# Displacement Feedback for Active Vibration Control of Smart Cantilever Beam

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**Abstract --Considerable work has been carried out in active vibration control of flexible smart structures. In the present study, active vibration control of a flexible aluminium cantilever beam using piezoelectric patches as sensor and actuator is investigated. A linear mathematical model is developed to predict the dynamics of a smart beam using system identification technique. The same model is used to design and simulate displacement feedback control law without any physical displacement sensor. The proposed feedback control algorithms is analyzed and implemented in real-time using National Instruments PXIe-1082 controller in LabVIEW graphical programming environment. The control law has demonstrated significant vibration suppression of the smart beam. It is observed that with this feedback law 65.23% and 70.27% reduction are obtained for first and second modes of vibration respectively and the experimental results show good correlations with simulations.**

***keywords;*** ***smart beam, system identification, active vibration control, and displacement feedback***

## I. INTRODUCTION

In the past couple of decades, different active vibration control techniques have been investigated to improve the performance and life of structures. Among those strategies the classical control techniques showed a good performance in controlling the unwanted vibration. S. M. Khot et al.[1], carried out PID based output feedback for active vibration control of cantilever beam using a reduced model extracted from a full (ANSYS) Finite Element model. T. C. Manjunath et al. [2], employed a robust decentralized controller for a multimode smart flexible system using a periodic output feedback control technique when there is a failure of one of the piezoelectric actuator. Manning et. [3], presented vibration control scheme of a smart structure using system identification and pole placement technique. System Identification technique and pole placement method are used to derive the mathematical model and to suppress the vibration respectively by Xiongzhu Bu et al. [4]. Similarly, Xing-Jian Dong et al. [5] presented a System Identification technique based on measured input and output data of the smart plate using observer/ Kalman filter identification technique in numerical simulation and experimental study for active vibration control of smart plate using the Linear Quadratic Gaussian (LQG) control algorithm. The effectiveness of Direct Strain feedback on suppression of undesired vibration of a beam structure is investigated recently by Riessom W. et al. [6] using a model derived from System identification technique. Fanson et. al. [7] demonstrated active vibration control of a beam with piezoelectric patches using positive position feedback. Fei J. [8] investigated both strain feedback and optimized PID compensator methods for active vibration control of cantilever beam bonded piezoelectric actuators. Shan J. et al. [9] analyzed and experimentally demonstrated PPF controller for suppressing multi-mode vibrations while slewing the single-link flexible manipulator. Moreover, the experimental robustness of PPF is studied for active vibration suppression of flexible smart structure by Song et al.[10].

The aim of the present study is to demonstrate and evaluate the performance of displacement feedback control law without a physical displacement sensor as applied to a smart beam to suppress unwanted vibration.

## II. MATHEMATICAL MODELING

The frequency equation of a continuous system like a beam structure is a transcendental equation that yields an infinite number of natural frequencies and normal modes. The dominant modes of beam vibration are identified experimentally using an Impact hammer test as shown in Figure1. The result shows that the first two modes are significantly dominant over the remaining higher mode frequencies in terms of their tip displacement amplitude. Hence, the first two dominant frequencies that relatively produce higher amplitude of vibration in the beam are only considered for the present study.

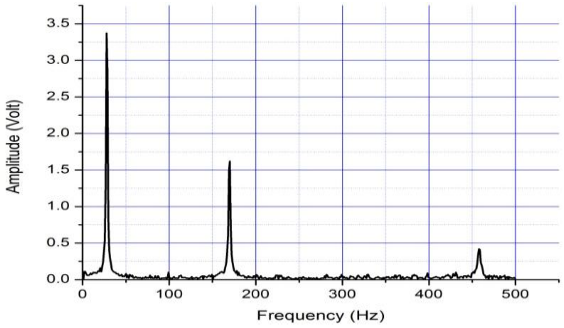


Fig.1 Frequency Response Function plot (Impact Hammer Test)

