

Dynamic Analysis of Composite Plate using Finite Element Analysis

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Abstract— The most common defects observed in composite laminates are Delaminations. These are due to their relatively weak interlaminar strengths. They may occur during fabrication, as a result of incomplete wetting, or during service, such as low-velocity impact. The existence of delaminations may effect the vibration parameters of a composite laminate, such as the natural frequency and the mode shape. In particular, delaminations decreases the natural frequency, which may cause resonance, if the reduced frequency is reaches the working frequency. It is crucial that we should be able to estimate the change in the frequency, as well as the mode shape, in a dynamic environment.

In this thesis dynamic response of composite plate with different stacking sequences and different boundary conditions with single mid-plane delamination is studied. The results of the present model agree well with the analytical and experimental data reported in the literature. Relating to parameter studies the sizes and locations of the delaminations have important effect on the natural frequencies and mode shapes.

Keywords— *Delamination; Dynamic analysis; Frequency; Mode Shapes*

I.INTRODUCTION

Laminated composite plates are greatly used in the construction of aerospace, civil, marine, automotive due to their high definite stiffness and strength, excellent fatigue resistance, long life and many other good properties compared to the conventional metallic materials. In general, these structures require reliability for which, the estimation of the maximum load that the structure can resist as well as the failure process is very important. Due to the anisotropy of composite laminates and the stress distribution is not uniform in laminate under flexural bending as well as other types of static/dynamic loading, the chance of failure in laminates is very high. Large variations in strength and stiffness values of the fibre and the matrix lead to different forms of defects caused during manufacturing process as well as service conditions.

Defect may be detected using frequency response of the structure. Defect induce a decrease in structural stiffness which in turn exhibit decrease in natural frequencies. The location of the defect can be predicted from the degree of change in natural frequency which depends on the position of the defect for a specific mode of vibration. In other words, local or distributed changes in stiffness provide changes in natural frequencies, which affect each mode separately depending on the defect location.

Presently available Non-Destructive Testing (NDT) methods are mostly non-model methods, i.e., either visual or localized experimental methods, such as acoustic or ultrasonic methods, magnetic field methods, radiographs, eddy-current methods or thermal field methods. Running these techniques is time consuming and costly. Some of them are also impractical in many applications such as in service aircraft testing, and space structure. Almost all of these techniques require that the vicinity of the damage is known in advance and that the portion of the structure being inspected is readily accessible for human beings. Subject to these merits, these non-model (experimental) methods can provide only local information and no response of the structural strength at a system level.

In this thesis dynamic response of composite plate with different stacking sequences and different boundary conditions with single mid-plane delamination is studied. The results of the present model agree well with the analytical and experimental data reported in the literature. Parametric studies show that the sizes and locations of the delaminations have tremendous effect on the natural frequencies and mode shapes.

II.LITERATURE REVIEW

Shen and grady [1] studied the delamination effect of a composite plate using varational principle. However, the ‘constrained mode’ model, failed to predict the opening in the mode shapes found in the experiments by Shen and Grady. To simulate the ‘open’ and ‘closed’ behavior between the delaminated layers.

Shu[2] presented an analytical solution to study a sandwich beam with double delaminations. His study draw attention towards the influence of the contact mode, 'free' and 'constrained', between the delaminated layers and the local deformation at the delamination fronts. Della and Shu further investigated the beam with double delaminations by using 'free mode', 'partially constrained mode' and 'constrained mode' models. Shu and Della presented 'free mode' and 'constrained mode' models to study composite beams with multiple delaminations. Their study focused on the influence of a second short delamination on the bending frequencies and the corresponding mode shapes of the beam.

Gim [3] developed a plate finite element to analyze a two dimensional single delamination with multiple constraint condition to find the strain energy release rate for the element. P.k.Parhi [4] Proposed a analytical model for arbitrarily located multiple delaminations.

To study the free vibration of a composite beam with a through-width delamination.Ramkumar [7] presented an analytical solution by assumng the delaminated beam as four Timoshenko beams that are connected at the delamination edges. The natural frequencies and the mode shapes were evaluated by a boundary eigenvalue solution. However, the predicted frequencies were much lower than the experimental values.

Similarly, Wang [8] presented an analytical model using the Euler–Bernoulli beam theory. They assumed the delaminated layers twist 'freely' without touching each other and have different transverse deformations ('free mode'). The coupling between the bending and vibrations along the axis was included in their analysis. Their study showed that for short and midplane delaminations, the estimated frequencies were close to the experimental values.

Mujumdar and Suryanarayanan [9] proposed a model based on the assumption that the delaminated layers are 'constrained' to have same transverse deformations ('constrained mode') but are free to slide over each other in the axial direction other than at their ends. The coupling between the bending and axial vibrations was considered by including the effect of the differential elongation between the delaminated layers.

Similar 'constrained mode' models were presented by Tracy and Pardoen [10] for composite beams, Hu and Hwu for sandwich beams, Shu and Fan [11] for bimaterial beams and Valoor and Chandrasekhar for thick composite beams.

Luo and Hanagud [12] presented an analytical model based on the Timoshenko beam theory by using piecewise-linear springs. Saravacos and Hopkins[13] developed an analytical Composites solution for predicting the natural frequency, mode shape and modal damping of a delaminated composite beam based on a general laminate theory which involves kinematic assumptions showing the discontinuities in the in-plane and through-the-thickness displacements across each delamination crack.

Krawczuk and Chakraborty [14] presented finite element models using the first-order shear deformation theory (FSDT).The above works are on one-dimensional beam-plates with a single delamination. Two- dimensional plates with a single delamination have mostly been numerically investigated. Zak presented finite element models using the FSDT. They modeled the delaminated region by using extra boundary conditions at the delamination fronts. Chattopadhyay and Hu presented finite element models using higher-order shear deformation theories. Different works have been presented for multiple delaminations.

Lestari and Hanagud [15] studied a composite beam with multiple delaminations using the Euler–Bernoulli beam theory with piecewise linear springs to reproduce the 'open' and 'closed' behavior between the delaminated surfaces. Lee [16] studied a composite beam with arbitrary lateral and longitudinal multiple delaminations by using a 'free mode' model and considered a constant curvature at the multiple-delamination tip.

Finite element models have been presented by Ju [17] using the Timoshenko beam theory and Lee using the layerwise theory. Similarly with the single delamination case, composite plates with multiple delaminations have been numerically investigated. Finite element models have been presented by Ju using the Mindlin plate theory, Kim[18] using the first-order zig-zag theory and Cho and Kim using the higher-order zig-zag theory. Three dimensional finite element models were presented by Teneket and Yam [19].

III.FINITE ELEMENT ANALYSIS.

3.1. Introduction to FEA

Finite element method can be explained through physical concept and hence it is most appealing to the engineer. And the method is manageable to systematic computer program and offers scope for application to a wide range of analysis cases. The basic concept is that a body or a structural may be divided into small elements of finite dimensions called finite elements. This process of distributing a continuum into finite elements is known as discretisation. The original body or the structure is then assumed as an assemblage of these elements connected at a finite number of joints called nodes or nodal points. Same concept is applied in finite difference method.

3.1.1. Concept of a Finite element:

The finite element method depends upon the general principle known as going from part to whole. In engineering, problems may come which cannot be solved in closed form that is as a whole. Therefore, we consider the physical medium as an assemblage of several small parts. Analysis of the basic part forms the first step towards a solution.

This notion which in mathematical rather than physical, does not consider the body or the structure to be sub-divided into different parts that are re-formed in the analysis procedure. Instead of that the continuum is zoned into regions by imaginary lines or planes marked on the body. Using this concept, variational-procedure is applied in the

analysis of the continuum by considering a patchwork of solution or displacement models each of which applies to a single region.

