









Where  $\mu_0$  is permeability of free-space ( $\mu_0 = 4 \times 10^{-7} \text{H/m}$ ),  $A_a$  is the cross-section of the air-gap which is assumed to be equal to pole face area,  $N$  is the number of turns per pole,  $G$  is a nominal air gap between stator and rotor,  $\alpha$  is half the included angle of a pole ( $\alpha = 22.5^\circ$ ),  $I_b$  is bias current which is kept fixed and  $i_n$  ( $n = 1, 2, 3$  and  $4$ ) are small variable current of corresponding poles which are varied according to control strategy. For the sake of simplicity currents in the diametrically opposite poles are kept equal in magnitude, but opposite in direction, i.e.,  $i_1 = -i_3$  and  $i_2 = -i_4$ . In the actuator design, the allowable displacement of the rotor is considered about one-tenth of the air gap size. At normal point of operation,  $i_1 = i_2 = i_3 = i_4 = 0$  and  $x = y = 0$ . By expanding with Taylor series and neglecting higher order terms, we get:

$$F_{ax} = \frac{4\mu_0 A_g N^2 I_b \cos \alpha}{G^2} i_1 + \frac{4\mu_0 A_g N^2 I_b^2 \cos^2 \alpha}{G^3} x = k_{1a} i_1 + k_{2a} x \quad (8a)$$

$$F_{ay} = \frac{4\mu_0 A_g N^2 I_b \cos \alpha}{G^2} i_2 + \frac{4\mu_0 A_g N^2 I_b^2 \cos^2 \alpha}{G^3} y = k_{1a} i_2 + k_{2a} y \quad (8b)$$

Here,  $k_{1a}$  and  $k_{2a}$  are force-current and force-displacement constants of electromagnetic actuator respectively. Computation of currents  $i_1$  and  $i_2$  as a function of instantaneous errors is the function of controller.

### 3.1 Design of PD Controller

To minimize the vibration amplitude of system appropriate magnetic force should applied by the actuator; force should act in the opposite direction of motion and should increase with displacement. As seen from above simplified equations, the magnetic force depends upon the pole winding current of an electromagnet; so by controlling pole winding currents, the system response can be controlled. Proportional-Derivative (PD) control is generally used for faster response. Proportional control increase the control input in proportion to the error  $e(t)$  within the acceptable range of error. Derivative control changes the control input in proportion to rate of change of error  $\dot{e}$ . Equation of control output of PD controller using proportional and derivative gains  $K_p$ ,  $K_d$  can be given as

$$u(t) = K_p e(t) + K_d \dot{e}(t) \quad (9)$$

The controlled output is currents and the error signal. Hence, the control currents for pole pair in horizontal direction ( $i_1$ ) and in the vertical direction ( $i_2$ ) are given as [12]:

$$i_1 = K_p e_x(t) + K_d \dot{e}_x(t) \quad (10 a) \quad i_2 =$$

$$K_p e_y(t) + K_d \dot{e}_y(t) \quad (10 b)$$

As the set point is having zero velocity and displacement, the terms  $e_x(t) = -x$  and  $e_y(t) = -y$  along with  $\dot{e}_x(t) = -\dot{x}$  and  $\dot{e}_y(t) = -\dot{y}$ . Designer has to select the appropriate gain constants  $K_p$  and  $K_d$  and obtain the control forces  $F_{ax}$  and  $F_{ay}$ .

### 3.2 Neural network based controller

As seen from Eq.(7), it is observed that the control forces are functions of the velocities of rotor and its radial displacements. The control function is non linear in nature. So, a control concept similar to an inverse dynamic model of the system is used. A nonlinear controller based on trained neural network model with instantaneous transient response as inputs and corresponding currents as outputs is proposed in present work. Fig.5 shows the block diagram of the control methodology.

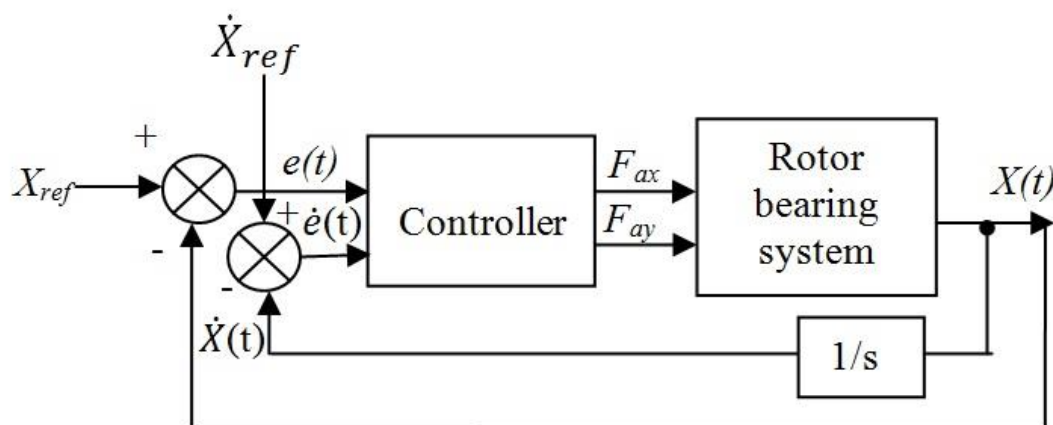


Fig. 5. Error based control scheme.