In general, the equation of motion of a smart beam can be expressed in lumped form as follows:

$$m\ddot{x} + c\dot{x} + kx = f_{ext} + f_c \qquad (1)$$

where m is the mass, $c$ is the structural damping, $k$ is the stiffness, $f_{ext}$ is the external disturbance force vector, $f_c$ and is the control force vector. Mathematical modeling of the system can be approached in two ways. One way to achieve the mathematical model in the form presented above is the utilization of Finite Element modeling technique. However, due to incomplete knowledge of the system dynamics especially the behavior of the piezoelectric material bonded to the structure at any instant of time, it is difficult to develop an accurate model of the system that describes the entire dynamics of the system. Controller design of smart structures relies on the accuracy of the system dynamic model for non robust controllers. Hence, FEM technique is considered to be less effective compared to the System Identification technique. Therefore, the system model uncertainties resulted from the other approach using FEM can be minimized by System Identification techniques. System Identification is a well known modeling tool used in building an accurate model of complex systems from time-series input and output data for numerous engineering applications. In this work, using MATLAB System Identification Toolbox the following mathematical model in the form of transfer function:

$$G(s) = \frac{-0.1743 s^3 - 392.7 s^2 - 9.61\times10^4 s - 8.419\times10^7}{s^4 + 9.007 s^3 + 1.171\times10^6 s^2 + 3.36\times10^6 s + 3.615\times10^{10}} \qquad (2)$$

Frequency response of the identified model clearly shows that it has resonance frequencies at 28 Hz (178 rad/s), and 170 Hz (1070 rad/s) for the first and second mode of vibration respectively. The mathematical model obtained is checked for sweep signal of same band of frequencies from 5 Hz to 200 Hz for 200 s simulation time.



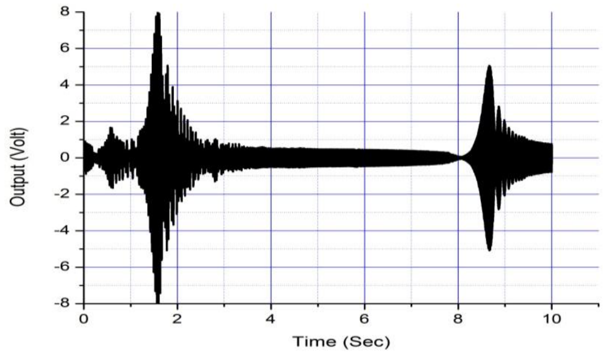


Fig.2 Sweep Input Response of the Beam

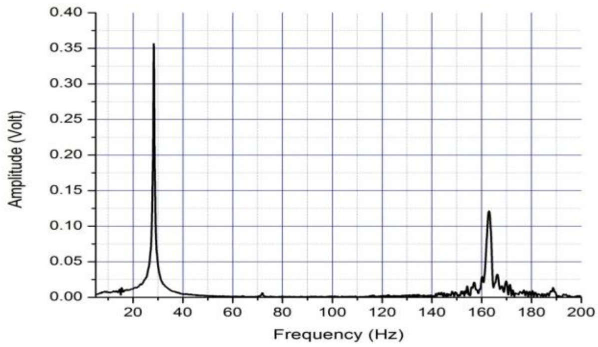


Fig.4 Identified Model Frequency Response

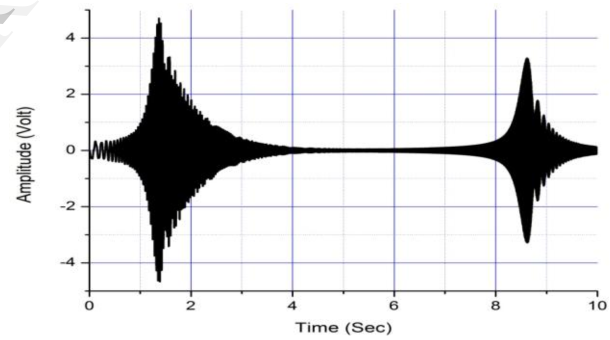


Fig.5 Identified Model Response to Sweep Input

As shown in Figure 5, the model responses to the first dominant frequencies only but for the rest frequencies there is relatively no response as expected, hence the model is a good estimation of the smart beam if the beam external excitation is within this frequency band. The model estimate to a best fit value of 82.49% the accuracy of modeling to a highest level as compared with the other models.