3.2 Modeling Composites

Composites are somewhat more difficult to model than an isotropic material such as iron or steel. A special care is to be taken in defining the properties and orientations of the various layers since each layer may have different orthotropic material properties. In this section, the following aspects of building a composite model are concentrated:

1. Choosing the appropriate element type
2. Defining the layered configuration
3. Specifying failure criteria

3.2.1 Following modeling and post processing guidelines

Different element types are available to model layered composite materials: SHELL99, SHELL91, SHELL181, and SOLID46 AND SOLID191. The type of element you choose depends on the application, the type of results that need to be calculated, and so on. Check the single element descriptions to determine if a specific element can be used in ANSYS product. All layered elements allow failure criterion calculations.

Choosing the Proper Element Type:

1. SHELL99

SHELL99 is an 8-node, three dimensional shell element with 6 DOF at each node. It is designed to model thin to comparatively thick plate and shell structures with a side-to-thickness ratio of roughly 10 or greater. For structures with smaller ratios, you may consider using SOLID46. The SHELL99 element allows a total of 250 uniform-thickness layers. Alternately, the element allows 125 layers with thickness that may vary bilinearly over the area of the layer. If more than 250 layers are required, you can input your own material matrix. It also has an option to offset the nodes to the top or bottom surface.

2. SHELL91

SHELL91 is quite similar to SHELL99 other than it allows only up to 100 layers and does not allow you to input a material property matrix. anywhere, SHELL91 supports plasticity, strain behavior and a special sandwich option, SHELL91 is also more robust for large deflection behavior.

3. SHELL181

SHELL181 is a 4-node three dimensional shell element with 6 DOF at each node. The element has full nonlinear capabilities including large strain and allows 255 layers. The layer information is input using the section commands rather than real constants. A failure criterion is available using the FC commands.

4. SOLID46

SOLID46 is a 8-node, three dimensional solid element, SOLID45, with three degrees of freedom per node (UX, UY, UZ). It is designed to model thick layered shells or layered solids and allows up to 250 uniform-thickness layers per element. possibly, the element allows 125 layers with thicknesses that may vary bilinear over the area of the layer. An advantage with this type of element is that you can stack several elements to model more than 250 layers to allow through-the-thickness deformation slope discontinuities.

5. SOLID191

SOLID191 is a layered version of the 20-node three dimensional solid element SOLID95, with 3 DOF per node (UX, UY, UZ). It is designed to model thick layered shells or layered solids and allows up to 100 layers per element. As with SOLID46, SOLID191 can be packed to model through-the-thickness discontinuities.

6. SOLID95

The 20-node structural solid element, functions similar to a individual layered SOLID191 including the use of an orientation angle and failure criterion. It allows nonlinear materials and large deflections.

3.2.2 Defining the Layered Configuration

The most significant characteristic of a composite material is its layered configuration. Each layer may be made of a unlike orthotropic material and may have its principal directions oriented differently. For laminated composites, the fiber directions determine layer orientation.

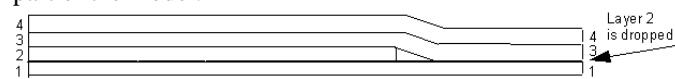
Two methods are available to define the layered configuration:

1. By specifying single layer properties
2. By defining constitutive matrices that consist generalized forces and moments to generalized strains and curvatures.

3.2.3 Specifying Individual Layer Properties

With this method, the layer configuration is defined layer-by-layer from bottom to top. The bottom layer is designated as layer 1, and extra layers are stacked from bottom to top in the positive Z (normal) direction of the element coordinate system. Define only half of the layers if stacking symmetry exists.

At times, a physical layer will extend over only part of the model. In order to model continuous layers, these dropped layers may be modeled with zero thickness. Below figure shows Layered Model Showing Dropped Layer shows a model with four layers, the second of which is dropped over part of the model.



Layered Model Showing Dropped Layer

For each layer, the following properties are specified in the element real constant table (Main Menu> Preprocessor> Real Constants) (accessed with REAL attributes).

- Material properties (via a material reference number MAT)
- Layer orientation angle commands (THETA)
- Layer thickness (TK)

3.2.4 Material Properties

Compared to any other element, the MP command (Main Menu> Preprocessor> Material Props> Material Models> Structural> Linear> Elastic> Isotropic or Orthotropic) is applicable to define the linear material properties, and the TB command is used to define the nonlinear material data tables.

3.2.5 Layer Orientation Angle

This defines the change of axis of the layer coordinate system with respect to the element coordinate system. It is the angle between X-axes of the two systems. By default, the layer coordinate system is parallel to the element coordinate system. All elements have a usual coordinate system which you can change using the ESYS element attribute (Main Menu> Preprocessor> Meshing> Mesh Attributes> Default Attribs).

3.2.6 Layer Thickness

If the layer thickness is uniform, you only need to define TK(I), the thickness at node I. Otherwise, the thickness at the four corner nodes must be input. Dropped layers may be represented with zero thickness.

3.2.7 Number of integration points per layer

This allows you to find out in how much detail the program should calculate the results. For very thin layers, when used with several other layers, one point would be appropriate. But for laminates with less layers, more points would be needed. The default is 3 points. This feature implies only to sections defined through the section commands.

3.2.8 Defining the Constitutive Matrices

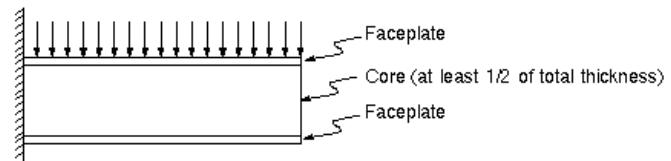
This is an another choice to specify the individual layer properties and is available as an optional [KEYOPTION (2)] for SOLID46 and SHELL99. The matrices, which define the force-moment and strain-curvature relationships for the element, must be computed outside the ANSYS program

The main advantages of the matrix approach are:

1. It allows incorporating an entire composite material behavior.
2. A thermal load vector may be supplied.
3. The matrices may represent an infinite number of layers.

3.2.9 Sandwich and Multiple-Layered Structures

Sandwich structures consists of two thin face plates and a thick, but relatively weak, core. Figure shows Sandwich Construction illustrates sandwich construction.

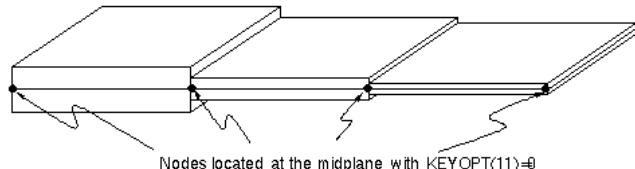


Sandwich Construction

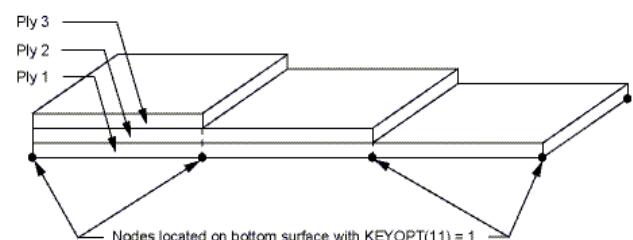
We can model sandwich structures with SHELL63, SHELL91, or SHELL181.

3.2.10 Node Offset

The figures below illustrate how you can model ply drop off in shell elements that are adjacent to each other. In Figure Layered Shell with Nodes at Midplane, the nodes are located at the middle surfaces (KEYOPT (11) = 0) and these surfaces are put in order. In Figure Layered Shell with Nodes at Bottom Surface, the nodes are located at the bottom surfaces (KEYOPT (11) = 1) and these surfaces are aligned.



