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Discretization Effect of Equivalent Sliding Mode Controlled DC-DC Buck Converter

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Abstract

This paper presents a simple and systematic approach to use of a practical voltage mode based sliding mode controlled for buck converters operating in continuous conduction mode. In the use of sliding mode control (SMC) in dc-dc converter, major problems arise like variable switching frequency. The switching frequency fixation can be achieved by implementing SM controller in discrete domain. The discretization behaviors of equivalent SMC of dc-dc buck converter are investigated. In particular, one of the most frequently used discretization schemes for digital controller implementation, the zero-order-holder discretization, is studied. Some inherent dynamical properties of the discretized SMC systems are studied. Based on MATLAB and SIMULINK model, the simulation results of such kind of dynamics behaviors are also addressed.

"1. Introduction"

Switched mode DC-DC converters are being used extensively for the purpose of efficient power conversion. These converters are nonlinear and time variant systems. Due to its nonlinear characteristic in nature, a nonlinear Sliding Mode Control (SMC) is developed. Although the analog control systems have proven successful in SMPS applications, there are several reasons that make digital control attractive. The digital controllers are less sensitive to external influences, re-programmable, more flexible and allow implementation of more sophisticated control laws.

The operation of DC-DC buck converter can be classified based on the flow of current in the inductor. They are called as continuous conduction mode (CCM) and discontinuous conduction mode (DCM). In CCM, the inductor current never falls to zero where in DCM, the inductor current falls to zero and remains at zero for some portion of the switching cycle. CCM of the buck converter is our study of interest here. Control system comes where some desired value is required. To maintain the output voltage at desired value with some constant value in DC-DC buck converter, we need control system. The main aim of the control system is to improve the stability, reducing the sensitivity to disturbances, improving the efficiency and system performances.

Sliding mode control is one of the best control techniques which were introduced initially for variable structure systems. A variable structure system is a dynamical system whose structure changes in accordance with the current value of its state. Since DC-DC converters are inherently variable structured systems, it is suitable to apply sliding mode controller to DC-DC buck converter. This control method has several advantages over other control methods such as stability even for large load and line variations, robustness, good dynamic response, and simple implementation. There are two ways of applying sliding mode control to the system. These are analog sliding mode control and digital sliding mode control. Sliding mode control in digital domain has not been studied as much as in analog domain. Here, we have applied both analog and digital sliding mode control to our system. We have studied the behavioral changes in both domains by parameter variations.

Because of its simplicity and low cost, DC-DC converters are controlled in analog controllers. In practical application, the analog controller requires a lot of external passive components and this architecture increases the overall size of the system. The analog components are sensitive to the environmental influence, such as temperature, ageing, noise, tolerance of fabrication, which results in lack of flexibility, low reliability. Again because of higher switching frequency operation in the system, the analog controller suffers from the limitation of bandwidth and large gain variation. Therefore, Digital controller came to overcome these problems.

Since the system is nonlinear in nature, it exhibits some nonlinear phenomena. We found this in digital sliding mode controller while applying to our system. So some restrictions are also there to use of digital controller. We have studied the effect of discretization of sliding mode controller of DC-DC buck converter. We have derived the mathematical analysis for this system and observed the undesirable phenomena by doing the simulation with help of MATLAB. We have observed that by varying the system parameter up to some extent, the system fulfills the properties of sliding mode control that means the system is robust to parameter variations.

But after crossing certain limits, the stability of the system is affected. Some sub-harmonic oscillation is coming which is the aim of this paper to study that behavior. This type of behavior brings the system into quasi-periodic state and chaotic state. We can say in one line that chaotic state is unstable but bounded. A lot of research work is going on chaos. The main objective is how to avoid the chaos and all other nonlinear behaviors from the system and make it smooth operation. This chaotic behavior also has some advantages in space applications. But this behavior is not always desirable. We need to optimize the controller to operate the system in an ideal manner.