## III. EXPERIMENTAL SETUP

The experimental setup consists of a $(0.3\times0.024\times0.005m)$ beam in free-fixed configuration with a pair of piezoelectric patches as sensor and actuator mounted on both faces of the beam. The optimal location of the piezoelectric pairs used as sensor and actuator is in the regions of higher nodal strain energies of the beam [11]. Hence, the two piezoelectric patches are mounted at a distance of 10 mm from the fixed end to be used as a sensor and actuator pair. Moreover, a third PZT is mounted at 44mm distance from the fixed end to excite the beam. The sinusoidal and sweep signals are generated by signal generator. This signal is applied as an excitation signal to set the beam into continuous vibration after being amplified to the level of 120V. The real-time control algorithm is coded on National Instruments PXIe-1082 processor using LabVIEW. NI PXI-6229 M series multifunction DAQ is used to acquire the sensor data from the charge amplifier. The acquired signal is filtered for high frequency noise using 3rd order Butterworth low pass digital filter.

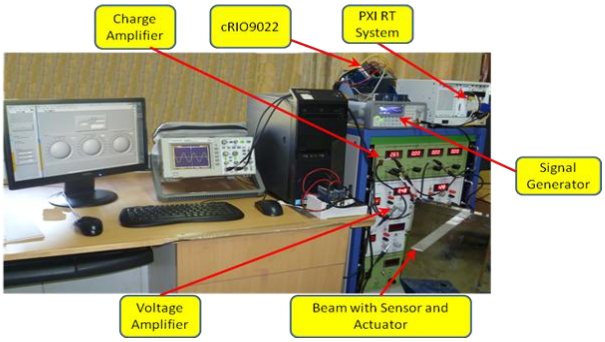


Fig. 6 Photographic View of the Experimental Set-up

This conditioned strain signal is then used for feedback as well as to calculate the equivalent tip displacement of the beam. The actuating signal generated by the controller is sent to the voltage amplifier using same DAQ module and supplied to PZT after amplification to actuate the beam.

## IV. STRAIN TO DISPLACEMENT CONVERSION

For Displacement feedback it is important to either to measure the tip displacement using displacement sensor or derive an equation that relates beam tip displacement to the strain induced at the location of the sensor. : Using Euler-Bernoulli theory, the equation of motion for forced lateral vibration of a uniform beam is obtained

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^t} = f(x,t) \quad (3)$$

Where $E$ is Young's modulus , $I$ is the cross sectional moment of inertia of the beam about $y$-axis, $\rho$ is the mass density, $A$ is the cross sectional area and $f(x,t)$ is the external input force per unit length of the beam.

For fixed-free boundary condition of the beam the solution of the differential equation for the infinite number of normal modes associated with each frequency becomes

$$w_n(x,t) = C_n[\sin\beta_n x - sinh\beta_n x - \frac{sin\beta_n l + sinh\beta_n l}{cos\beta_n l + cosh\beta_n l}(cos\beta_n x - cosh\beta_n x)] \quad (4)$$

$$\omega = (\beta_n l)^2\sqrt{\frac{EI}{\rho A l^4}} \quad (5)$$

where $w_n(x,t)$ is the normal vibration mode, $l$ is the beam length and $\omega$ is the natural frequency of the beam.

The relationship between strain and displacement is quite obvious and straight forward in the case of beam. Here, the tip displacement of the beam is measured with the knowledge of the strain without any physical displacement sensor as follows:

The beam displacement $w_i(x,t)$ for the i$^{th}$ mode of vibration for any point in the span of the beam as a function of corresponding amplitude $C_i$ is expressed as

$$w_i(x,t) = C_i \times y_i(x,t) \quad (6)$$

$$y_i(x,t) = \left[\sin\beta_i x - sinh\beta_i x - \frac{sin\beta_i l + sinh\beta_i l}{cos\beta_i l + cosh\beta_i l}(cos\beta_i x - cosh\beta_i x)\right] \quad (7)$$

where $C_i$ is amplitude of vibration which can be determined from equation (9) for each mode and