Layered Shells with Nodes at Midplane



Layered Shells with Nodes at Bottom Surface

3.2.11 Specifying Failure Criteria

Failure criteria is employed to learn if a layer has damaged due to the applied loads. Any one can be chosen from three predefined failure criteria for specifying up to six failure criteria of user-written criteria.

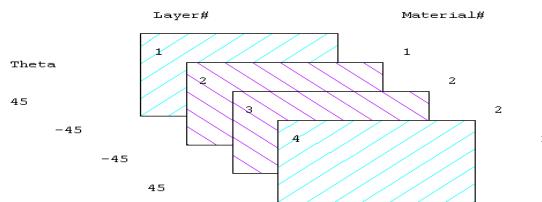
The three predefined criteria are:

1. Maximum Strain Failure Criterion, which allows nine failure strains.
2. Maximum Stress Failure Criterion, which allows nine failure stresses.
3. Tsai-Wu Failure Criterion, which allows nine failure stresses and three additional coupling coefficients.

3.2.12 Additional Modeling and Post processing Guidelines

Some additional guidelines for modeling and post processing of composite elements are presented below.

1. Composites present different types of coupling effects, such as coupling between bending and twisting, coupling between extension and bending, etc. This is due to stacking of layers of differing material properties. Therefore, if the layer stacking sequence is not symmetric, you may not be able to use model symmetry even if the geometry and loading are symmetric, because the displacements and stresses may not be symmetric.
2. Interlaminar shear stresses are usually significant at the free edges of a model. For relatively accurate interlaminar shear stresses at such locations, the element size at the boundaries of the model should be almost equal to the total laminate thickness. For shells, increasing the number of layers per actual material layer does not improve the accuracy of interlaminar shear stresses. With elements SOLID46, SOLID95 and SOLID191, however, stacking elements in the thickness direction should result in more accurate interlaminar stresses through the thickness. Interlaminar transverse shear stresses in shell elements depends on the assumption that no shear is carried at the top and bottom surfaces of the element. These interlaminar shear stresses are only calculated in the interior and are not suitable along the shell element boundaries. Use of shell-to-solid sub modeling is recommended to accurately compute all of the free edge interlaminar stresses.
3. Since a large amount of input data is necessary for composites, you should recheck the data before proceeding with the solution.
4. By default, only data for the bottom of the first layer, top of the last layer and the layer with the maximum failure criterion value are written to results file.
5. Use the ESEL, S, LAYER command to select elements that have a limited layer number. If an element has a zero thickness for the requested layer, the element is not selected.



Sample LAYPLOT Display for [45/-45/-45/45] Sequence

3.3 Composite Plate

Depending upon the finite element procedure and delamination modeling, computer programs are developed to generate numerical solutions to study the behavior of free and forced vibrations of delaminated composite plates with different boundary conditions and stacking sequences. For calculating the numerical solutions, four cases are considered, where delaminations are assumed to have different sizes. Case 1 shows a plate without delamination and cases 2, 3 show those with centrally located delaminations of 1/16 and 1/4, respectively of total plate area.

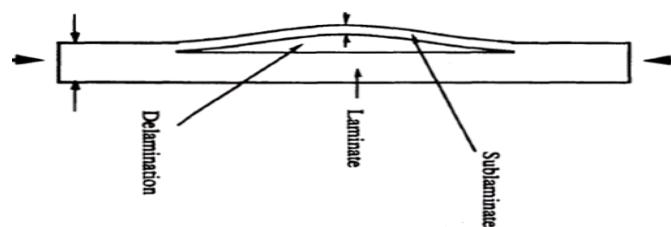
3.3.1 Free Vibration of Delaminated Composite Plates

To establish the accuracy of the finite element formulation, bending and free vibration solutions for an undelaminated composite plates. They are first compared with those existing in the literature. It is observed that the present non-dimensional fundamental frequencies compare well with the published ones. The carbon-epoxy lamina properties used for obtaining other numerical results are as follows:

Material properties of Carbon Epoxy

E11	E22=E33	G12=G13	G23	v12	a=b	a/h
172Gpa	6.9Gpa	3.45Gpa	1.38Gpa	0.25	0.5m	100

3.3.2. Modeling delamination of Plate



A Composite Plate with a Delamination

The finite element delamination analysis is been done by using ANSYS finite element software. There are different ways in which the plate can be modeled for the delamination analysis. For the present study, a three dimensional model with 8-node SHELL 99 composite element of ANSYS is used. The plate is divided into two sub-laminates. Thus, the present finite element model would be referred to as two sub-laminate model. The two sub-laminates are modeled individually using 8-node SHELL 99 composite element, and then joined face to face with appropriate interfacial constraint conditions for the corresponding nodes on the sub-laminates, based on whether the nodes lie in the delaminated region. Typically, nodes in the delaminated region of bottom sub-laminate and corresponding nodes on the top sub-laminate are declared to be coupled nodes using Contact Elements facility of ANSYS. The nodes in the delaminated region, whether in the top or bottom laminate, are connected by Contact 174 element. This would mean that the two sublaminates are free to move away from each other in the delaminated region, and constrained to move as a single laminate in the undelaminated region.

3.3.2.1 Mesh Generation

The mesh generation for Dynamic analysis of plate is modeled using element of ANSYS library. The Three dimensional model is made through key points, lines and areas of solid modeling technique of ANSYS. All the areas created are merged and then the edge divisions on each surface of entities are assigned for meshing. Mapped meshing of ANSYS is used for mesh generation. The model is then extruded into three dimensional models.

3. 3.2.2 Boundary Conditions and Loading:

The plates are assumed to be clamped and simply supported at the four edges of bottom plate and loads are applied at all the nodes of the face plates. A uniform pulse load of 10N/cm^2 is applied on all the nodes of the top and bottom plates.

3. 3.2.3 Material Properties-

The composite plates are considered having made composite carbon/epoxy, which is an orthotropic material. Its material properties are as given

Material Properties of Carbon/Epoxy

E11	E22=E33	G12=G13	G23	v12	a=b	a/h
172Gpa	6.9Gpa	3.45Gpa	1.38Gpa	0.25	0.5m	100

IV. FINITE ELEMENT FORMULATION

4.1. Composite Plate

A laminated composite plate of length a , breadth b and h with n arbitrary layers is considered. The plate axes and the layer details are illustrated in figure. The $x-y$ plane coincides with the plate at middle plane and the z axis is oriented along the thickness direction. The displacements u , v and w at any point (x, y, z) in the laminate are given by

$$\begin{aligned} u(x, y, z) &= u^\theta(x, y) + z\theta_x(x, y), \\ v(x, y, z) &= v^\theta(x, y) + z\theta_y(x, y), \\ w(x, y, z) &= w^\theta(x, y), \end{aligned}$$

Where u^θ , v^θ and w^θ are the mid-plane displacements and θ_x and θ_y are the rotations along the x and y axes, respectively. By first order shear deformation, the strain components in a lamina for an 8-noded isoparametric quadratic plate element with 5DOF per node are expressed as

$$\begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} = \epsilon_x^o + zk_x, \\ \epsilon_y &= \frac{\partial u}{\partial y} = \epsilon_y^o + zk_y, \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{xy}^o + zk_{xy}, \\ \gamma_{xy}^o &= \frac{\partial w}{\partial x} + \theta_x \\ \gamma_{yz}^o &= \frac{\partial w}{\partial y} + \theta_y, \end{aligned}$$

where ϵ_x , ϵ_y and γ_{xy}^o are mid-plane strains, k_x , k_y and k_{xy} are the plate curvatures and γ_{xy}^o and γ_{yz}^o are the transverse shear strains, respectively.

The strains in the k th lamina at a distance z from the mid-plane in the matrix form are given by

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

Here k_{xz} and k_{yz} are taken as zero. The shape functions N_1 (Cook et al 1989) used for different nodes are as follows.

