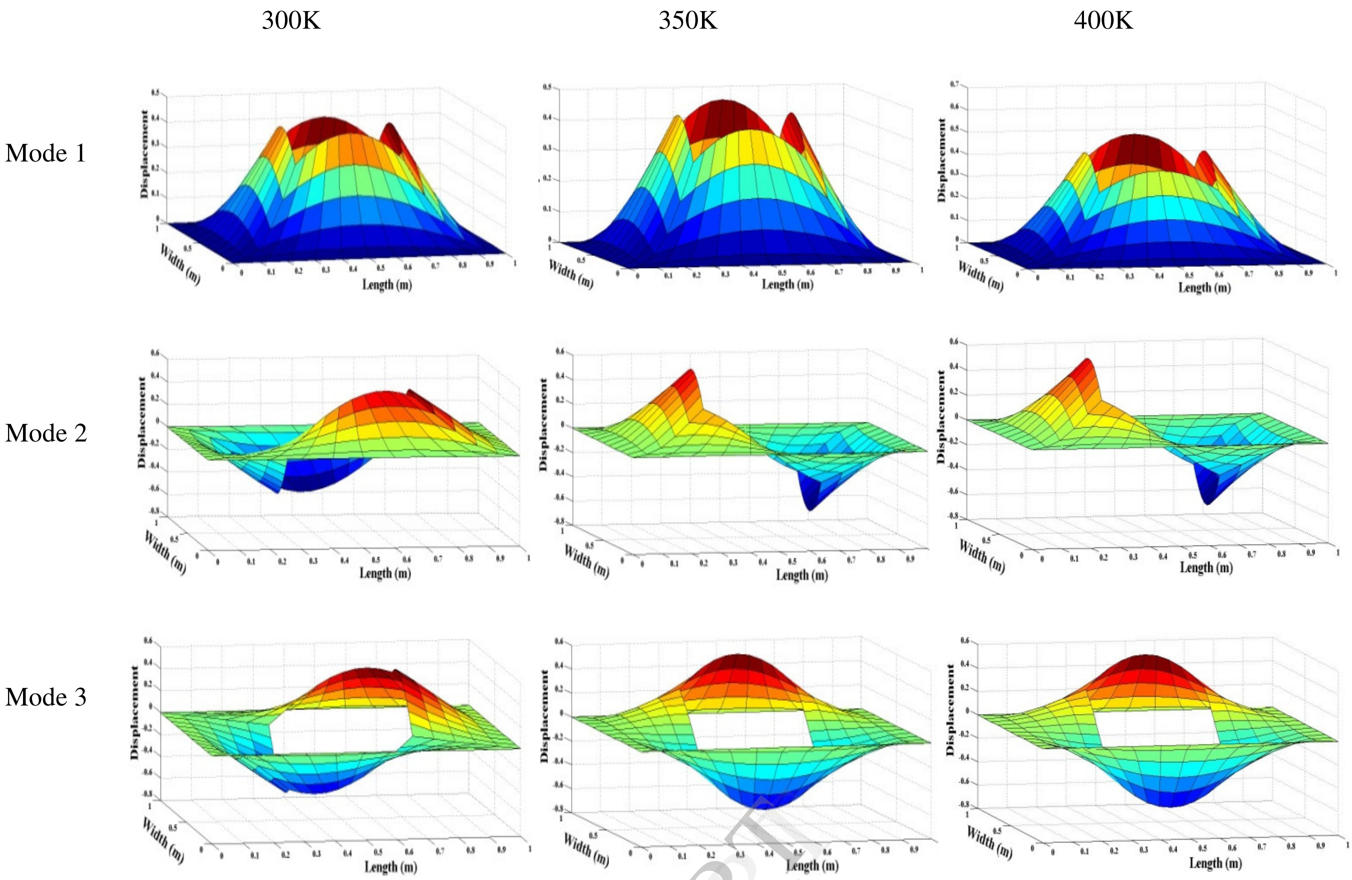

Fig. 5. First three mode shapes of vibration of (0/90/90/0) laminate for increase in temperature

3) Study of Dynamic Behavior of Laminated Composite Plate Structure under Hygral Load

Here the effect of increased hygral load on the dynamic behaviour of laminated composite plate of same material and geometry, as previous case has been analysed. The moisture concentration is varied from 0 to 1.5%. Graphite-epoxy material with the following material properties (Table 7) are used in the analysis.

Table 7: *Degraded Elastic Moduli for various Moisture Concentrations*

Elastic Moduli (GPa)	**Moisture Concentration, C (%)**						
	0.00	*0.25*	*0.50*	*0.75*	*1.00*	*1.25*	*1.50*
E_1	130	130	130	130	130	130	130
E_2	9.5	9.25	9.0	8.75	8.5	8.5	8.5
G_{12}	6.0	6.0	6.0	6.0	6.0	6.0	6.0

First three natural frequencies (Hz) for different cases are shown in Table 8 and a plot of fundamental frequency and changing moisture concentration is presented in Fig. 6.

