Different Techniques of Secondary Path Modeling for Active Noise Control System: A Review

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Abstract—Acoustic noise problem becomes notable with the development of industries, electronics appliances, etc. Active noise control (ANC) system using adaptive algorithms is an effective technique for acoustic noise reduction. FxLMS is an adequately potent algorithm for ANC system. As estimation of secondary path in FxLMS is done before applying ANC system, hence, it gives inferior results for time varying secondary path. As a result, online modeling of secondary path has been done. In this paper, a detailed study has been done which focuses on various techniques of online modeling of secondary path.

Keywords—ANC, FxLMS algorithm, Online secondary path modeling, Eriksson’s method, Zhang’s method, Akhtar’s method, Akhtar’s modified method, Carini’s method, Ahmed’s method.

1. INTRODUCTION

Noise is an unpleasant sound that causes disturbance in the natural balance of human being which badly affects the human health both mentally and physically. The acoustic noise pollution causes problems like hearing loss, psychiatric disorder, insomnia, high blood pressure, cardiac arrest and stress in human beings and also responsible for low productivity at work places. Thus, it is essential to reduce noise pollution from the surroundings of human being [1]. Due to the increasing use of industries, electrical, transportation and medical equipment’s (like transformers, fans, vacuum cleaner, washing machines, airplanes, trains, MRI (Magnetic Resonance Imaging) and ambulance etc.) in our daily routine, the acoustic noise problem becomes notable. To control acoustic noise, approaches like passive noise control and active noise control are used. Passive approach is conventional approach and uses sound absorbing materials, barriers, enclosures, silencers and ear muff etc. to mitigate the acoustic noise. This approach is effectively reducing level of noise signal having high frequency component but not fit for low frequency component. Thus, active noise control (ANC) approach is preferred to passive noise control approach. That’s why ANC approach has gained popularity over last two decades to mitigate low frequency signal i.e. acoustic noise signal. Superposition principle is the fundamental principle for an ANC system in which an anti-noise signal is superimposed with primary noise signal so that they cancel each other. The secondary noise source generates an anti-noise signal which has exact magnitude and 180° out of phase with respect to the primary noise signal. The estimation of the phase and amplitude of this signal should be done precisely because the estimation of anti-noise shows the extent of elimination of the primary noise [2-9]. The schematic diagram of a single channel feed-forward ANC system is shown in Fig. 1. Lueg [10] firstly proposed a technique to produce an anti-noise signal using loudspeaker.

The introduction of adaptive filter theory in 1980’s led to the advancement of adaptive algorithms based on signal processing and hardware for ANC system which makes ANC system useful for processing of real-time signal. In 1981 first time Bugress [10] designed a noise cancellation system based on filter theory for a duct. After that ANC systems using adaptive filters become more popular because of efficient working and less cost. ANC can be used in many applications such as headsets, MRI system, airplanes, motorcycle helmet, infant incubator, ambulance, ducts etc. The least mean-squared (LMS) adaptive algorithm using digital filter is applied to an ANC system. But, the presence of time delay caused by secondary path is the disadvantage of LMS algorithm [11-12].

To overcome this situation, secondary path’s estimation is added in the way of primary noise signal. The modified LMS algorithm is familiar as Filtered-x Least Mean Square algorithm (FxLMS). This modification is done by Widrow and Stearns [13]. The estimation of secondary path is done before activation of an ANC system in FxLMS. Hence, FxLMS algorithm based ANC system is a technique of offline modeling but for real-time applications secondary path is time varying. The secondary path consists power amplifier, pre-amplifier, anti-aliasing filter, ADC, DAC, reconstruction filter and a secondary source (loudspeaker) and with the change in time the characteristics of these components vary so it is difficult to achieve good estimation of time-varying secondary path. Therefore, the estimation of secondary path needs online modeling of path which can be done in two ways.
In first method, a random noise is taken and feed into the ANC system by using a modeling filter for restructuring of secondary path. The second method takes the output of an ANC system to model secondary path, thus there is no need of an additional random noise signal and hence, this modeling is dependent of output signal. Among these two methods, first way is better than second in terms of convergence rate, computational complexity and updating period, therefore, by the time with the use of first method so many methods have been evolved by researchers. Comparison of above methods elaborated in [14]. The ANC system with online modeling of secondary path by using a white noise generator is first time developed by Eriksson et al. [15]. After method given by Eriksson for online modeling of secondary path of an ANC system, many methods have been evolved by many researchers like Zhang [18], Akhtar [19-20], Carini [21] and Ahmed [22].

