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Dominating Functions in Semigraphs

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Abstract:- **Let $G = (V, X)$ be a semigraph. A function $f_a: V(G) \to \{-1, 1\}$ and $w_a(f) = \sum_{v \in V} f_a(v)$ then the function f_a is a signed adjacent dominating function for the semigraph G if for every vertex $v \in V, f_a[v] \geq 1$. The signed adjacent domination number of a semigraph G denoted by $\gamma_{sa}(G)$ is the minimum weight of a signed adjacent dominating function on G. In this paper, we study the properties of signed adjacent dominating function for a class of semigraphs and present their signed adjacenct domination number.**

Keywords— Dendroids, Semigraphs, Star, Strongly Complete semigraph

I. INTRODUCTION

The works on "Semigraphs" by E. Sampathkumar [12] introduced the concept of semigraph, which has given scope for emerging trends in the feild of Graph Theory. Domination in Semigraphs has many practical applications such as providing a city with minimum number of security officers, possible light arrangements in the offices, etc. defined the terms such as open adjacent neighbor set, open consecutive adjacent neighbour set, closed adjacent neighbour set, closed consecutive adjacent neighbour set, adjacency domination number and consecutive adjacency domination number of a semigraph. Various parameters of domination in semigraphs such as adjacency domination number, consecutive adjacency domination number was introduced by S.S.Kamath and R.S.Bhat [9]. Strong and weak domination was introduced by S.S.Kamath and Saroja R. Hebbar [10]. S.Gomathi [6] introduced (m,e)-strong domination in semigraphs. Xa-dominating set, Ya-dominating set, Hyperdomination number work was contributed by Y.B.Venkatkrishnan and V. Swaminathan [14].

In this paper we define signed adjacent dominating function (SADF), signed adjacency domination number (SADN), signed consecutive adjacent dominating function (SCADF) and signed consecutive adjacenct domination number (SCADN) for semigraph. Also find the signed adjacenct domination number for star semigraph, strongly complete semigraphs**.**

II. PRELIMINARIES

Definition 2.1 [12] : Semigraph

A *semigraph* G is a pair $= (V, X)$ where V is a nonempty set whose elements are called vertices of G, and X is a set of ordered n-tuples, called edges of G denoted by $X = (E_1, E_2, \ldots\ldots, E_n)$ of distinct vertices, for various $n \geq 2$, satisfying the following conditions: (i) Any two edges have atmost one vertex in common (ii) Two edges $(u_1, u_2, \ldots\ldots\ldots, u_n)$ and $(v_1, v_2, \ldots\ldots\ldots, v_m)$ are considered to be equal if and only if (a) $m = n$ and (b)Either $u_i = v_i$ for $1 \leq i \leq n$, or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$. Thus, the edge $E = (u_1, u_2, \ldots\ \ldots\ldots, u_m)$ is the same as $(u_m, u_{m-1}, \ldots\ldots\ldots, u_1)$. The vertices u_1 and u_m are the end vertices of E, while $u_2, u_3, \ldots\ldots\ldots, u_{n-1}$ are called the middle vertices of E.

Definition 2.2 [12] : Subedge

A *subedge* of an edge E = $(v_1, v_2, \ldots\ldots\ldots, v_n)$ is a *k*-tuple E'= $\left(v_{i_1}, v_{i_2}, \ldots\ldots\ldots, v_{i_k}\right)$, where $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ or $1 \leq i_k < i_{k-1} < \cdots < i_1 \leq n$.

Definition 2.2 [12] : Partial edge

A *partial edge* of E is a $(j - i + 1)$-tuple $E(v_i, v_j) = (v_i, v_{i+1}, \ldots\ldots\ldots, v_j)$, where $1 \leq i \leq n$. Thus a subedge E' of an edge E is a partial edge if and only if, any two consecutive vertices in E are also consecutive vertices of E.

Example 2.3:

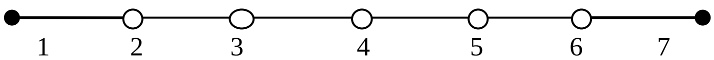

Figure 1

E=(1,2,3,4,5,6,7) is an edge which contains the middle vertices (2,3,4,5,6) and (1,7) as the endvertices. Here *E*$_1$={1,3,5,7} is a subedge and *E*$_2$={3,4,5,6,7} is a partial edge.

Definition 2.4 [12] : fs-edge and fp-edge

fs-edge is an edge which is either a full edge or a subedge and *fp-edge* is an edge which is either a full edge or a partial edge.

Example 2.5: Let $G = (V, X)$ be a semigraph where V=$\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $X = \{(v_1, v_2, v_3), (v_1, v_6, v_5, v_4), (v_2, v_4), (v_3, v_4), (v_3, v_6)\}$ as shown in the Figure 2.