$\beta_i l$ is a constant and its values depends on the mode of vibration. This study focuses on the effectiveness of the proposed control algorithm for the first two dominant frequencies only. Hence, for the first two natural modes of vibration the corresponding values of $\beta_i l$ are 1.875104 and 4.694091 respectively. The displacement of each and every point along the entire beam length for the first two modes can be determined using equation (3). This facilitates to derivate relation between the tip displacement to stain. Considering the two extreme edges of the PZT along the beam length as two end points, the strain induced at the PZT due to the beam vibration can be theoretically calculated. The extreme end points of the sensor PZT patch are at $x_1 = 0.01m$ and $x_2 = 0.034$ from the fixed end. The strain at the PZT theoretically calculated using the knowledge of the nodal displacement is as follows:

$$\varepsilon = \frac{w(x_2) - w(x_1)}{x_2 - x_1} = \frac{w(x_2) - w(x_1)}{l_p} \quad (8)$$

where $\varepsilon$ is the strain induced, $l_p$ is the PZT length, $w_1(x)$ and $w_2(x)$ are the transverse displacement of the PZT first and second extreme ends respectively.

$$c_i = \frac{w(x_2) - w(x_1)}{y_i(x_2) - y_i(x_1)} = \frac{\varepsilon l_p}{y_i(x_2) - y_i(x_1)} \qquad (9)$$

Therefore, the tip displacement of the beam for any mode of vibration as a function of induced strain becomes

$$w(x_t) = \frac{\varepsilon l_p}{y_i(x_2) - y_i(x_1)} y_i(x) \qquad (10)$$

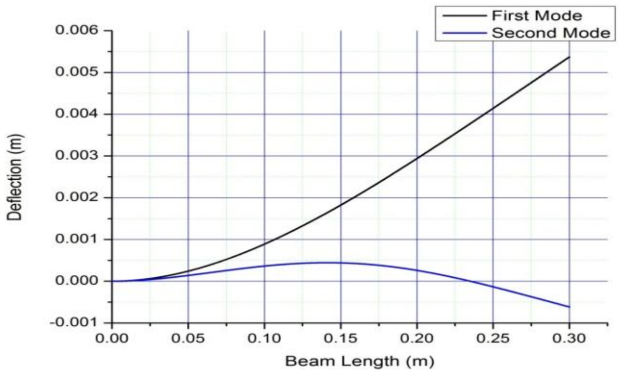


Fig.7 Beam Mode Shape

## V. CONTROL AND SIMULATION

To prove the effectiveness of Displacement feedback for vibration suppression strategies, numerical simulation is carried out prior to experimentation. These control law is implemented in this study because the output of a system is always available for measurement unlike state feedback techniques in which all the states usually may not be available for measurement. Besides, these control law is among the computationally simple and effective feedback techniques which can be easily implemented in a real time. In the present analysis, Displacement feedback control law is considered. Once the tip displacement of the smart beam is known from the sensed strain as per the relation derived in the above section, the closed loop simulation for displacement feedback is carried out based on the following control law:

$$V_a(t) = K v_d(t) \qquad (11)$$

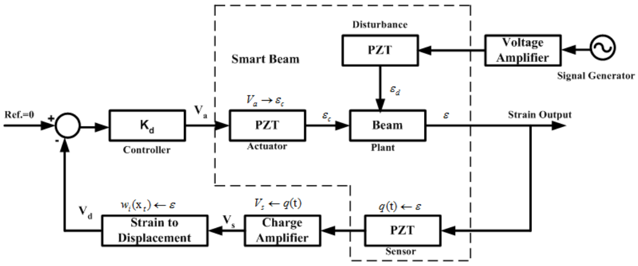


Fig. 8 Closed Loop Schematic Diagram

where $K$ denotes the feedback control gain, $V_a(t)$ is the actuating signal, and $v_d(t)$ is the tip displacement signal proportional to the stain induced due to beam vibration (output of the charge amplifier). The effectiveness of the controllers for resonance frequency is more important than the remaining frequencies because the amplitude levels of non resonance frequencies does not cause any malfunctioning of systems or catastrophic failures of structures in reality. To demonstrate the proposed approach, a closed loop control simulation was performed on the system model for the first two dominant modes of vibration in Matlab/ SIMULINK environment.