2. FxLMS ALGORITHM

The secondary path transfer function introduces delay which causes instability in application of LMS algorithm to ANC system. This instability eliminates by using FxLMS algorithm as it utilizes the pre-determination evaluation of the secondary path. The schematic diagram of FxLMS algorithm for feed-forward ANC system is shown in Fig. 2 [13]. Estimation of the secondary path and primary path are represented as $S(z)$ and $H(z)$ respectively. Compared to the LMS algorithm, FxLMS algorithm achieves faster convergence. The algorithm is resistant to errors made by the modeling filter $\hat{S}(z)$ in the determination of the secondary path $S(z)$. $x(n)$ the primary noise signal measured at reference microphone. The output signal $y(n)$ is evaluated as:

$$y(n) = w(n) \ast x(n)$$

(1)

where, $c(n)$ is the coefficient vector. The error signal $e(n)$ is evaluated as:

$$e(n) = d(n) - y'(n)$$

(2)

In FxLMS algorithm, coefficients are updated as follows:

$$w(n + 1) = w(n) + \alpha \ast e(n) \ast x(n)$$

(3)

where, $\alpha$ is the step-size and $x'(n)$ is filtered input signal which is estimated as:

$$x(n) = \hat{S}(n) \ast x(n)$$

(4)

Estimation of secondary path in FxLMS is done before implementing ANC system but in many real time implementations, secondary path is time varying. Due to presence of time varying path leads to instability of system which also effects the convergence of filter.

3. ONLINE SECONDARY PATH MODELING METHODS

1. To mitigate the consequences caused by time varying secondary path, an online modeling of secondary path is done to ensure the stability of an ANC system and to get desired results.

3.1 Eriksson’s Method For Online Modeling Of Secondary Path

The concept of online secondary path modeling was first time proposed by Eriksson et. al.[15]. In this method, random white noise signal $v(n)$ is used as a training signal. The block diagram for this method is shown in Fig. 3. Eriksson’s method contains two adaptive filters. The former is based on Fx-LMS algorithm is named as a control filter and the latter is named as a modeling filter which is based on LMS algorithm. For Eriksson’s method, the error signal $e(n)$ is evaluated as:

$$e(n) = y(n) - y'(n)$$

(5)

where, $y(n) = s(n) \ast y(n)$, $y'(n) = s(n) \ast v(n)$, and $v(n)$ is the white Gaussian noise signal.

$f(n)$ is the error signal generated by $y'(n)$ for the modeling filter $\hat{S}(z)$.

$$f(n) = [d(n) - y'(n) + v'(n)]$$

(6)

The modeling filter coefficients are updated as:

$$\hat{S}(n + 1) = \hat{S}(n) + \beta \ast f(n) \ast v(n)$$

(7)

where, $\beta$ is step-size for modeling filter.

The updated control filter coefficients are given in equations (3) and (4).
The drawback of Eriksson’s method is that the step-size used for modeling filter is fixed which may lead to the continuous occurrence of white random noise signal, \( v(n) \) in the error signal \( e(n) \) when \( v(n) \) is amplitude is high. So, this restricts the amplitude of \( v(n) \) to be a low level such that it becomes responsible for slow convergence of modeling filter.

Another drawback is that the different error signal is used by both adaptive filters so there will be an interference between these signals.

### 3.2 Zhang’S Method For Online Modeling of Secondary Path

The results of Eriksson’s method are enhanced by various methods discussed in [16]-[18]. An additional adaptive filter is introduced by Bao et. al. [16]. This auxiliary filter is used to cancel out the interference introduced by Eriksson’s method and also improves rate of convergence. But, control filter’s performance is not improved by this method. Kuo et. al. [17] uses an additional adaptive filter for prediction of error signal to remove the effects of interference. The determination delay is to be optimized for prediction filter otherwise performance of overall system degraded. This method is only suitable for narrowband ANC system where primary noise is predictable. Zhang’s [18] method uses three filters, among these filters two filters are similar as Eriksson’s method and third filter \( h(n) \) which is cross-updated to decrease the interference among other filters. This method gives better results than Bao’s and Kuo’s methods by suppressing the perturbation effect which is caused by white noise signal.

The residual error signal, \( e(n) = d(n) - s(n) * y(n) \) \hspace{1cm} (8)

A new error signal, \( e'(n) = e(n) - \delta(n) * v(n) \) \hspace{1cm} (9)

The updating equation for control filter as:

\[ w(n + 1) = w(n) + x(n) * e'(n) \] \hspace{1cm} (10)

The filter coefficients of modeling filter are updated as:

\[ \delta(n + 1) = \delta(n) + \beta * v(n) * [g(n) - \hat{u}(n)] \] \hspace{1cm} (11)

The third filter \( h(n) \) decrease the interference caused by \( v(n) \) and can be updated as:

\[ h(n + 1) = h(n) + \delta * x(n) * e'(n) \] \hspace{1cm} (12)

Due to the use of an additional adaptive filter, there is increase in design complexity in Zhang’s method as compare to the basic method. It is hard to find optimum step-sizes for three filters simultaneously. The amplitude of \( v(n) \) signal is to be a low level therefore it is not able to model secondary path accurately.