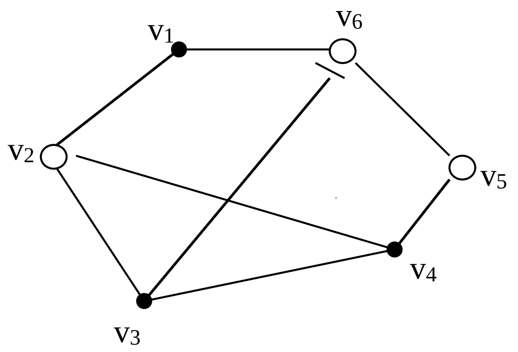

Figure 2: Semigraph G

Let G = (V, X) be a semigraph and E=$(v_1, v_2, \dots \dots \dots, v_n)$ be an edge of G. Then v_1 and v_n are the *end vertices* of E and v_i, $2 \le i \le n-1$ are the *middle vertices* (or *m*-vertices) of E. A vertex v is an *end vertex* of G if it appears only as an *end vertex*. A vertex v is a *middle vertex* of G if it appears only as a *middle vertex.* A vertex v is called *middle-cum-end*((*m*,*e*)) vertex if it is a *middle vertex* of some edge and an *end vertex* of some other edge. Two vertices are adjacent if both of them belong to an edge, and two edges are adjacent if they have a common vertex.

Definition 2.6 [12] : End vertex graph

The end vertex graph denoted by *G*e is a graph in which two vertices in *G*e are adjacent if and only if, they are end vertices of an edge in *G*.

Definition 2.7 [12] : Adjacency graph

The adjacency graph denoted by *G*a is a graph in which two vertices in *G*a are adjacent if and only if, they are adjacent in G.

Definition 2.8 [12] : *Consecutive adjacency graph*

*The consecutive adjacency graph G*ca is a graph in which two vertices in *G*ca are adjacent if and only if, they are consecutively adjacent vertices in *G*.

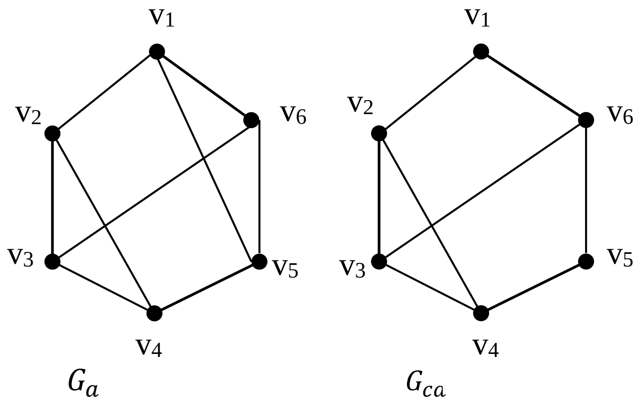

Figure 3: Adjacency graph and Consecutive adjacency graph

Definition 2.9 [12] : Dendroid

A star is a *dendroid* in which all edges have a common vertex.

Definition 2.10 [12] : Pendant vertex and pendant edge

A vertex v in a semigraph G is a *pendant vertex* if $\deg v = deg_e\, v = 1$. An edge E containing a pendant vertex is a *pendant edge*.

Definition 2.11 [12] : Complete and Strongly complete Semigraph

A semigraph G is *complete* if any two vertices in G are adjacent, and *strongly complete* if G is complete and every vertex in G appears as an end vertex of an edge.

Definition 2.12 [9]: Open and Closed Neighbour set

The set $N_a(v) = \{x \in V /\, x\ is\ adjacent\ to\ v\}$ is the *open adjacent neighbour set* and $N_a[v] = N_a(v) \cup \{v\}$ is the closed adjacent neighbour set. Also, $N_{ca}(v) = \{x \in V /\, x\ is\ consecutive\ adjacent\ to\ v\}$ is the consecutive adjacent neighbour set and $N_{ca}[v] = N_{ca}(v) \cup \{v\}$ is the *closed consecutive adjacent neighbour* set.

Definition 2.13 [9] : Adjacent and consecutive adjacent dominating set

A set $D \subseteq V$ is said to be an *adjacent dominating set* if for every $v \in V - D$, there exists $u \in D$ such that u is adjacent to v in G and D is a *consecutive adjacent dominating set* if for every $v \in V - D$, there exists $u \in D$ such that u is consecutively adjacent to v in G.

Definition 2.14 [9] : Adjacent and consecutive adjacent domination number

The adjacent domination number $\gamma_a(G)$ is defined as the minimum cardinality of an adjacent dominating set and the *consecutive adjacent domination number* $\gamma_{ca}(G)$ is the minimum cardinality of a consecutive adjacent dominating [9].