"2. Buck Converter"

Buck converter is the most widely used DC-DC converter topology in power management and microprocessor voltage-regulator applications. Those applications require fast load and line transient responses and high efficiency over a wide load current range. The output voltage of this converter is less than the input voltage. The converter is shown in Fig. 1.

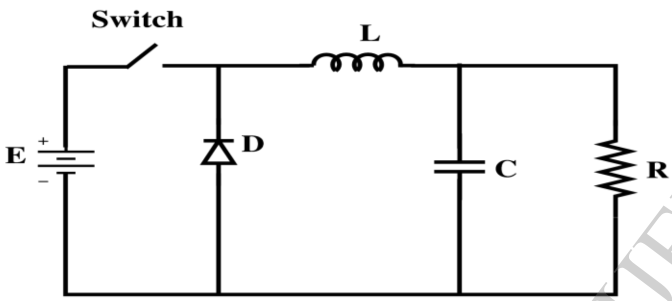

Fig. 1 Buck converter

The theory of operation of buck converter is divided into two states. They are ON state and OFF state. In ON state, the switch is turned ON. The output voltage is equal to the input voltage. In OFF state, the switch is turned OFF. The output voltage is zero. During switch ON condition, the current through the inductor increases and the energy stored in inductor increases. During switch OFF condition, the inductor acts as a source and maintains the current through the load resistor. During this state, the energy stored in the inductor decreases and its current also decreases. Since we are operating it in continuous conduction mode, the current never falls to zero. Current flows continuously in the inductor during the entire switching cycle in steady state operation. Since buck converter is a non-linear system, we are using nonlinear controller to control this converter.

"3. Sliding Mode Control"

The basic principle of sliding mode control is to design a certain sliding surface in its control law that will direct the trajectory of the state variables toward a desired origin when coincided. State variables mean minimum number of variables that uniquely specify the state of the dynamic system. Many control laws are used in sliding mode controller. The control parameters denote the desired state feedback variables to be controlled. A sliding surface can be obtained by making S(x) equals to zero which is shown in Fig. 2.

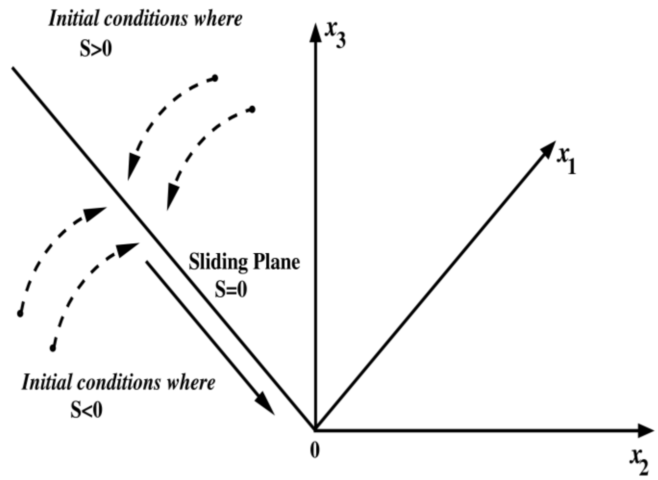

Fig. 2. State trajectory converging the sliding plane irrespective of its initial condition.

Where x_1, x_2, and x_3 are three state variables of a system. SMC process can be divided into two phases. In the first phase, which is called reaching phase, regardless of the starting position, the controller will perform a control decision that will drive the trajectory of the state variables to converge to the sliding surface. This is possible through the compliance of the hitting condition. Hitting condition assures that, regardless of the initial condition, the state trajectory of the system will always be directed towards the sliding surface. When the trajectory is within a small vicinity of the sliding surface, it is said to be in sliding mode operation, which is called the second phase of the control process. The sliding mode controller is performing its control decision by utilizing the sliding plane as a reference path, on which the trajectory will track and eventually converge to the origin to achieve steady-state operation which is shown in Fig. 3