- From the figure, it is observed that fundamental frequency decreases with increase in moisture concentration. This is due to reduction in stiffness with increasing hygral load.
- The decrease in natural frequency is nearly nonlinear for thinner plates (s/t=40).
- Instability occurs at 1.0% moisture concentration for the thinner plate at simply supported boundary condition.
- Clamped boundary condition imparts higher stiffnesses to the structure than simply supported conditions.
- From the figure, it is also observed that, the less stiff a structure is, chances of instability is more with increase in moisture concentration.
- The mode shapes are plotted in Fig. 7. Here also similar frequency of vibration for second and third modes is observed as in Table 8 due to similarity in mode shapes.

Table 8: *First Three Natural Frequency (Hz) for varying Moisture Concentrations*

	Mode No.	**Moisture Concentration, C (%)**													
		SSSS							**CCCC**						
		0.0	*0.25*	*0.50*	*0.75*	*1.00*	*1.25*	*1.50*	*0.0*	*0.25*	*0.50*	*0.75*	*1.00*	*1.25*	*1.50*
s/t = 40	1	116.4	97.6	74.3	39.2	-	-	-	241.5	228.4	214.4	199.3	182.8	162.3	136.6
	2	230.5	219.7	208.8	198	-	-	-	343.7	334	324.4	314.9	305.6	295.8	285.5
	3	230.5	219.8	209.5	199.6	-	-	-	343.7	334.1	325	316.3	308.1	299.9	291.7
s/t =25	1	182.6	171.2	159.5	147.3	134.7	119.1	100.8	375.8	367.3	358.8	350.3	341.8	333.3	324.3
	2	353.3	346	338.9	332.1	325.5	318.7	311.7	521.7	514.9	508.2	501.8	495.6	489.8	484
	3	353.3	346	339.1	332.5	326.2	319.8	313.4	521.7	514.9	508.4	502.2	496.2	490.9	485.5

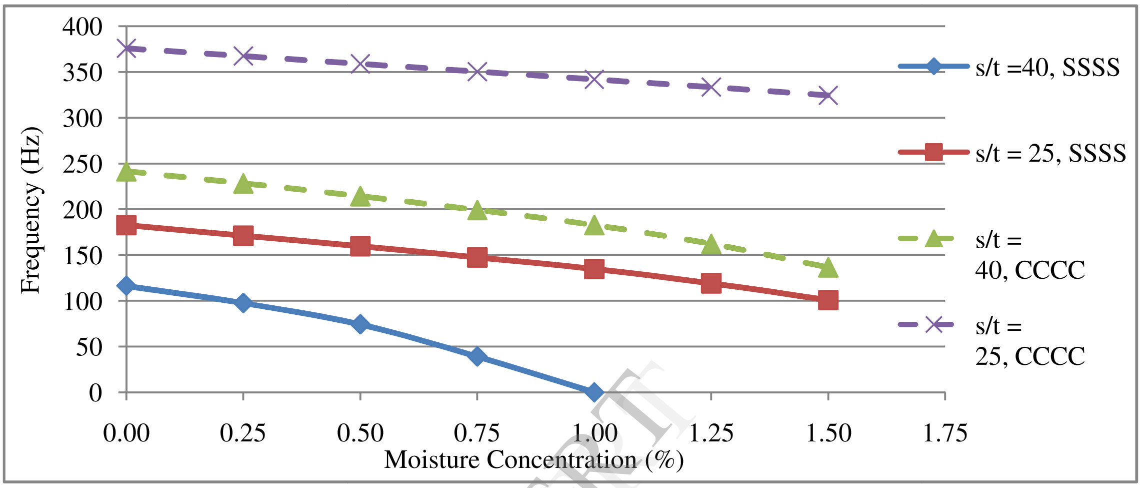

Fig. 6: *Graphical plot for fundamental frequency vs. varying moisture concentration for various boundary conditions and side to thickness ratio*