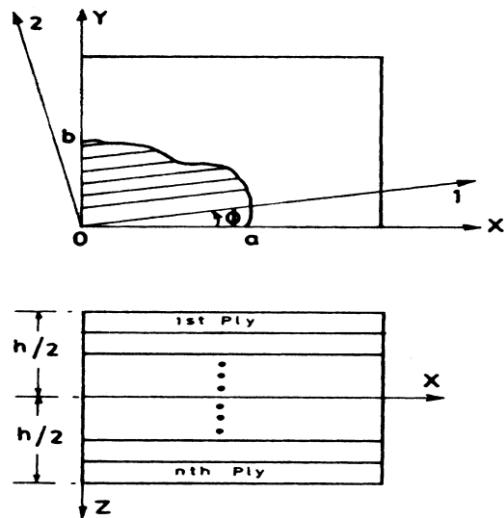


Plate axes and layer details.

$$N_i = 0.25(1 + \xi_i \xi)(1 + \eta_i \eta)(\xi_i \xi + \eta_i \eta - 1), \quad i = 1 \text{ to } 4,$$

$$N_i = 0.5(1 - \xi^2)(1 + \eta_i \eta), \quad i = 5, 7,$$

$$N_i = 0.5(1 - \eta^2)(1 + \xi_i \xi), \quad i = 6, 8,$$

with ξ and η , the local natural axes of the element and ξ_i and η_i , the natural coordinates at node i . Strain-displacement relations are expressed in the form of midplane nodal DOFs as

$$\begin{aligned} \{\epsilon\} &= \sum_{i=1}^8 [B_d] \{d_i\} \\ \{\epsilon\} &= [\epsilon_x \epsilon_y \epsilon_z \gamma_{xz} \gamma_{yz} \gamma_{xy}]^T \end{aligned}$$

Here, B_d is the strain-displacement matrix. The stresses at any point in the k th lamina are

$$\{\sigma\} = [Q_{ij}]_K \{\epsilon\}_P, \quad i, j = 1 \dots 6$$

$$\{\sigma_1\} = [\sigma_x \sigma_y \sigma_z \tau_{xz} \tau_{yz} \tau_{xy}]^T, \quad \{\epsilon_1\} = [\epsilon_x \epsilon_y \epsilon_z \gamma_{xz} \gamma_{yz} \gamma_{xy}]^T$$

$[Q_{ij}]_K$ is the off-axis stiffness of the k th lamina. Here, E_2 is zero. The elasticity matrix of the undelaminated composite plate is given by

$$[D] = \begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & A_{sij} \end{bmatrix}$$

The element stiffness and mass matrices are given by

$$[K_e] = \int_{-1}^1 \int_{-1}^1 [B_d]^T [D] [B_d] j d\xi d\eta,$$

$$[M_e] = \int_{-1}^1 \int_{-1}^1 [N]^T [N] j d\xi d\eta,$$

$$[N] = \sum_{i=1}^8 \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix}$$

$$[\rho] = \begin{bmatrix} P & 0 & 0 & 0 & 0 \\ 0 & P & 0 & 0 & 0 \\ 0 & 0 & P & 0 & 0 \\ 0 & 0 & 0 & P & 0 \\ 0 & 0 & 0 & 0 & P \end{bmatrix}$$

With

$$P = \sum_{k=1}^n (z_k - z_{k-1}) \rho_k$$

$$I = \frac{1}{3(z_k^3 - z_{k-1}^3)} \text{ and } \rho_k$$

the mass per unit volume of the k th lamina. Combining element mass and stiffness matrices and the force vector with respect to the common global axes, the resulting equilibrium equation for bending is

For free vibration

$$[K]\{u\} = \{F\},$$

For forced vibration

$$[M]\{\ddot{u}\} + [K]\{u\} = \{F\},$$

where $\{u\}$ and $\{\ddot{u}\}$ are global displacement and acceleration vector, respectively.

For the impact problem, $\{F\}$ is given by

$$\{F\} = [000 \dots F_c \dots 000]^T,$$

Where, F_c is the contact force corresponding to the contact point. The dynamic equilibrium of the impactor is given by

$$m_i w_i + F_c = 0,$$

Where m_i and w_i are impactor mass and acceleration, respectively.

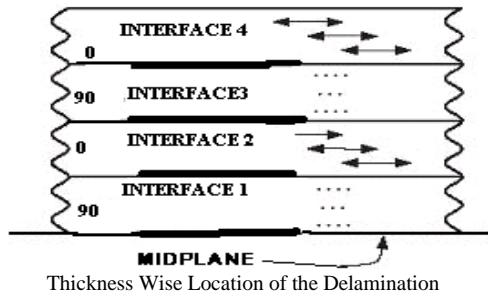
Newmark's constant average acceleration method is used to solve the dynamic forced vibration equations of the plate and the impactor in each time-step during impact.

V.RESULT AND DISCUSSIONS

5.1. Cantilever plate

A cantilever composite plate with the four-ply [0/90,45/-45,30/-30]2s model and dimensions of 500mm x 0.5m x 100mm is considered. Thickness direction delamination locations are defined as in Shen and Grady [1], i.e., bending and tensile test is examined for a composite plate. The Thickness wise locations of the Delaminations are shown in below figure.

Natural frequencies and mode shapes are compared with the experimental results provided in Shen and Grady (1992)[1]. The ANSYS Multiphysics program is employed to obtain the natural frequencies.



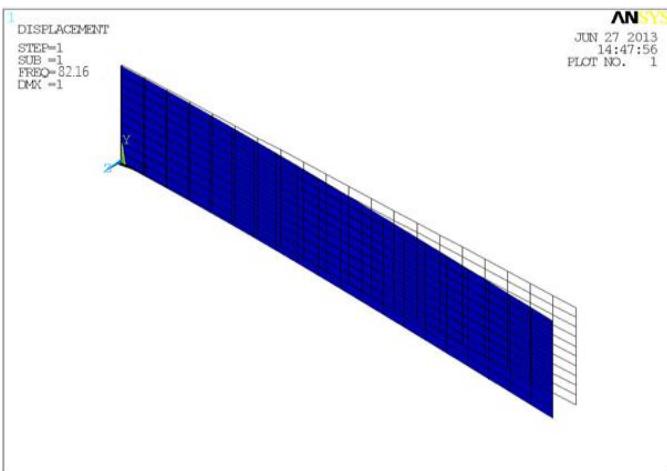
It is found that good agreement was obtained between the frequencies predicted by present model and the analytical and experimental results presented in the literature. The only exception is the case with delamination located at interface 4 (uppermost lamina). The large difference in the natural frequency that was observed experimentally is most likely due to buckling of the upper sublaminate.

5.1.1 Mode Shapes:

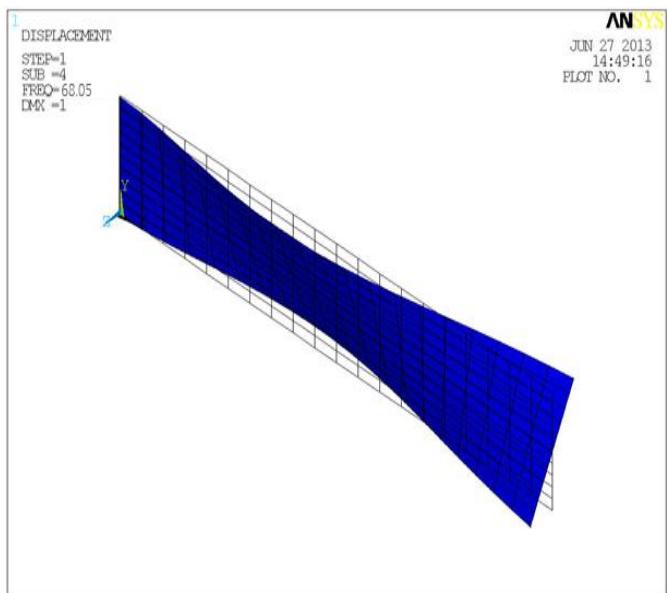
The mode shapes of composite plates are also affected by the size and location of embedded delaminations. In this section, the mode shapes obtained from Ansys are shown. From Fig. 5.2, we can see that first vibration modes do not show any opening in the cases of Interface 1 and 2 delaminations, while in cases of Interface 3 and 4 delaminations, we can clearly see the delamination opening modes except for the one inch delamination. This is in good agreement with the results of Luo and Hanagud [12].