Conventional 3-layer back propagation neural network is used to train the data. The inputs for the neural network are errors (vector of negative  $X(t)$  values) and error rates (vector of negative  $\dot{X}(t)$  values) over a sampling period as the values

of  $i_1$  and  $i_2$  are varied randomly from 0 to 100 milli-amperes according to equations (7). A 4 input ( $-x$ ,  $-y$ ,  $-\dot{x}$  and  $-\dot{y}$ ) and two output ( $i_1$ ,  $i_2$ ) training patterns generated inversely have been used in training. Learning rate of 0.4 and momentum

factor of 0.01 are implemented. Optimum hidden layer nodes were found to be 4 and the neural network is trained using an interactive computer program to find the set of weight matrices. Further, these are employed in estimating the unknown currents corresponding to a computed nodal displacement and velocity vectors during every time step. The detailed backpropagation algorithm for 3-layer multilayer perceptron network can be found in open literature [34].

#### 4 RESULTS AND DISCUSSIONS

The coupled set of differential equations is evaluated in time-domain series with aid of Runge-Kutta equation of order four with initial value as zero. The dimensional

properties of the rotor used in the analysis shown in Table 1. A speed ratio of 1.5 is implemented between HP and LP rotors. The material for the spools is steel with modulus of elasticity value  $E=210$  GPa and density  $\rho=7.8$  g/cm<sup>3</sup>. All four disks have polar moment of inertia ( $I_P$ ) twice the diametric mass moment of inertia ( $I_D$ ). Further, an eccentricity of 10 microns is considered in the disks. The resultant stiffness, mass, damping (damping factor of 0.01) and gyroscopic matrices are used. The SFD parameters used are [35]:  $\mu=5 \times 10^{-3}$  PaS,  $c=2 \times 10^{-4}$  m,  $L=8.3 \times 10^{-3}$  m,  $R_1=0.03$  m. The value of  $\omega$  is set as 2000 rpm and angular acceleration  $\alpha$  is varied.

Table I Geometric and material data for rotor (Referring to Fig.2)

Node	Axial dist.(mm)	d <sub>outer</sub> (mm)	d <sub>inner</sub> (mm)	Disc mass(kg)	I <sub>p</sub> (kg-mm <sup>2</sup> )
1	0	30.4	0	-	-
2	76.2	30.4	0	4.904	0.02712
3	323.85	30.4	0	-	-
4	406.4	30.4	0	-	-
5	457.2	30.4	0	4.203	0.02034
6	508	30.4	0	-	-
7	152.4	50.8	38.1	-	-
8	203.2	50.8	38.1	3.327	0.01469
9	355.6	50.8	38.1	2.227	0.0972
10	406.4	50.8	38.1	-	-

Fig.6 depicts the frequency response plot of a system considering ball bearing forces and not considering the forces. Fig.6(a) coincides well with the available results. It is seen that there are multiple ball passing frequencies (VC) along with harmonics in the system, when ball bearing forces are accounted.