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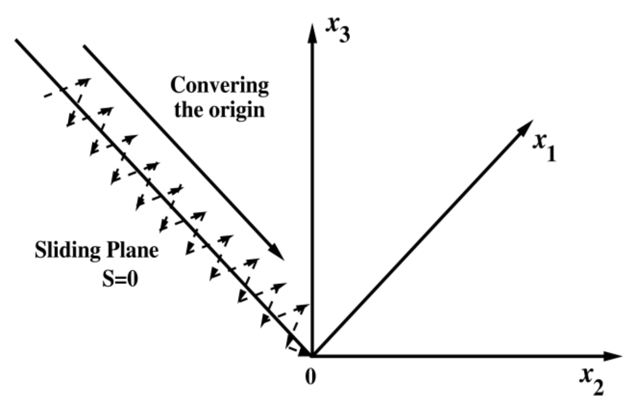

Fig. 3. Phase trajectory being maintained within a small vicinity from the sliding plane and concurrently being directed to converge to the origin.

"4. Discretized Sliding Mode Controller"

Study of SMC in discrete-time can be divided into two parts. One is the discrete-time sliding mode control. Here both the system and controller are in discrete domain. Another approach is the discretization of SMC designed in the continuous-time domain. In simulations, SMC systems are usually discretized using the Euler method, while a zero-order holder (ZOH) is used in practical implementation of the SMC strategy. Discretization of SMC may cause irregular behaviors such as periodic trajectories and strange attractors. In Euler method, the sampled SMC suffers more from the sampling process, as it would lose the high gain property nearby the vicinity of the switching surface. To compensate for this, disturbance prediction is indispensable, which is feasible under the hypothesis that the disturbance is slow time-varying.

In [17], the properties of equivalent control based two-dimensional SMC system discretized using ZOH have been studied in detail. It has been shown that if the discretization step is sufficiently small then in the steady state trajectories spend at most two iterations on each side of the sliding surface. In [21], the zero-order holder (ZOH) discretization of the double integrator system has been studied. It is shown that if the switching frequency is high enough, then the system converges to a small neighborhood of the origin, while for low switching frequencies, the system sustains periodic oscillations with arbitrarily large magnitudes.

"5. Design of Digital Controller"

A digital control system uses digital hardware, usually in the form of a programmed digital computer, as the heart of the controller. A typical digital controller has analog components at its periphery to interface with the plant. It is the processing of the controller equations that distinguishes analog from digital control.
One way of designing digital control system for continuous-time systems is first to design an analog controller for the system, then to derive a digital counterpart that closely approximates the behavior of the original analog controller. It is shown in the Fig. 4
This type of control design is required when the designer replaces all or part of an existing analog controller with a digital controller. However, even for small sampling periods, the digital approximation usually less well than the analog controller from which it is derived. In Fig. 4, the analog to digital converter (A/D) samples the analog sensor signals and produce equivalent binary representations of these signals. The sampled sensor signals are then modified by the digital controller algorithms, which are designed to produces the necessary digital control inputs.
Consequently, the control inputs are converted to analog signals using digital to analog converters (D/A). The D/A transforms the digital codes to signal samplers and then produces step reconstruction from the signal samples by transforming the binary-coded digital input to voltages. These voltages are held constant during the sampling period T until the next sample arrives. The process of holding each of the samples is termed zero-order-holder. Then the analog control signals are applied to control the behavior of the plant. The other way of designing digital control system for continuous time systems is first to derive a discrete-time equivalent of the system and then to design a digital controller directly to control the discretized system. It is shown in Fig. 5.

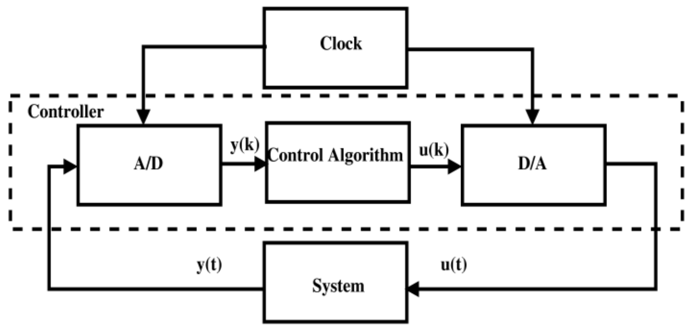

Fig. 4. Closed loop system formed by a discrete-time controller and a continuous-time system.