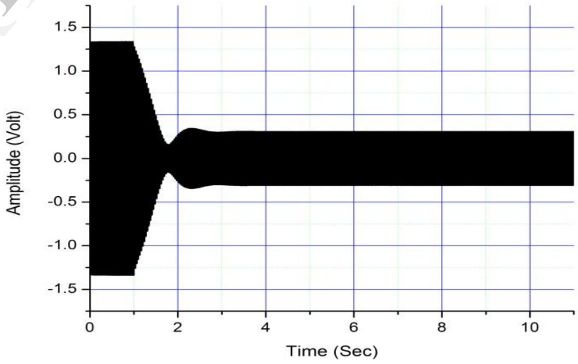


Fig.9 Displacement Feedback Simulation Result for First Mode

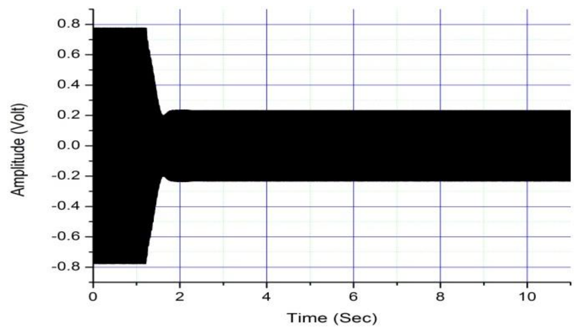


Fig.10 Displacement Feedback Simulation Result for Second Mode

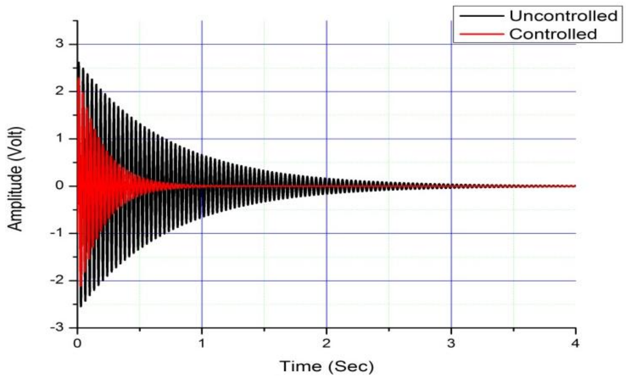


Fig.11 Simulation Result of Displacement Feedback for Impulse Input

## VI. RESULTS AND DISCUSSION

The frequencies of the two dominant modes of vibration of the beam experimentally identified using Impact Hammer test and it is found to be 27.65 Hz and 170 Hz for first and second modes of vibration respectively. The simulation and experimental results show that the beam responds significantly to those excitation frequencies corresponding to the resonance frequencies. Displacement feedback control law is simulated first in MATLAB/SIMULINK to obtain the corresponding controller gains using the mathematical model and relations derived. For the purpose of illustration, the effectiveness of the controller is analyzed by comparing the open and closed loop system dynamics in terms of system natural frequency and damping. It is obvious that active vibration control technique modifies the transient dynamics of the structure in terms of introducing additional damping and improving the system response speed (natural frequency). In other words, the transient and steady state dynamics of the smart beam depends on the location of system poles and zeros. However, the transient effect of the poles located relatively far towards to the left of the S plane decays faster compared to the one closer. Hence, the transient response of the smart beam mainly depend on the dominant poles. Therefore, using the controller gain obtained from simulation in the previous section the locations of closed loop poles are identified. The dominant open loop poles are $-1.39 \pm 178.13i$.

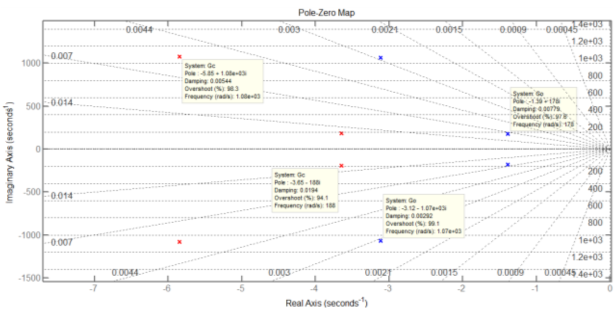


Fig.12 Open and Closed loop poles map

These poles play a major and critical role in deciding the transient response of the smart beam. The open loop effective natural frequency and damping ratio of smart beam is found to be **178.13** rad/s and **0.00779** respectively. In closed loop, the locations of the open loop poles are shifted towards left side of the imaginary axis using the Displacement feedback gain. This pair of dominant pole is shifted to new location as a result of the feedback technique to $-3.65 \pm 188.06i$.