### 3.3 Akhtar’s Method For Online Modeling Of Secondary Path

The design complexity of Zhang’s method is rectified by method applied by Akhtar et al. [19], which uses only two adaptive filters and gives better performance than Zhang’s method.

The Akhtar’s method varies the step-size of modeling filter to restrict the presence of white noise signal in desired output so this algorithm as (Variable Step Size) VSS-LMS and the control filter uses Modified-FxLMS algorithm. In Modified-FxLMS, an additional fixed filter is used to modify error signal for control filter which is used to increase the upper bound of step size. This additional filter is responsible for fast convergence of an ANC system as compare to previous methods. A schematic diagram of this method is shown in Fig. 4.

The step-size parameter, \( \beta(n) \) of VSS-LMS algorithm is varied according to the ratio \( r(n) \)

\[ r(n) = p_f(n)/p_e(n) \text{such that } r(n) \approx 1 \text{ when } n=0 \] \hspace{1cm} (13)

Where, \( p_f(n) \) and \( p_e(n) \) are power of modeling error signal \( f(n) \) and power of residual error signal \( e(n) \). The power of error signals \( f(n) \) and \( e(n) \) are calculated as:

\[ p_f(n) = \mu * p_f(n-1) + (1-\mu) * f^2(n) \] \hspace{1cm} (14)

\[ p_e(n) = \mu * p_e(n-1) + (1-\mu) * e^2(n) \] \hspace{1cm} (15)

where, \( \mu \) is the forgetting factor (0.9 < \( \mu < 1 \)). Now, the step-size can be estimated as:

\[ \beta(n) = r(n) * \beta_{min} + (1-r(n)) * \beta_{max} \] \hspace{1cm} (16)

where \( \beta_{min} \) and \( \beta_{max} \) are minimum and maximum values of step-size.

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**Figure 4:** Zhang’s method based feedforward ANC system [18]
The modeling filter coefficients are updated as:
\[ \hat{s}(n+1) = \hat{s}(n) + \beta(n) \ast v(n) \ast f(n) \]  
where \( \beta(n) \) is the step-size for modeling filter. The control filter coefficients are updated as:
\[ w(n+1) = w(n) + \alpha \ast f(n) \ast x'(n) \]  

\[
\sigma^2(\nu_1) \ast \gamma \ast \alpha = \times \binom{\mu}{\mathbf{r}} \text{for } \mathbf{r}(1) \text{ where } \mu = x - \nu_1 \text{ is never zero} \}
\]

\[ v(n) = \sqrt{(1 - r(n)) \ast \sigma^2_{\min} + r(n) \ast \sigma^2_{\max}} \ast v_o(n) \]  
where \( v_o(n) \) is noise signal of unity variance. \( \sigma^2_{\max} \) and \( \sigma^2_{\min} \) are maximum and minimum values for the variances of \( v(n) \), respectively. The random noise signal variance, \( v(n) \), i.e., power is varied by using ratio \( r(n) \) which is known as noise-power scheduling.

The value of the parameter \( r(n) \) is never zero in case of steady state which affects the noise power value and the method has slow convergence due to implementation of VSSLMS algorithm.

3.4 CARINI’S METHOD FOR ONLINE MODELING OF SECONDARY PATH

Two advancements to the previous methods are introduced by Carini’s method: 1) Both control and modeling filters are using optimal step-size parameter, and 2) a new approach of self-tuning is proposed for scheduling of noise power [21].

The basic structure of Carini’s method resembles with Akhtar’s modified method except that Carini’s method used normalized LMS algorithm with optimal variable step-size parameters. In this approach, step size of both filters is made to be varied. The schematic diagram of implementation of Carini’s method is shown in Fig. 7.

The method is used to avoid tuning of step size by designer. However, there is a requirement of choosing an appropriate ratio of error noise power to the auxiliary noise power.

A delay coefficient technique is used here which provides a delay \( D \) to obtain optimized value of \( \beta(n) \) for \( \hat{s}(n) \). For this delay a vector of length \( (D+L) \) is given by technique, where \( L \) is the tap-length of an filter.