5.1.2 Variation in frequency with change in mode shapes at different lengths of delamination by considering different angle plies.(0/90, 45/-45, 30/-30)

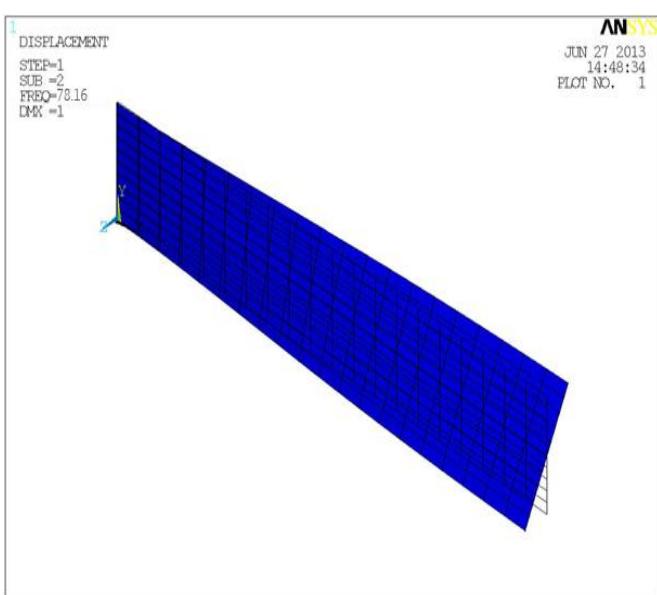
Variation of different modes of frequencies with the change in the length of the delamination for case i:



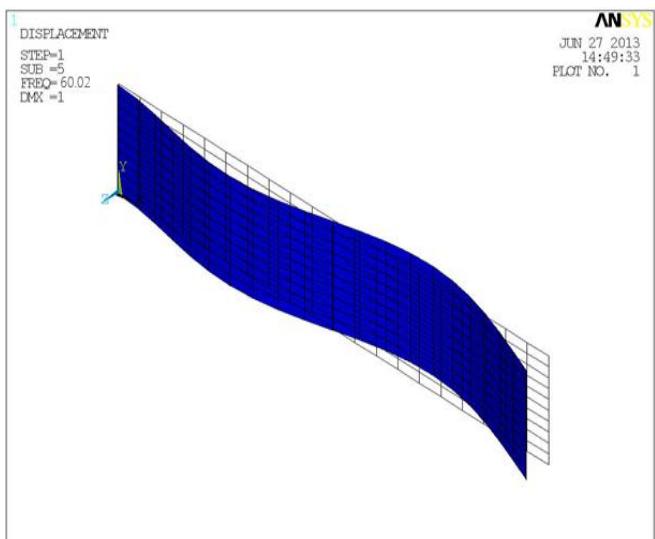
a) First mode frequency of cantilever plate at interface 1 with [0/90] angle
ply laminate



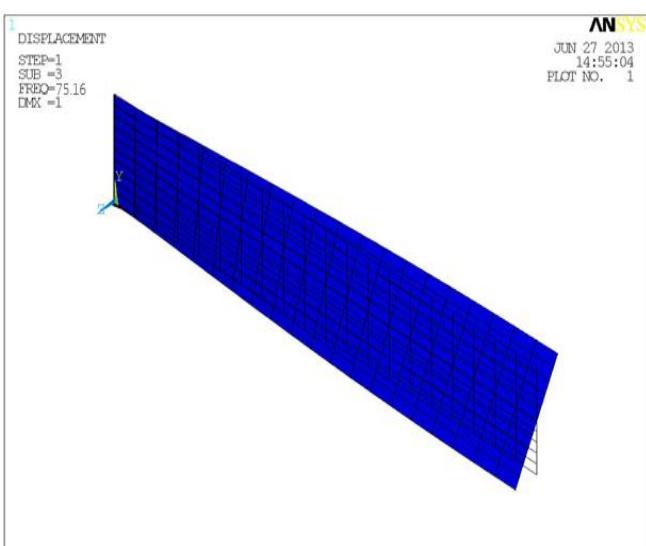
d) fourth mode frequency of cantilever plate at interface 1 with [0/90] angle
ply laminate



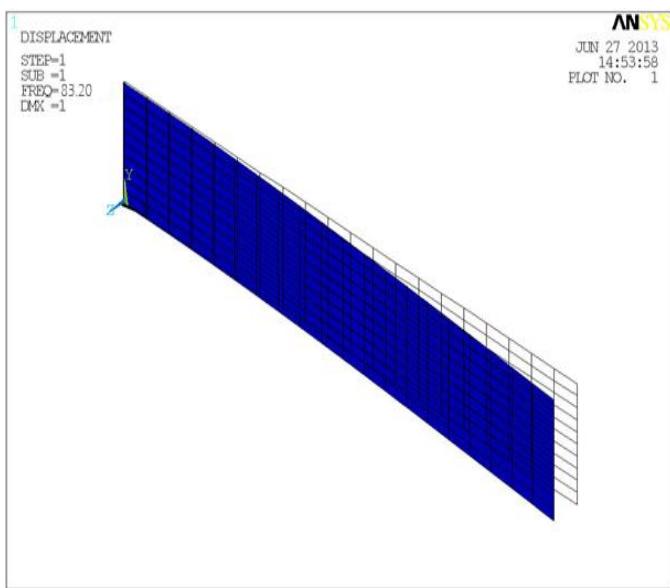
b) Second mode frequency of cantilever plate at interface 1 with [0/90] angle
ply laminate



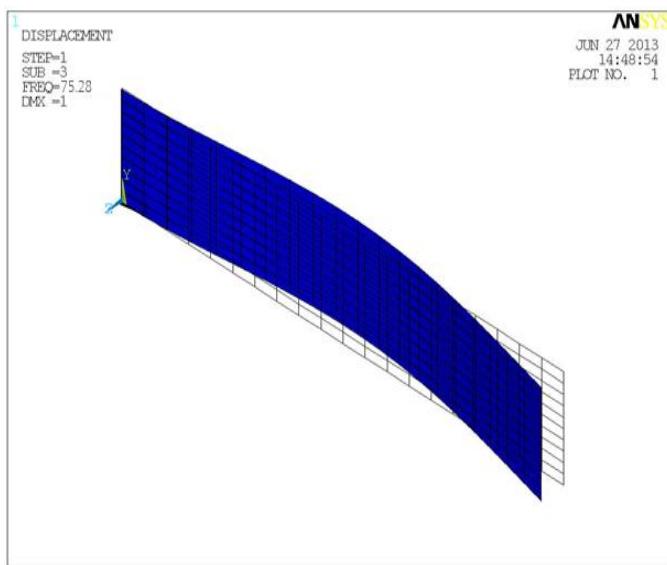
e) fifth mode frequency of cantilever plate at interface 1 with [0/90] angle
ply laminate