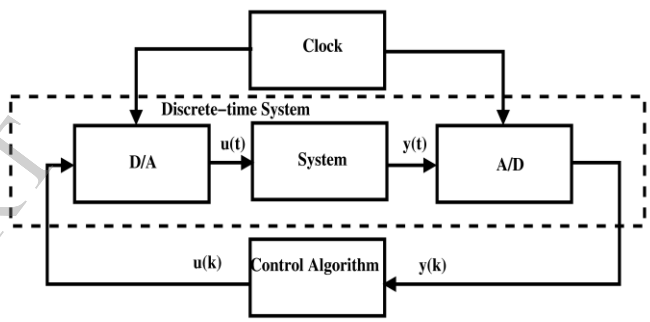

Fig. 5. Closed loop system formed by a discrete-time controller and a continuous-time system. The D/A and A/D converters have been combined with the continuous-time system illustrating the discrete-time description of the system.

Both way of designing is acceptable when the sampling is fast enough. However, first approach has been used more in industrial applications.

"6. Stability of DSMC"

The properties of discrete-time SMC system are different from the continuous-time SMC system. In the continuous-time SMC system, the sliding mode existence condition is $\dot{S}S < 0$, whereas, it is different for discrete-time SMC system. The equivalent form of the continuous sliding mode existence condition is to give a discrete sliding mode existence.

$$(S(k+1) - S(k))S(k) < 0 \quad (1)$$

DSMC can undergo only quasi-sliding mode control (QSMC). The quasi-sliding mode is the motion that satisfies the following conditions:

- Starting from any initial state, the trajectory will move monotonically towards the switching plane and cross it in finite time.
- Once the trajectory of the system first crosses the switching plane, it will cross it again at every successive sampling time, resulting in a zigzag motion about the switching plane.
- The size of each zigzagging step is not increasing and hence the trajectory stays within a specified band.

The condition in equation (1) is found out not sufficient for the existence of a quasi-sliding mode. A new condition is that:

$$|S(k+1)|<|S(k)| \quad (2)$$

which is decomposed into the following inequalities:

$$(S(k+1)-S(k))\,\mathrm{sgn}(S(k))<0 \quad (3)$$
$$(S(k+1)+S(k))\,\mathrm{sgn}(S(k))>0$$

Using the equivalent form of a Lyapunov-type of continuous-time condition, the quasi sliding mode existence condition with the Lyapunov function candidate $V(x(k))=S^2(k)$,

$$\Delta V(k)=S^2(k+1)-S^2(k)=2S(k)\Delta S(k)+\Delta S^2(k)<0 \ (4)$$

A more suitable approach called reaching law approach for quasi sliding mode is:

$$S(k+1)-S(k)=-qhS(k)-\varepsilon h\,\mathrm{sgn}(S(k)),$$
$$\varepsilon>0, q>0, 1-qh>0 \quad (5)$$

Where h is the sampling period.

"7. DSMC System Model"

Let us consider the continuous-time system

$$\dot{x}=Ax+Bu \quad (6)$$

The switching function defined as $S(x)=C^T x$.

where $x\in R^n, u, S\in R^1, A$ is an n×n matrix,

B and C are both n-dimensional vectors.

S is the switching surface to represent a desired asymptotically stable dynamics.

The equivalent control based SMC is $u=u_{eq}+u_s$.

where $u_{eq}=-(CB)^{-1}C^T Ax, u_S=-\alpha(CB)^{-1}\mathrm{sgn}(S(x))$

Under ZOH, this system (6) is converted into the following discrete form, that is $u=u_k$ over the time interval [*kh*, *(k+1 h)*], where *h* is a sampling period.