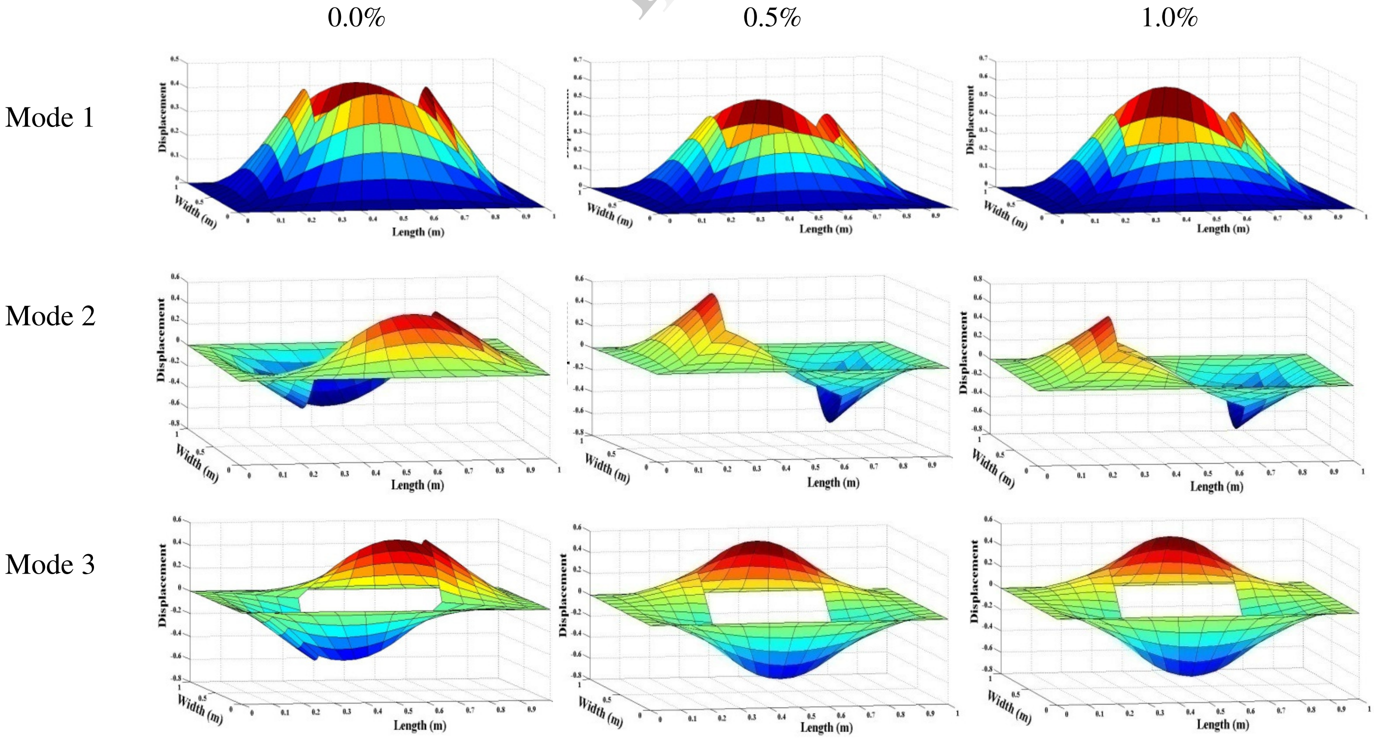

Fig. 7. *First three mode shapes of vibration of (0/90/90/0) laminate with increase in moisture concentration*

4) Study of Dynamic Behavior of Laminated Composite Plate Structure with a cutout under increasing hygrothermal load for various Layup Sequences

Here the effect of hygrothermal load on the dynamic behaviour of simply supported laminated composite plate (1.0m x 1.0m, s/t=25 and s/t=40) with a square cutout with (a/s=0.4), for different symmetric fibre angle, has been analyzed.

The temperature is varied from 300K to 400K and the moisture concentration is varied from 0.0% to 1.5%. The fundamental frequencies (Hz) are plotted in Fig. 8 and Fig. 9.

- From the figures, it can be seen that $(45/\text{-}45)_S$ is the stiffest, whereas $(0/90)_S$ have the least stiffnesses.
- Also, for increasing moisture concentration, symmetric cross ply shows instability much earlier than others.
- The rate of drop of natural frequency is less for $(60/\text{-}60)_S$.
- The rate of degradation of material is more pronounced due to increased moisture effects than due to thermal changes.