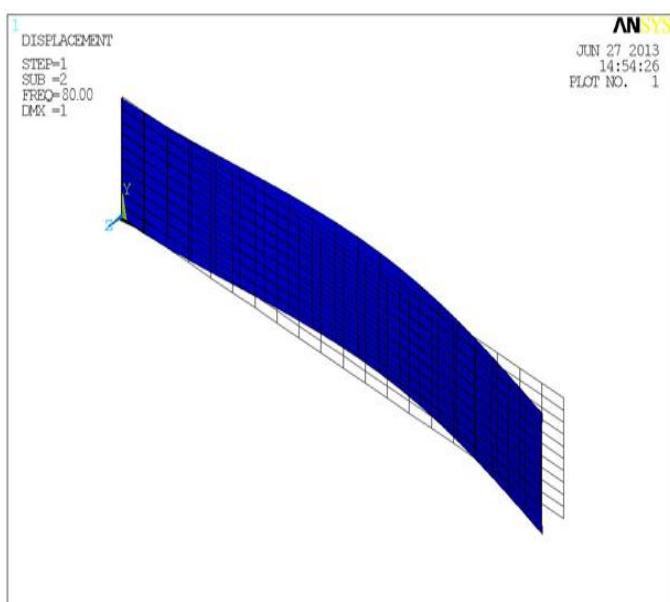
c) Third mode frequency of cantilever plate at interface 1 with [0/90] angle
ply laminate



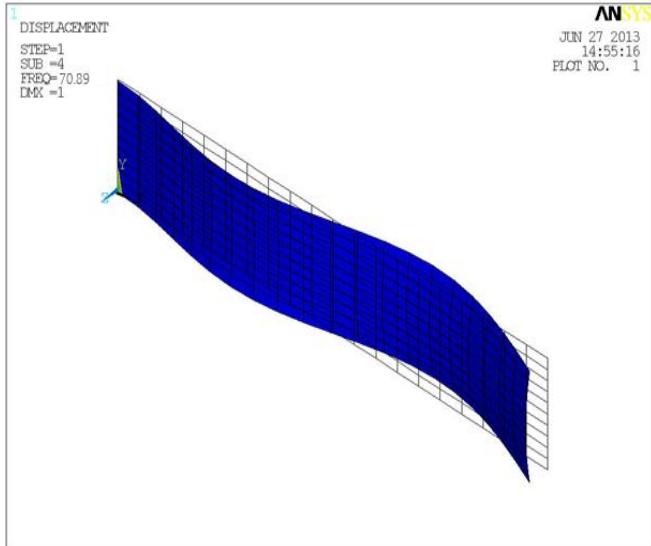
a). First mode frequency of a cantilever plate with [45/-45] angle ply laminate



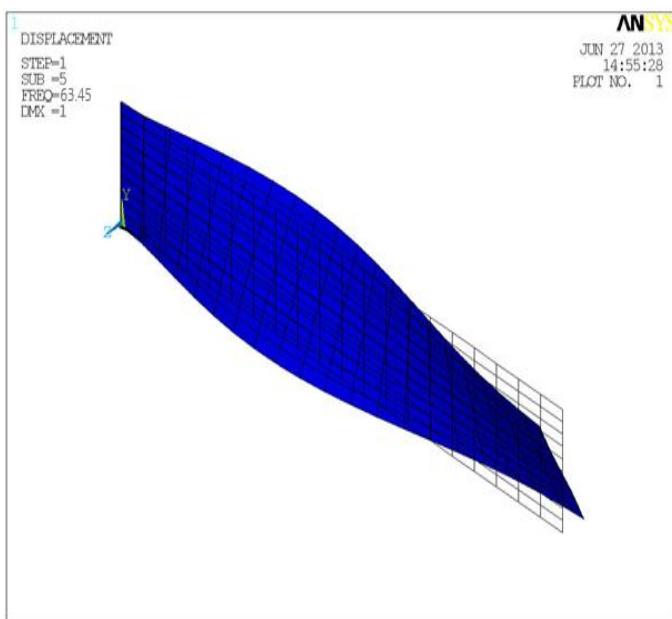
c)Third mode frequency of a cantilever plate with [45/-45] angle ply laminate



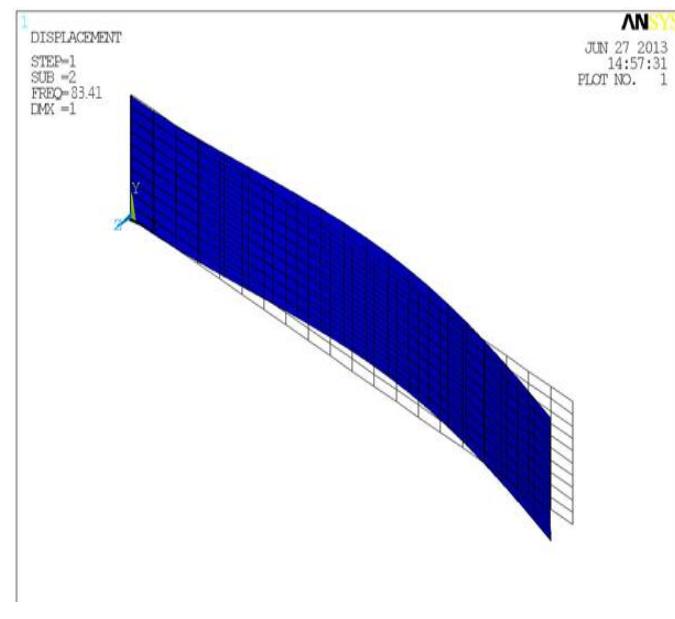
b) Second mode frequency of a cantilever plate with [45/-45] angle play laminate



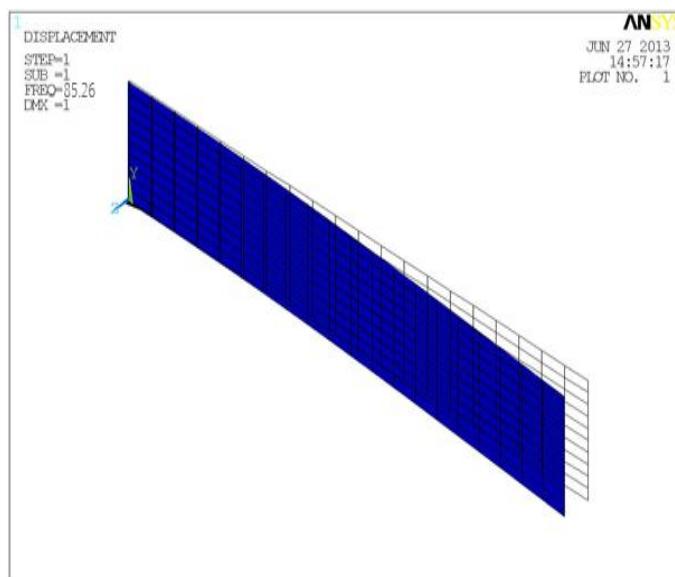
d)Fourth mode frequency of a cantilever plate with [45/-45] angle ply laminate



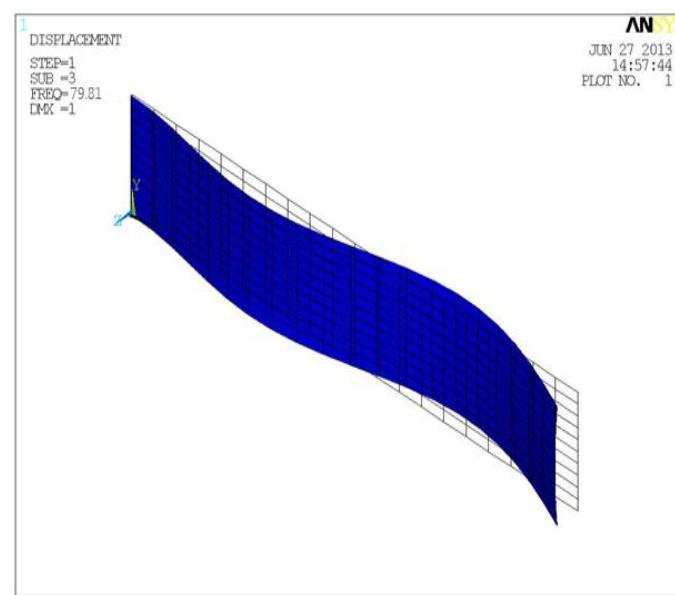
e) Fifth mode frequency of a cantilever plate with [45/-45] angle ply laminate