The continuous-time system (6) under the ZOH into the discrete form is

$$x(k+1)=\Phi_1 x(k)+\Gamma u_k \quad (7)$$

where $\Phi_1=e^{Ah}$ and $\Gamma=\int_0^h e^{A\tau}d\tau B.$

The equivalent control is

$$u_k=u_{eq}(k)+u_S(k)=-CAx(k)-\alpha S_k, \quad (8)$$

where $S_k=\mathrm{sgn}(S(x(k)))$. Applying (8) to the sampled data system, (7) leads to the following discrete system

$$x(k+1)=\Phi x(k)-\alpha\Gamma S_k \quad (9)$$

$$\text{where } \Phi=e^{Ah}-\int_0^h e^{A\tau}d\tau BCA \quad (10)$$

$$\text{and } \Gamma=\int_0^h e^{A\tau}d\tau B \quad (11)$$

"8. Analysis of DSMC using Simulation"

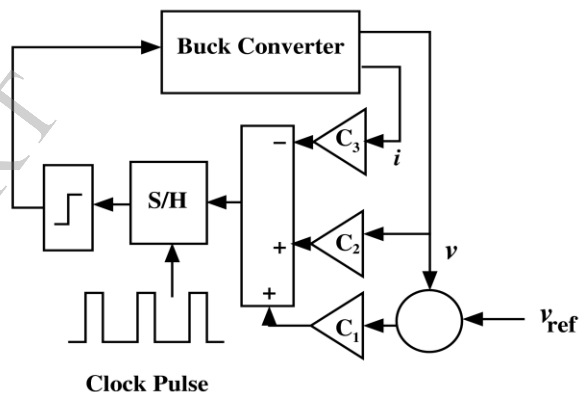

Fig. 6 Block diagram of buck converter with DSMC

$$\dot{x}=Ax+Bu+D$$

Where $A=\begin{bmatrix}0 & 1\\ \frac{-1}{LC} & \frac{-1}{RC}\end{bmatrix}, D=\begin{bmatrix}0\\ \frac{v_{ref}}{LC}\end{bmatrix}, B=\begin{bmatrix}0\\ \frac{-E}{LC}\end{bmatrix}$

Sliding surface S(x) is defined as

$$S(x)=c_1x_1+c_2x_2=Cx$$
$$=c_1(v_{ref}-v)+c_2\left(\frac{-i}{C}+\frac{v}{CR}\right).$$

For existence condition, $\dot{S}(x)=C^T\dot{x}=0$

$$\dot{S}(x)=C^TAx+C^TBu+C^TD$$

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The condition for the sliding surface to exist results

$$\dot{S}(x) = \begin{cases} C^T Ax + C^T Bu + C^T D < 0 \text{ for } S(x) > 0 \\ C^T Ax + C^T Bu + C^T D > 0 \text{ for } S(x) < 0 \end{cases}$$

The switching plane is defined as

$$u(t) = \begin{cases} 1; \text{ if } S(x) < 0, \\ 0; \text{ if } S(x) > 0, \end{cases}$$

Therefore, switching surface is S(*x*)=0.

According to SMC,

$$\frac{dS(x)}{dt} = 0$$

$$C\frac{dx}{dt} = 0$$

$$C[Ax + Bu + D] = 0$$

$$CAx + CBu + CD = 0$$

$$CBu = -CAx - CD$$

$$u = -(CB)^{-1}(CA)x - (CB)^{-1}(CD).$$

Equivalent control law based SMC is $u = u_{eq} + u_s$

$$u_{eq} = -(CB)^{-1}CAx - (CB)^{-1}(CD)$$

$$u_s = -(CB)^{-1}\operatorname{sgn}(S(x))$$

The resulting dynamics can be written as

$$\dot{x} = Ax + B[u_{eq} + u_s] + D.$$

We consider the effect of ZOH for digital implementation during two successive sampling instant t_k and t_{k+1}. Control signal is constant. The state trajectory moving on the switching line is defined by the switching logic which is shown in Fig. 7.