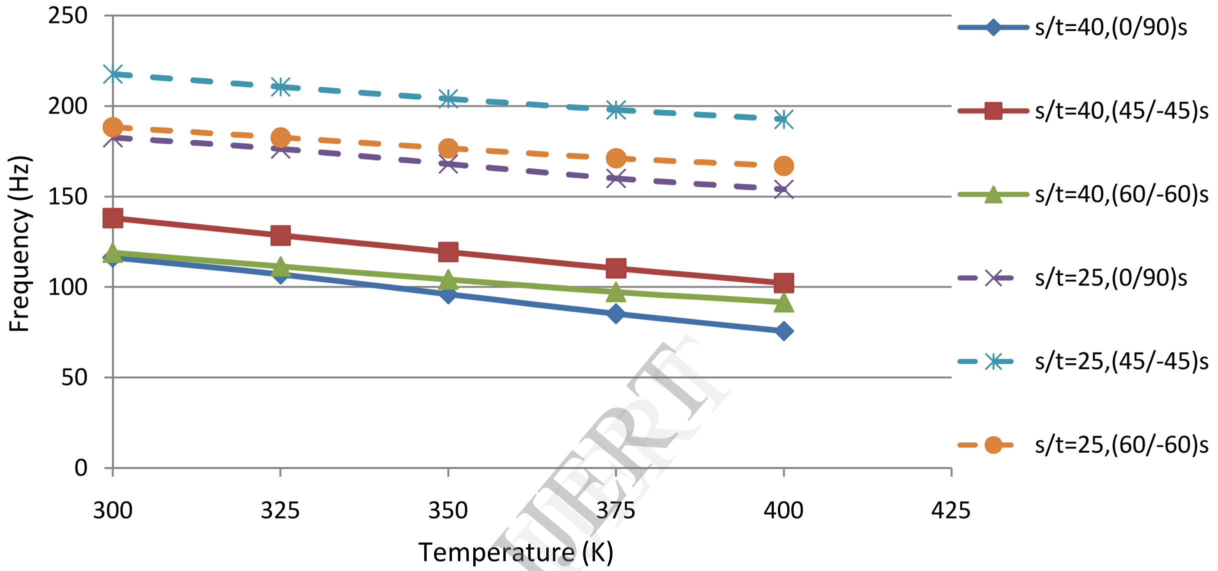

Fig. 8: *Graphical plot for fundamental frequency vs. varying temperature for various layup sequences*

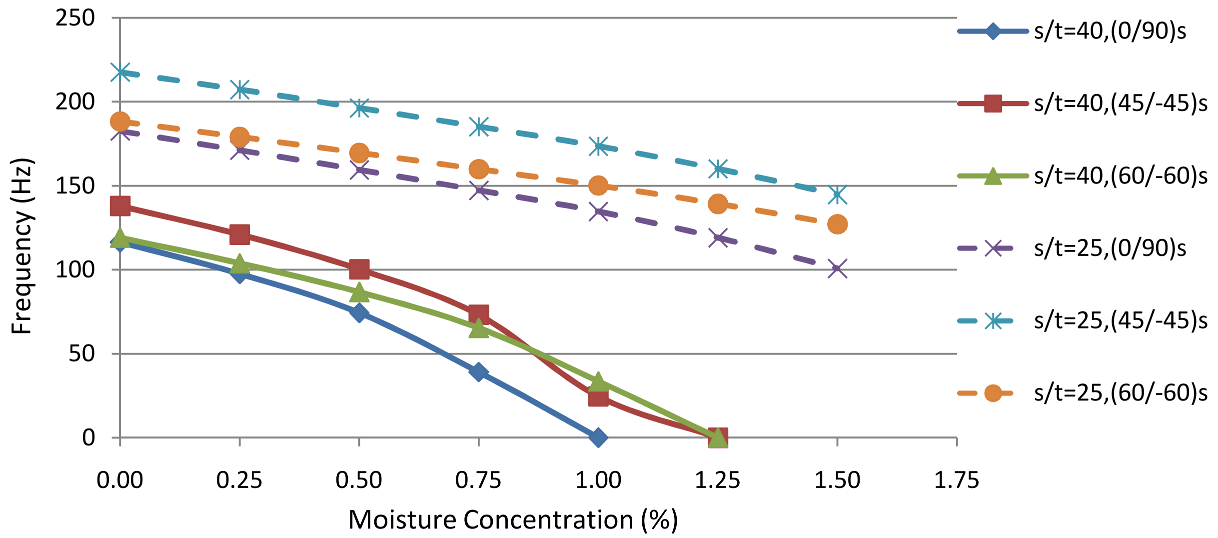

Fig. 9: *Graphical plot for fundamental frequency vs. varying moisture concentration for various layup sequences*

IV. CONCLUSION

This paper is concerned with the dynamic analysis of laminated composite plate structures with a square cutout under hygrothermal load. For the numerical simulation of the problem finite element technique has been used. Various cutout sizes, plate thicknesses, hygral and thermal load, lay up sequences and boundary configurations have been considered in the present study. A set of new results are presented. It has been shown that boundary condition plays an important role in providing stiffness of a plate structure with a cutout. This is also observed that higher moisture concentration and higher temperature turns the structure soft and there is reduction in stiffnesses as well as natural frequency. Less stiff plate has higher chances of instability under higher hygral load. Also instability of a structure largely depends on the layup sequence of the laminated composite structure.

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