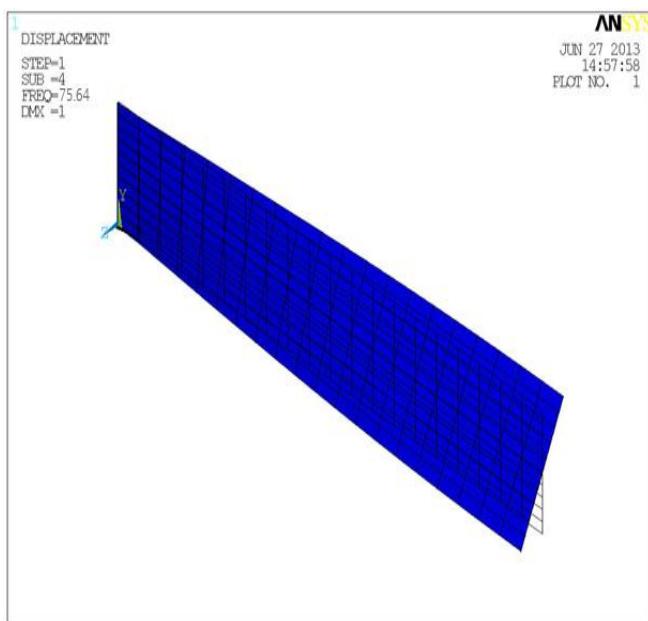
b) Second mode frequency of a cantilever plate with [30/-30] angle ply laminate



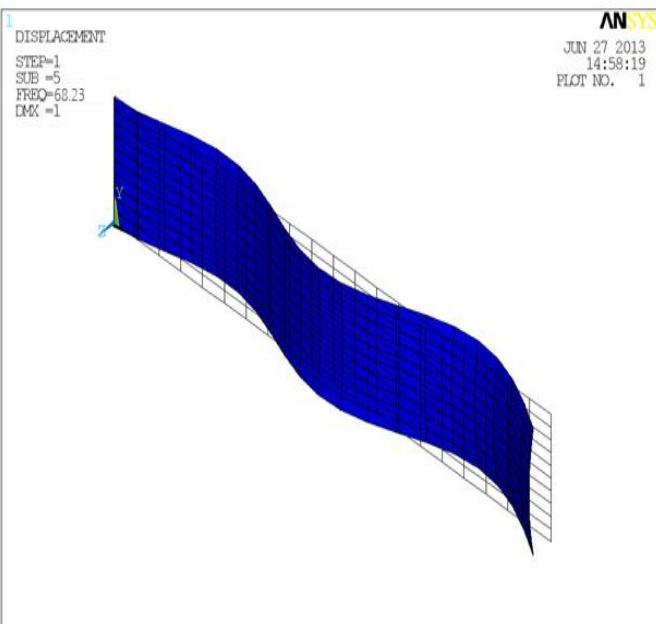
a) First mode frequency of a cantilever plate with [30/-30] angle ply laminate



c) third mode frequency of a cantilever plate with [30/-30] angle ply laminate



d) fourth mode frequency of a cantilever plate with [30/-30] angle ply laminate

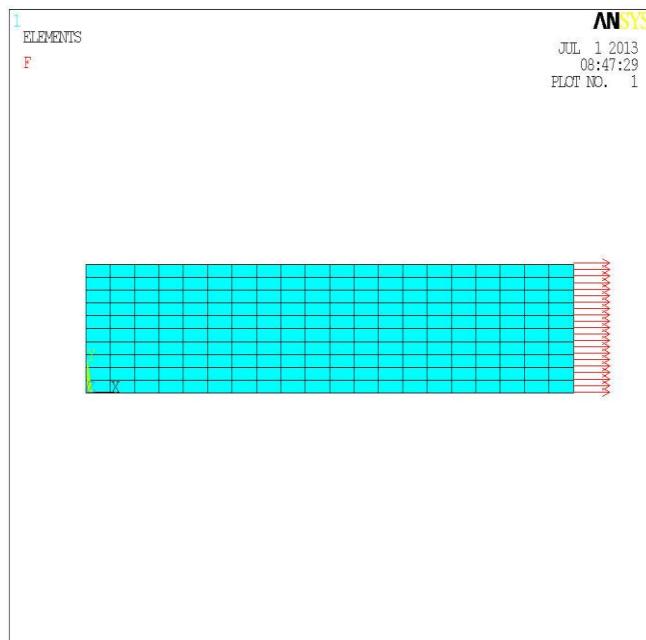


e) fifth mode frequency of a cantilever plate with [30/-30] angle ply laminate

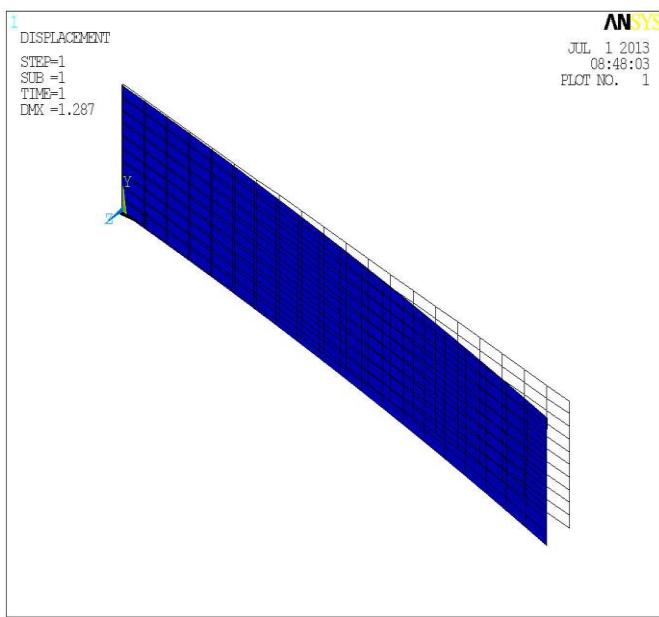
Delamination length	Case1 (frequency) Hz	Case2 (frequency) Hz	Case3 (frequency) Hz
0	82.16	83.2	85.26
0.0254	78.16	80.00	83.41
0.0508	72.16	75.28	79.81
0.0762	68.05	70.89	75.64
0.1016	60.02	63.45	68.23

Among the above three cases, composite plates with (0/90) angle ply is weaker compared to composite plates with (45/-45), (30/-30) angle ply.