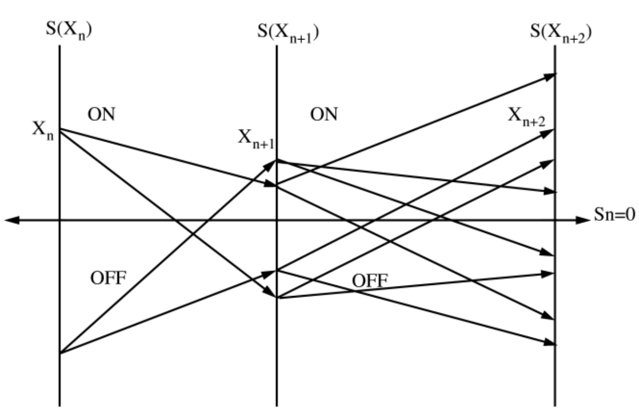


Fig. 7. State trajectory moving on the switching line

This switching logic helps the state trajectory to follow the desired path. Here, we have done the simulation in analog and discrete domain. We vary the parameter i.e. load in both cases. We have observed that for same parameter variation, the system behavior does not change whereas in discrete domain, we found the system behavior changes. Due to discretization effect of SMC, subharmonic-2 behavior comes to the picture. The limit cycle breaks and enters into period-2 cycle. We use the MATLAB environment for simulation. Voltage waveform, current waveform, phase plane diagram and limit cycle are obtained from the simulation.

"8. Simulation Results"