Normalized LMS adaptation is used for both filters are:
\[ w(n+1) = w(n) + \frac{\hat{v}(n)f(n)}{\hat{s}(n)} \times \alpha(n) \]  
\[ \hat{s}(n+1) = \hat{s}(n) + \frac{v(n)f(n)}{p^T(n)p(n)} \times \beta(n) \]  
\[ \beta(n) = \begin{cases} \frac{\hat{M}_f(n)}{p_f(n)} & \text{when } \frac{\hat{M}_f(n)}{p_f(n)} > \beta_{\min} \\ \beta_{\min} & \text{otherwise} \end{cases} \]

where, \( \hat{M}_f \) is obtained by technique which uses delay coefficient and can be obtained by using given equation:
\[ \hat{M}_f(n) = \hat{M}_f(n-1) + (1 - \mu) \left[ \frac{\sigma^2_{\min}(n)\hat{\nu}(n)\nu^T(n)}{D} \right] \]  
\[ \text{Now, } a(n) = \frac{\hat{M}_a(n)}{p_f(n)} \]  
where, 
\[ \hat{M}_a(n) = \mu \hat{M}_a(n-1) + (1 - \mu)g(n)\hat{\nu}^T(n)\hat{\nu}'(n) \]

\[ \hat{\nu}(n) \] is calculated as:
\[ \hat{\nu}(n) = \hat{\nu}(n-1) + \frac{(1 - \mu)g(n)\hat{\nu}'(n)}{x^T(n)\hat{\nu}'(n)} \]  
\( \hat{\nu} \) is chosen in the range \([0.6,0.9]\).

A self-tuning for scheduling of noise power is done to obtain the desired results by using ratio \( K \) which is given as:
\[ K = \frac{E[|d(n) - y(n)|^2]}{E[|y(n)|^2]} = \text{constant} \]

The ratio \( K \) is constant because power of signal \( e(n) \) and the power of signals \( y'(n) \) is almost constant with respect to change in signal \( x(n) \).
By utilizing the property of fast convergence deployed with optimized step size, the approximation of noise gain \( G(n) \) can be obtained.

\[
G(n) = \frac{p_f(n)}{\sqrt{(K+1)p_e(n)}} \tag{28}
\]

where \( p_e(n) \), power approximation of \( e(n) \) which is estimated using equation (14) and \( p_f(n) \), power estimation of \( \delta_1(n) \) which is given as:

\[
p_f(n) = \mu p_s(n-1) + (1-\mu)^2 \delta_1^T(n) \delta_1(n) \tag{29}
\]

where, \( \mu \) is the forgetting factor and \( \mu \) should be between 0.9 and 1.

The evaluation of optimal normalized parameters for step-size leads to increase in complexity due to lot of computation required with respect to Akhtar’s method. The technique used in obtaining a delay coefficient to compute optimal value for normalized variable step size is not efficient for highly time-varying secondary path.

### 3.5 Ahmed’s Method For Online Modeling Of Secondary Path

To overcome the problems due to computational complexity and obtaining a delay coefficient in Carini’s method, a new two stage approach to vary gain is applied [22]. This method uses normalized FxLMS and normalized LMS for control and modeling filters respectively. The schematic diagram is shown in Fig. 8. Two stages used to compute gain are:

1) When ANC system is not working in stable state. This situation occurs either at initialization of an ANC system or there is sharp variation in acoustic path. For this stage, gain parameter \( G(n) \) can be written as:

\[
G(n) = \frac{p_f(n-1)}{\| \delta(n) \|^2} \tag{30}
\]

\( p_f(n) \) can be evaluated as:

\[
p_f(n) = p_{\mu-y}(n) + p_{\mu-y}(n) \tag{31}
\]

Therefore, \( G(n) \) can be re-written as:

\[
G(n) = \frac{1}{\| \delta(n) \|^2} \tag{32}
\]

2) When ANC system is in stable state. For this stage, gain parameter \( G(n) \) can be computed as:

\[
G(n) = \begin{cases} 
\frac{p_f(n)}{p_e}; & \delta(n) > \frac{p_e}{p_f} \\
\delta(n) & \text{otherwise}
\end{cases} \tag{33}
\]

The parameter \( \delta(n) \) computed as:

\[
\delta(n) = \gamma \delta(n-1) + k(n) \left( \frac{p_f(n)}{p_e(n)} \right)^2 \tag{34}
\]

where \( 0 < \gamma < 1 \) and \( k(n) > 0 \) are control parameters. The autocorrelation function of \( f(n) \) and \( f(n-1) \) is represented by \( p_f(n) \). The estimation of \( p_f(n) \) is done as:

\[
p_f(n) = \mu p_f(n) + (1-\mu) f(n-1) \tag{35}
\]
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