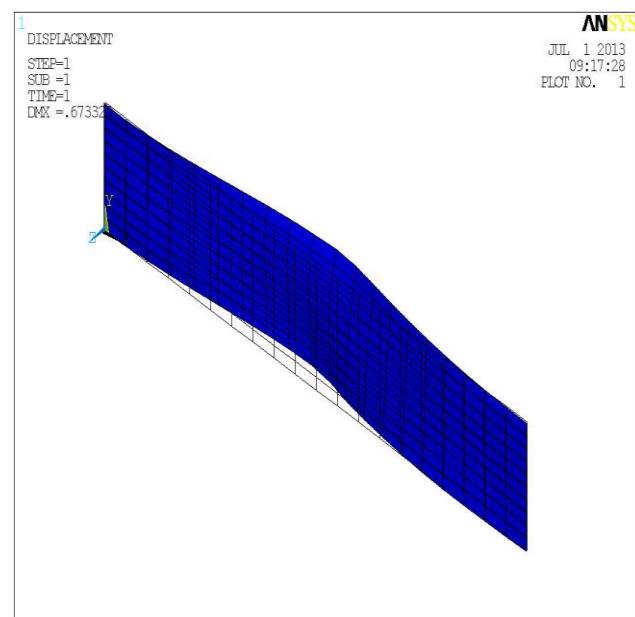
5.1.5 Deformation of the composite plate during tensile test and bending test at the mid plane of the plate:



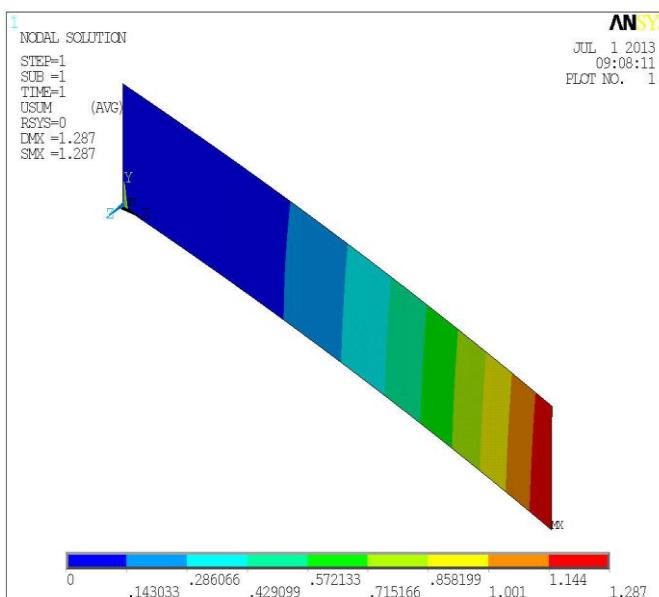
Mesh model of a composite plate when tensile load is applied.



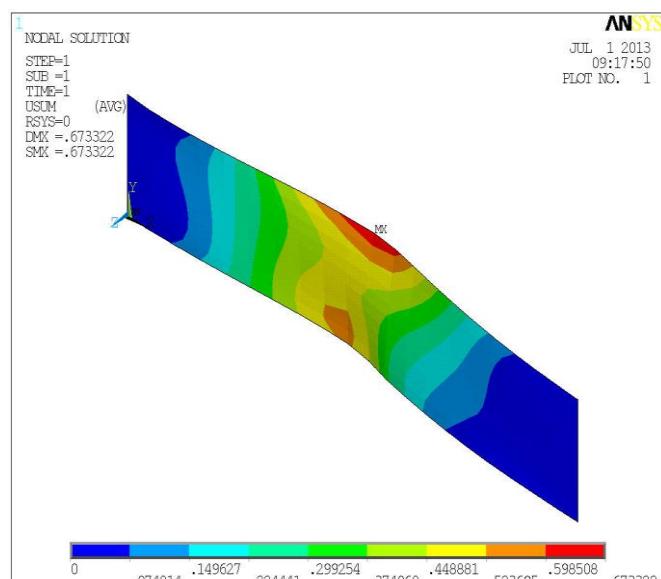
a) mode shape of a composite plate showing deformation when load is applied



c) mode shape of a composite plate when a load is applied at the mid-plane of the plate during bending test.



b) Mode shape of a composite plate in vector sum representation.



d) mode shape of a composite plate in vector sum representation.(Bending test)

Tabular form showing Deformation of a composite plate using Tensile & Bending test :

Tensile load(N/cm ²)	Deformation(mm)	Bending load(N/cm ²)	Deformation(mm)
1960	1.287	500	0.6733

VI.CONCLUSION AND FUTURE SCOPE

6.1 Conclusions

The following conclusion is drawn from present work.

1. A finite element method for the free vibration analysis of laminated composite plates with arbitrarily located single delamination is developed based on a simple delamination model. The present results compare well with those existing in the literature, numerical results and associated plots show how the increase on the number and sizes of delaminations and other parameters influence the first natural frequency and dynamic response of delaminated composite plates.

i).The increase in the number and sizes of delaminations has, in general, a deteriorating effect on the plate dynamic stiffnesses.

ii).The (0/90) laminates are weaker compared to angle-ply laminates of (30/-30), (45/-45).

6.2 Future scope

Analysis can be carried out on different composites materials with different stacking sequences and boundary conditions and arbitrarily located multiple delaminations. Further analysis can be done for prediction of delamination stresses. Multi layer theory and finite element method can be used to analyze delamination stress concentrations in composite materials.

REFERENCES

- [1] M.H.H. Shen, J.E. Grady, "Free vibrations of delaminated beams", AIAA Journal 30,1992, 1361–1370.
- [2] D.Shu, H.Fan and Della, "Free vibration of a bimaterial split beam", Composites, Part B 27, 1996 79–84.
- [3] Gim, "To Analyze a two dimensional single delamination with multiple constraint condition to find the strain energy release rate for the element". J Sound Vib 1982, 125(3),441–61.
- [4] Parhi P K, Bhattacharyya S K and Sinha P K,"Dynamics analysis of composite plate", J. Reinf. Plast. Compos, 1999.
- [5] Reddy, J.N. "Dynamic (Transient) Response of Laminated Anisotropic Composite Material Plates," Int, J.Numer.Meths.in Engg., 1983.
- [6] Memaris C. and J.D.Day, "Dynamic Response of Laminated Anisotropic Plates" comput.Struct. , 1995.
- [7] Ramkumar RL, Kulkarni SV, Pipes RB,"Free vibration frequencies of a delaminated beam",34th Annual Technical Conference Proceedings, Reinforced/Composite Institute, Society of Plastics Industry, Section 22-E, 1979. p. 1–5.
- [8] Wang JTS, Liu YY, Gibby JA. "Vibration of split beams", J Sound Vib 1982, 84(4),491–502.
- [9] Mujumdar PM, Suryanarayanan S," Flexural vibrations of beams with delaminations" J Sound Vib, 1982, 125(3),441–61.
- [10] Tracy JJ, Pardoen GC,"Effect of delamination on the natural frequencies of composite laminates", Compos Mater 1989, 23(12), 1200–15.
- [11] Hu JS, Hwu C. "Free vibration of delaminated composite sandwich beams". AIAA J 1995, 33(10), 1911–8.
- [12] Luo H, Hanagud S. "Dynamics of delaminated beams". Int J Solids Struct 2000, 37(10),1501–19.
- [13] Saravacos DA, Hopkins DA. "Effects of delaminations on the damped characteristics of composite laminates analytical and experiments", J Sound Vib 1996, 192(5)977–93.
- [14] Zak A, Krawczuk M, Ostachowicz W,"Numerical and experimental investigation of free vibration of multilayer delaminated composite beams and plates". Comp Mech 2000, 26(3), 309–15.
- [15] Lestari W, Hanagud S, "Health monitoring of structures: multiple delamination dynamics in composite beams". Proceedings of the 40th AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics and materials conference and adaptive structures forum, St Louis, MO, April 1999.
- [16] Lee S, Park T, Voyatzis GZ, "Vibration analysis of multi-delaminated beams". Compos, Part B 2003, 34(7),647–59.
- [17] Ju F, Lee HP, Lee KH," Finite element analysis of free vibration of delaminated composite plates". Compos Eng 1995, 5(2), 195–209.
- [18] Kim SH, Chattopadhyay A, Ghoshal A, "Characterization of delamination effect on composite laminates using a new generalized layerwise approach". Comp Struc 2003, 81(15),1555–66.
- [19] Tenek LH, Hennekke II EG, Gunzburger MD," Vibration of delaminated composite plates and some applications to nondestructive testing". Compos Struct 1993, 23(3), 253–62.