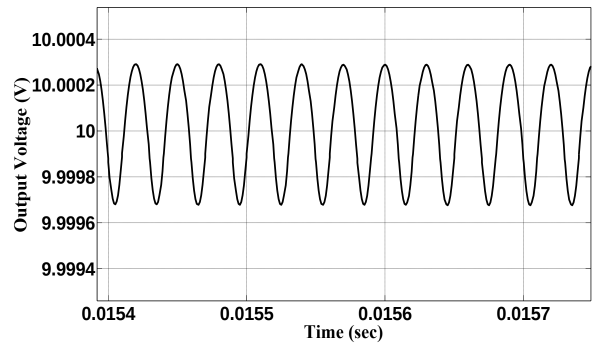


Fig.8(a) Output voltage with R=5Ω in analog domain

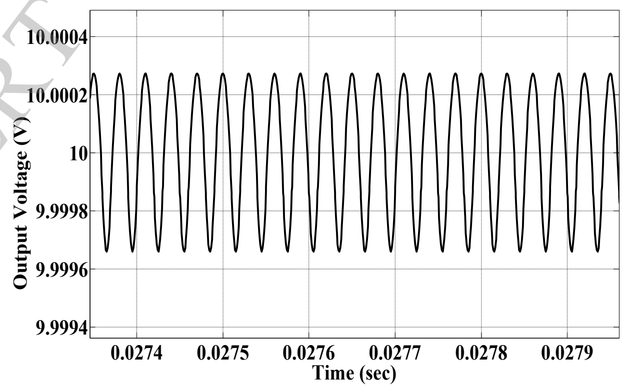


Fig.8(b) Output voltage with R=50Ω in analog domain

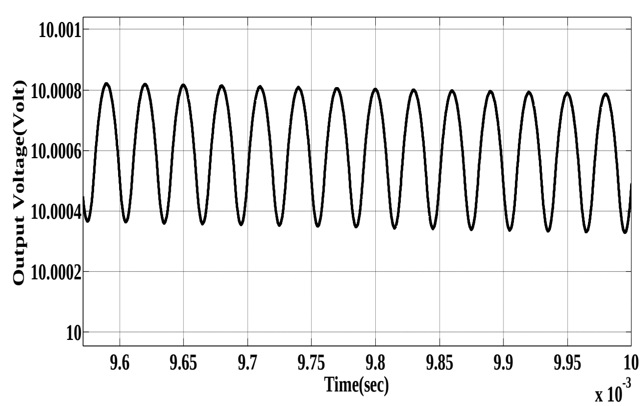


Fig.8(c) Output voltage with R=5Ω in discrete domain

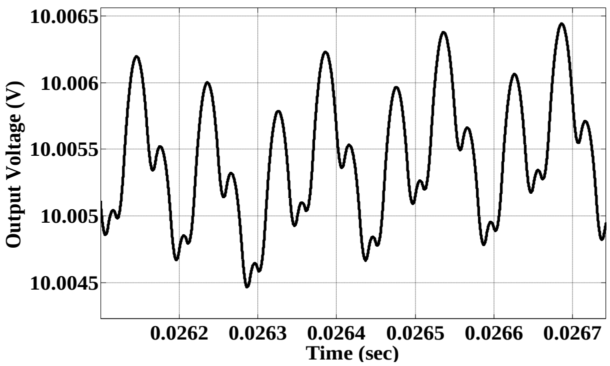


Fig.8(d) Output voltage with R=50Ω in discrete domain

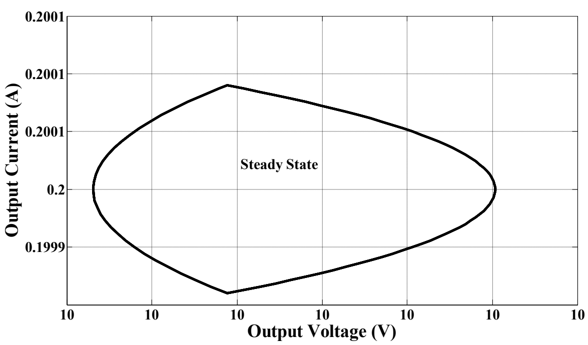


Fig. 9(a) Limit cycle with R=50Ω in analog domain

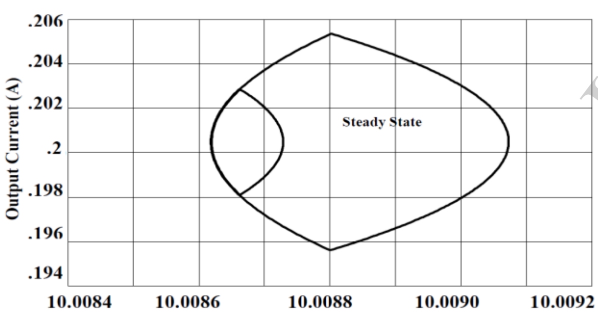
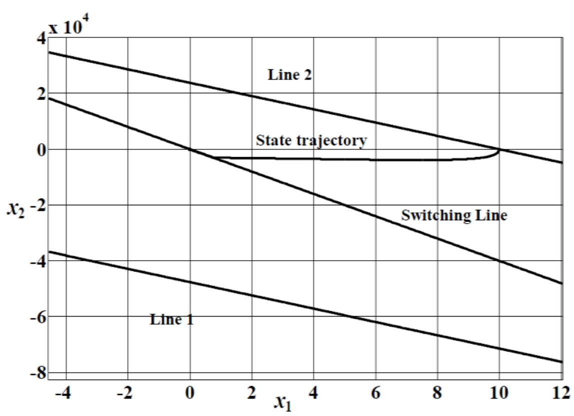
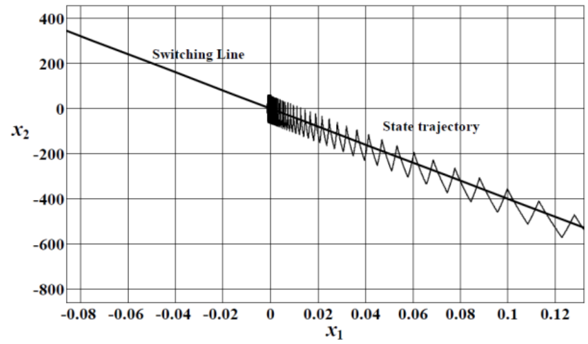