Proposition 2.15 [9] : For any semigraph G, $\gamma_a(G) = \gamma(G_a)$ and $\gamma_{ca}(G) = \gamma(G_{ca})$

III. MAIN RESULTS

Definition 3.1: Signed adjacent dominating function

Let $G = (V, X)$ be a semigraph. A function $f_a: V \to \{-1,1\}$ is a signed adjacent dominating function (SADF) if $f_a[v] \geq 1, \forall\ v \in V$ where $f_a[v] = \sum_{x \in N_a[v]} f(x)$.

Definition 3.2: Signed adjacent domination number

The *signed adjacent domination number* (SADN) of a semigraph G is defined by $\gamma_{sa}(G) = \min\{ f_a(v)/f_a$ is a signed adjacent dominating function on $G\}$

Definition 3.3: Signed Consecutive adjacent dominating function

Let $G = (V, X)$ be a semigraph. A function $f_{ca}: V \to \{-1,1\}$ is a *signed consecutive adjacent dominating function* (SCADF) if $f_{ca}[v] \geq 1, \ \forall\ v \in V$ where $f_{ca}[v] = \sum_{x \in N_{ca[v]}} f(x)$.

Definition 3.4: Signed consecutive adjacent domination number

The *signed consecutive adjacent domination number* (SCADN) of a semigraph G is defined by $\gamma_{sca}(G) = \min\{ f_{ca}(v)/f_{ca}$ is a signed consecutive adjacent dominating function on $G\}$

As a consequence of the above definitions **3.3** and **3.4** we have the following result.

Proposition 3.5: Signed consecutive adjacent domination number of a semigraph G is equal to the signed domination number of consecutive adjacency graph i,e. $\gamma_{sca}(G) = \gamma_s(G_{ca})$.

Proposition 3.6 : If T is a star then $\gamma_{sa}(T) = 1$.

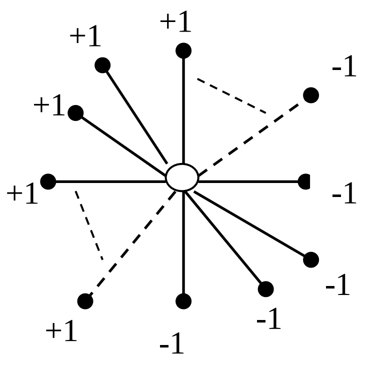

Figure 4

Proof: Let T be a star of order n. By the definition of a star [12], it contains a middle vertex x which is common to the rest of the end vertices. Let $\frac{n}{2}$ end vertices be assigned with +1 and the remaining $\frac{n}{2}$ end vertices with -1. The only vertex left to be assigned with either +1 or -1 is the middle vertex x. If $f(x) = -1$, then $f_a[v] < 1$.Hence$f(x) = +1$. Therefore, we have $\gamma_{sa}(T) = \sum f_a(v) = 1$.

Definition 3.7: (m, e)-pendant edge

A pendant edge of cardinality two containing a (m, e)-vertex and a pendant vertex is called a (m, e)-pendant edge.

Remark 3.8: A star containing an (m, e)-pendant edge is denoted by T^*.

Proposition 3.9: For a star T^* containing an (m, e)-pendant edge, the signed adjacent domination number $\gamma_{sa}(G) = k + 1$, where k is the number of (m, e)-pendant edges.

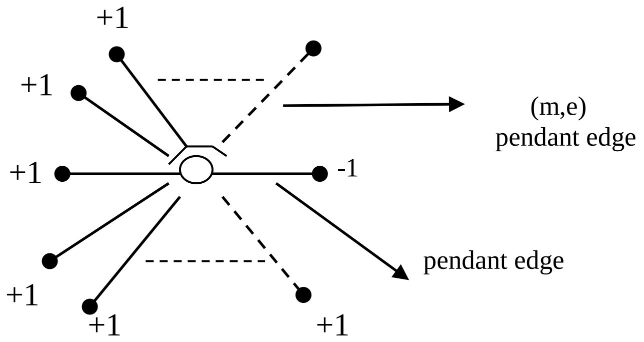

Figure 5

Proof: Let T^* be a star containing atleast one (m, e)-pendant edge. The end vertex and the (m, e)-vertex of the pendant edge is assigned with +1, since the satisfying the definition of SADF. The remaining end verticex is assigned with -1. As such if we consider k number of (m, e)-pendant edges, each contributes 1 to the SADN. Hence such k number of (m, e)-pendant edges along with the remaining pendant edge gives $\gamma_{sa}(G) = k + 1$.