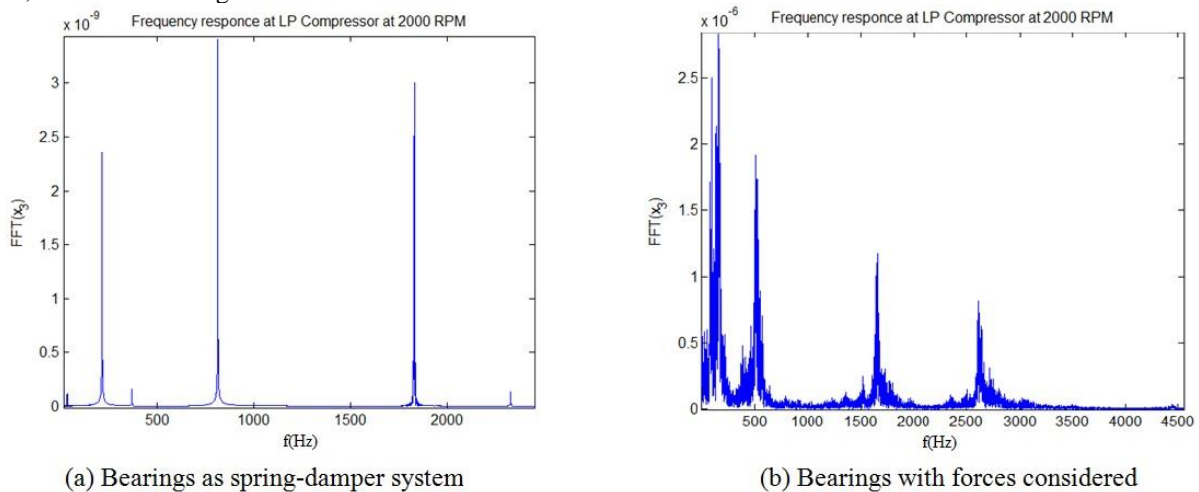


Fig. 6. Frequency plots of rotor at speed of 2000 rpm.

Fig.7 shows the transient analysis results at different acceleration conditions in terms of frequency responses. It is obvious that the multiplicities of ball passing frequencies increase with acceleration of rotor. This may be due to a randomly changing bearing excitation.

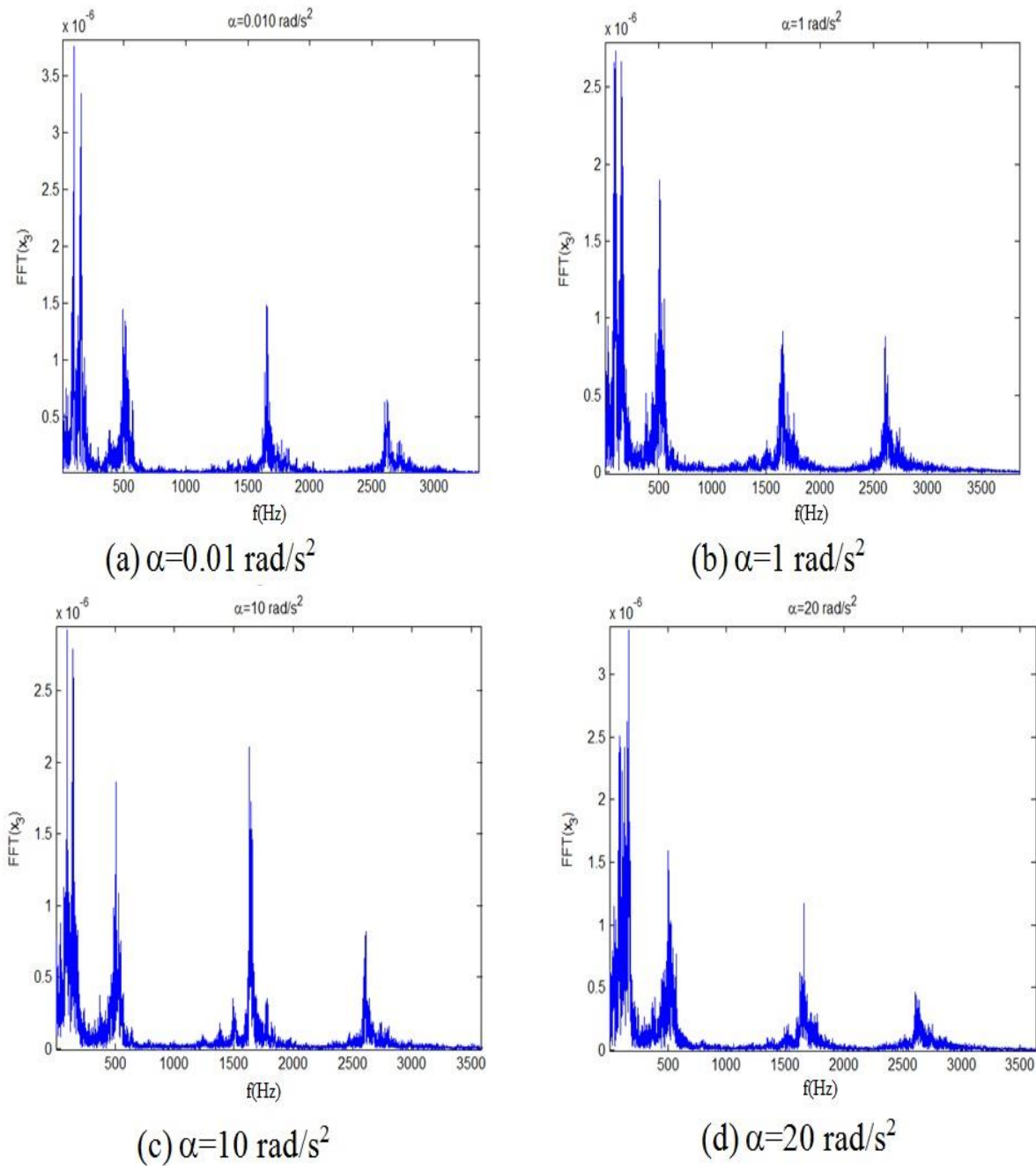


Fig. 7. Effect of acceleration of rotor at LP compressor disk.