The percentage increment in the effective system natural frequency and damping ratio using this pair of dominant poles is found to be 5.47% and 149% for Displacement feedback. From the analysis, it can be inferred that Displacement feedback control law makes the system to response faster and it also introduces more addition damping to the system as well.

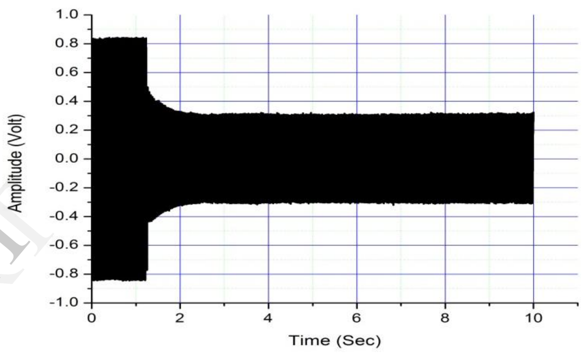


Fig. 13 Displacement feedback experimental result for first mode

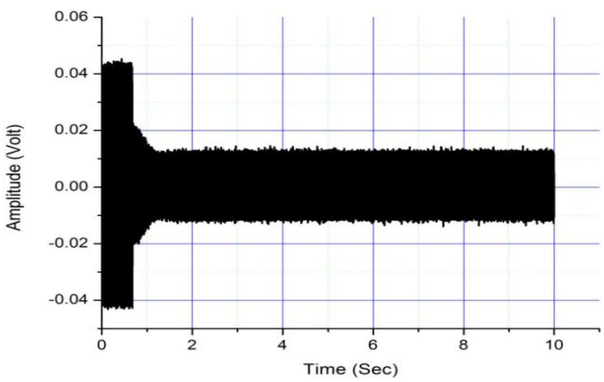


Fig. 14 Displacement feedback experimental result for second mode

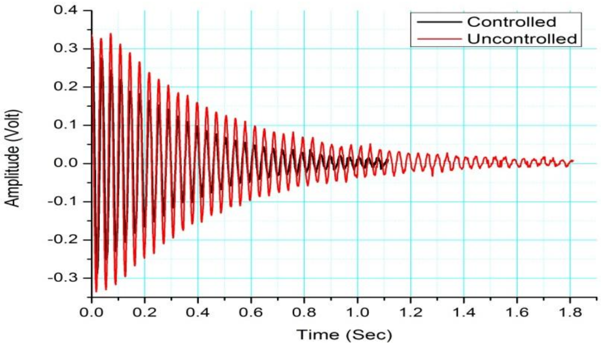


Fig. 15 Displacement feedback experimental result for impulse input

Experimental results demonstrate that the Displacement feedback is found to be more effective control logic compare to direct Strain feedback in terms of reduction in amplitude. It is observed that with a Displacement feedback a percentage reduction of 65.23% and 70.27% in amplitude for first and second modes of vibration is achieved respectively as it shown. Moreover, the controller showed its effectiveness to settle the beam faster for impulse input.

## VII. CONCLUSION

The mathematical model of the smart beam derived using System Identification technique was successfully simulated for controller design. The derived expression to relate strain to tip displacement shows that the tip displacement is directly proportional to the strain for both modes of vibration. From the derived relation proved it is possible to estimate exactly the tip displacement of the beam from the strain without having physical non-contact displacement sensor. Displacement feedback control law has been implemented to actively suppress the vibration of the beam. This control law significantly altered the damping ratio of the beam as a result of change in natural frequency. The effectiveness of this feedback law for the first two dominant frequencies has been demonstrated experimentally. From the results, it has been observed that Displacement feedback showed a stable system response due to minimum computational time delays. The experimental result shows that with this control law a percentage reduction of 65.23% and 70.27% % in amplitude is attained for the first and second mode of vibrations respectively. The performance this control law is fund to be relatively better than the Direct strain feedback law. Moreover, the beam transient response shows that it settled to the equilibrium position faster for impulse input.

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