Proposition 3.10: If $G \equiv K_n^{n-1}$ is a strongly complete semigraph of order $n \geq 3$, then

$$\gamma_{sa}(G) = \begin{cases} 1, & \text{if } n \text{ is odd} \\ 2, & \text{if } n \text{ is even} \end{cases}$$

Proof: Let K_n^{n-1} be a strongly complete semigraph of order n. The $n - 3$ end vertices are assigned with +1. To produce the SADF of weight 1, we assign the remaining $\left\lceil \frac{n-3}{2} \right\rceil$ (m, e) vertices with -1 and $\left\lfloor \frac{n-3}{2} \right\rfloor$ (m, e) vertices with +1.
We have the following two cases:
(i) For n= even number we have

$$3 + \left\lceil \frac{n-3}{2} \right\rceil - \left\lfloor \frac{n-3}{2} \right\rfloor = 2$$

(ii) For n= odd number we have

$$3 + \left\lceil \frac{n-3}{2} \right\rceil - \left\lfloor \frac{n-3}{2} \right\rfloor = 1$$

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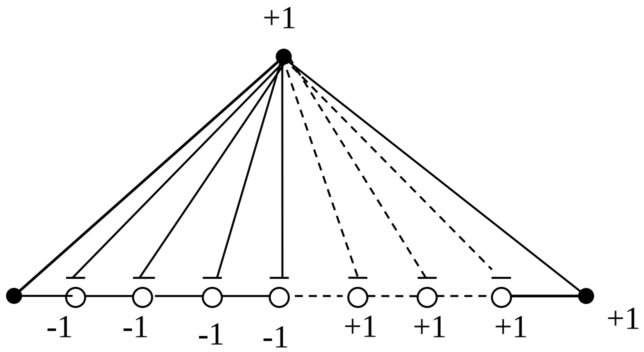

Figure 6: Strongly complete semigraphs

Observations:

Case(i): Let the end vertices be assigned with +1 and the (m,e)-vertices with -1.

For, $n = 4$ and $n = 5$ the above result holds good.

For, $n > 5$ the result does not hold good.

Case(ii): Let the end vertices be assigned with -1 and (m,e)-vertices with +1.

For, $n = 4$ and $n = 5$ the above result is not true.

For, $n > 5$ the result is true.

IV. CONCLUSION

The concept of dominating functions in semigraphs was defined. The signed adjacency domination number was calculated for various classes of semigraphs. We introduced signed consecutive adjacency dominating functions in semigraphs. Another classification of star containing an (m,e) pendant edge was given in this paper. In semigraphs, introducing algorithmic aspects will enrich its existing properties. Further this paper provides scope to establish relationship between Matching and Signed adjacency domination number of semigraphs.

REFERENCES

[1] B.Y.Bam, N.S.Bhave, *On Some problems of Graph Theory in Semigraphs*, Ph.D Thesis, University of Pune.

[2] J.E.Dunbar, S.T. Hedetniemi, M.A.Henning and P.J. Slater, *Signed Domination in Graphs. Graph Theory, Combinatorics, and Applications*, Jhon Wiley & Sons, Inc. 1 (1995) 311-312

[3] J.E.Dunbar, S.T. Hedetniemi, M.A.Henning and A.A. McRae, *Minus Domination in Regular Graphs*. Discrete Math. 49 (1996) 311-312

[4] C.M.Deshpande and Y.S.Gaidhani, Adjacency Matrix of Semigraphs. International Journal of Applied Phy. And Math. (2012), 250-252

[5] D.K.Thakkar and A.A.Prajapati, Vertex covering and independence in semigraph, *Annals of Pure and Applied Mathematics*, 4(2) (2013) 172-181.

[6] S.Gomathi, R.Sundareswaran and V.Swaminathan, *(m,e)-domination in Semigraphs*, Electronic Notes in Discrete Mathematics 33 (2009) 75-80

[7] T.W.Haynes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs* (Advanced Topics), Marcel Dekker Inc; New York

[8] P.R.Hampiholi and J.P.Kitturkar, Strong Circuit matrix and Strong Path matrix, *Annals of Pure and Applied Mathematics*, *Vol. 10, No. 2, 2015, 247- 254*

[9] S.S. Kamath and R.S.Bhat, *Domination in Semigraphs*, Electronic notes in Discrete Mathematics 15, 2003, pp. 112.

[10] S.S. Kamath and Saroja R. Hebbar, *Strong and Weak Domination, Full Sets and Domination Balance in Semigraphs*, Electronic notes in Discrete Mathematics 15, 2003, pp. 106-111.

[11] Surajit Kr. Nath and P.Das, *Matching in Semigraphs*, International Journal of computer Application, Issue 3, Volume 6 (November-December 2013).

[12] E. Sampathkumar, *Semigraphs and Their Applications*, Report on the DST Project, May 2000

[13] Y.B.Venkatkrishnan and V.Swaminathan, *Bipartite theory of Semigraphs*, WSEAS Transactions of Mathematics, 11(1), 2012, pp. 1-9

[14] Y.B.Venkatkrishnan and V.Swaminathan, *Hyper domination in Bipartite Semigraphs*, WSEAS Transactions of mathematics, volume 11, 866-875, October 2012.