In order to minimize the amplitude levels, the following electromagnetic actuator parameters are considered:  $N=106$ ,  $G=9 \text{ cm}$ ,  $A_g=2.4 \text{ cm}^2$ ,  $\alpha=22.5^\circ$ , and  $I_b=3 \text{ A}$ . The instantaneous current components  $i_1$  and  $i_2$  depend on the displacements  $x$  and  $y$  at the disk node-1. The software code is modified to incorporate an extra component of magnetic force on the force vector at right side located at disk-1 Fig.8 shows the frequency response plots of rotor computed when controlled with bearing force and when uncontrolled with force. It indicates that the amplitudes from second mode have reduced drastically and the ball passing frequency amplitudes have been dropped. The methodology can be extended for other nodes of interest.

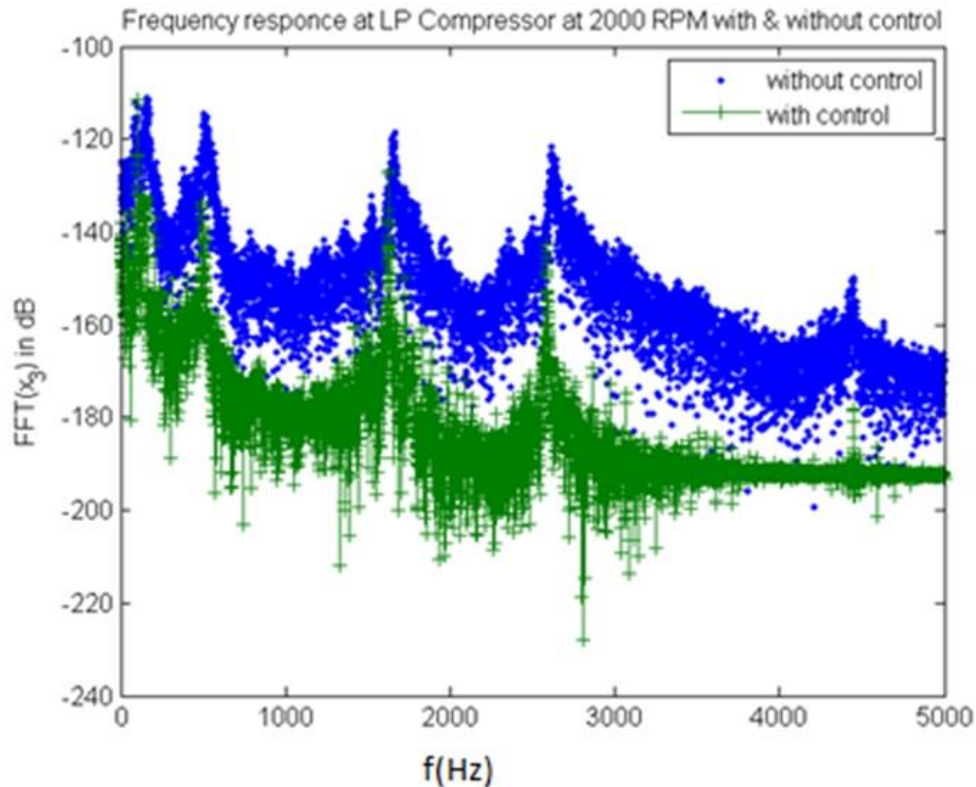


Fig. 8. Amplitude reduction at disk-1 node with EM controller.

## 5 CONCLUSIONS

In current study, a process to reduce vibration with electromagnetic actuators has been briefly presented for a twin-spool rotor case study. The considered system has unbalance and gravity loading along under accelerating conditions. Gyroscopic effects arising due to combined effect of the shaft and disks were accounted, and for all disks unbalance was taken into consideration. Frequency response diagrams were used to distinguish the dynamic response at different angular accelerations of the rotor. It was observed that the squeeze-film damper at the front and rear ends of LP rotor have drastically reduced the amplitudes at critical speeds of operation. Further, the neural-network based electromagnetic actuator designed to provide the nonlinear control forces has considerably reduced the response. Unlike the conventional PD control scheme, the present approach, do not search for optimum set of controller gains. However, it requires huge set of input-output training data before its implementation in the plant. Although the results were illustrated for a selected speed of LP rotor, the focus may be given at the critical speeds of operation in practice so as to identify the effective operational speed range for amplitude attenuation using EM actuator.

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