Fig. 9(b) Limit cycle with R=50Ω in discrete domain

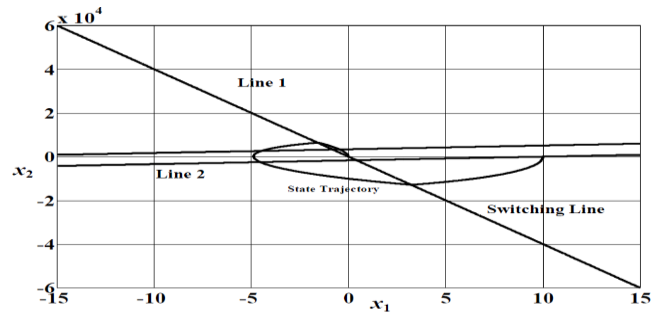


(a)

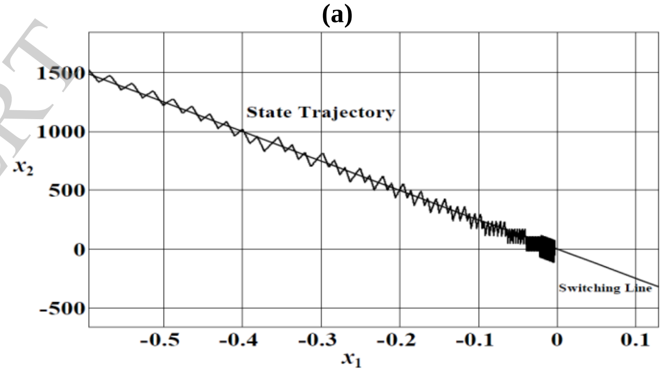


(b)

Fig. 10(a) Phase plane with R=5Ω in discrete domain, (b) State trajectory converging towards origin.



(a)



(b)

Fig. 11(a) Phase plane with R=50Ω in discrete domain, (b) State trajectory converging towards origin.

The simulations are done for buck converter using SMC. First the system is simulated in continuous SMC. After that, SMC is converted into discrete form using ZOH and applied to buck converter. The output results are shown in the above Figures. In Fig. 8(a), the output voltage is shown in analog domain. By varying load resistance, we observed that the behavior of output voltage shown in Fig. 8(b) does not change. We got the desired output voltage. Now we simulate in discrete environment. The output voltages with two different load resistances are shown in Fig. 8(c) and Fig. 8(d) respectively. We observed from these Figures that, no behavioral change occurs upto certain limits. Beyond certain limits, the system shows behavioral changes. The system enters from period one to period two. The state trajectory along the switching line is plotted which is shown in Fig. 10. The existence region is defined by line 1 and line 2. The state trajectory is starting from the point (v_{ref}, 0) with initial condition zero and converging towards the origin. We can observe that the periodicity breaks while converging towards the origin by parameter variation in discrete domain which is shown in Fig. 11. In the switching converter, the steady state operating condition is a periodic orbit, not an equilibrium point. The periodic state, the so-called limit cycle is obtained for both continuous and discrete domain with varying parameter. In the state of limit cycle, the trajectory keeps circulating along the same closed loop. But due to the discretization effect, limit cycle losses its stability, a period-1 orbit becomes period-2 orbit as shown in Fig. 9(b).

"9. Conclusion"

We have discussed the use of DSMC in this paper. SMC is mainly discretized in two ways. Here, discretization of SMC using zero-order-hold is studied in details. There are two methods for designing of digital controller. Both are described using the block diagram. The conditions for stability of DSMC are mentioned. The system model is derived for DSMC. The block diagram of buck converter with controller is shown. We define the control law for equivalent SMC. SMC is discretized using zero-order-hold for the proposed system. The simulation results are mentioned both for analog domain and discrete domain by varying parameters to study the behavioral changes in discrete domain. We can observe that by varying parameter, the limit cycle in discrete domain changes from period one to period two. Voltage waveform and phase plane are shown in the simulation results.

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