

Different Models of Memristor

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Abstract—Memristor is regarded as a fourth passive circuit element with the advantages of variable resistance, flexibility, no leakage current, and compatibility with CMOS. The element memristor exhibits different characteristics for different applications which results into the formations of different models of memristor. This paper gives the brief explanation of different models of memristor.

Keywords—fitting parameters, memristance, threshold, window function.

I. INTRODUCTION

In 1971 L.O. Chua presented the memristor which is to be esteemed as the fourth passive circuit element after resistor, inductor and capacitor [1]. The uniqueness of memristor lies in its property of having variable resistance. One can define memristor as a two terminal passive non volatile device which is characterized by its memristance. The memristance yields the relation in the time integrals of voltage and current. Generally current controlled time invariant memristor is explicated as

$$\frac{dw}{dt} = f(w, i) \quad (1)$$

$$v(t) = R(w, i).i(t) \quad (2)$$

Where $v(t)$ is the voltage device $i(t)$, is the current of device, $R(w, t)$ is the memristance, w is the state variable and t is the time.

In 2008, HP labs claimed the existence of the physical working model of the memristor [2]. Afterwards the new element memristor opens the doors to the new types of electronics with wide range of applications which including logic design, neuromorphic systems and memory [3]-[6]. Since HP labs promulgated their work on the implementation of memristor, different models have been proposed to analyze, design and simulate memristor based circuits and applications. In this paper the major models of memristor will be discussed.

II. MODELS OF MEMRISTOR

Memristor can be used in wide range of applications. Hence different characteristics from memristor is required for each application some characteristics of memristor includes good scalability, low power consumption, flexibility and compatibility with CMOS. The major models of memristor are explained in this section.

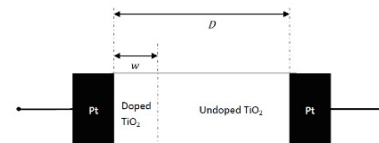


Figure1: HP memristor model [1]

A. Linear Ion Drift Model

This is the first physical model of memristor developed by R.S Williams in HP labs [2]. As shown in fig. 1, the width D of this model is shared into two regions. One region with width w is doped (with positive ions of oxygen, typically TiO_2) has low resistance and is therefore more conductive and another region is undoped. It is assumed that memristor have ohmic conductance, uniform field and average ion mobility. Hence the state variable equation is as follows :

$$\frac{dw}{dt} = \frac{\mu_v R_{ON}}{D} i(t) \quad (3)$$

$$v(t) = \left(R_{ON} \frac{w(t)}{D} + R_{OFF} \left(1 - \frac{w(t)}{D} \right) \right). i(t) \quad (4)$$

In (3) and (4) R_{ON} is resistance of devices at $w(t)=D$ and R_{OFF} is the resistance of device at $w(t)=0$. In this case $w(t)$ state variable is bounded within the limits of intervals $[0, D]$

B. Non Linear Ion Drift Model

Even the linear ion drift model is simple and fulfill the basic memristor equations but according to the experiments of the fabricated memristor device, it behaves in different manner and high non-linearity results are depicted [7] [8]. This leads to the development of another memristor model which is named as non linear ion drift model [9]. The current voltage relationship for this model is expressed as;

$$i(t) = w(t)^n \beta \sinh(\alpha v(t)) + \chi [\exp(\gamma v(t)) - 1] \quad (5)$$

In (5), α , β , γ and χ are called as fitting parameters and parameters n defines the shape of w state variable over the currents. The state variables are standized in the interval $[0, 1]$. The differential equation of state variable is

$$\frac{dw}{dt} = a. f(w). v(t)^m \quad (6)$$

Where a is a constant $f(w)$ is window function and m is odd integer.

The non-linearity of the device is dependent on voltage. The major application of this memristor model is in logic gates.

C. Simmons Tunnel Barrier Model

In 2008 D.B Strukov and M.D Pickett proposed another memristor model having higher accuracy than previously explained model [10]. This model also assumed the non-linearity and anti symmetric switching property. As shown in Fig. 2 the model has a resistor in series with electron tunnel barrier instead of two resistors in series as in the case of linear ion drift model. In this model, x is the simmons tunnel barrier width also known as state variable. Differentiation of x can be defined as oxygen vacancy drift velocity and is given as

$$\frac{dx(t)}{dt} = \begin{cases} C_{off} \sinh\left(\frac{i}{i_{off}} \exp\left[-\exp\left(\frac{x-a_{off}}{w_c} - \frac{|l|}{b}\right) - \frac{x}{w_c}\right]\right), & i < 0 \\ C_{on} \sinh\left(\frac{i}{i_{on}} \exp\left[-\exp\left(\frac{x-a_{on}}{w_c} - \frac{|l|}{b}\right) - \frac{x}{w_c}\right]\right), & i > 0 \end{cases} \quad (7)$$

Where $b, c_{on}, c_{off}, i_{on}, i_{off}, a_{on}, a_{off}$ are called as fitting parameters of the device provided $c_{on} > c_{off}$ and both put effect on the magnitude of change of x . i_{off} and i_{on} gives the value of current threshold. The parameters a_{off} and a_{on} confines the upper and the lower boundaries of x respectively. Since state variable derivative is effectively small than state variable in the defined range, this model does not require any window function. This is the main advantage of this model. Based upon the assumption of simmons tunnel barrier model, the current-voltage relation for this model is written as

$$i(t) = A(x, v_g) \phi(v_g, x) \exp\left(-B(v_g, x) \phi_1(v_g, x)^{0.5}\right) - \overline{A}(x, v_g) (\phi_1(v_g, x) + e|v_g|) \exp\left(-B(v_g, x) (\phi_1(v_g, x) + e|v_g|)^{0.5}\right) \quad (8)$$

$$v_{\square} = v - i(t) \quad (9)$$

As the control mechanism of this model is current, therefore it is widely used in the digital applications.

D. TEAM Model

The limitation of predefined model includes complexity, inexplicit VI relation leads to the requirement of another memristor model, which must have high accuracy with simplicity. Therefore in 2013, Shahar Kvatinsky developed the new model ‘TEAM’ (Threshold Adaptive Memristor Model) to overcome the above-mentioned limitation [11]. This model is similar to Simmons Tunnel Barrier Model but with much simpler expressions. This model needs some assumptions for simplification the analysis and computational efficiency. The assumptions are: state variable does not change below threshold and, polynomial; dependency involved between internal state derivative and current of the device. As per assumptions, state variable derivative can be written as

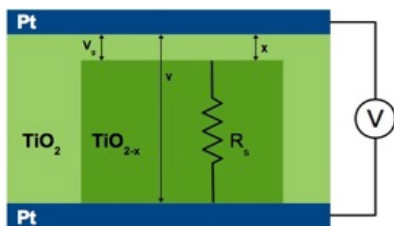


Figure 2: Physical memristor structure based on the Simmons tunnel barrier model [10].

$$\frac{dx(t)}{dt} = \begin{cases} k_{off} \cdot \left(\frac{i(t)}{i_{off}} - 1\right)^{\alpha_{off}} \cdot f_{off}(x), & 0 < i_{off} < i \\ k_{on} \cdot \left(\frac{i(t)}{i_{on}} - 1\right)^{\alpha_{on}} \cdot f_{on}(x), & i < i_{on} < 0 \\ 0, & otherwise \end{cases} \quad (10)$$

Where a_{on}, a_{off}, k_{on} , and k_{off} are constants (k_{off} is positive and k_{on} is negative) in (10). The parameters $f_{off}(x)$ and $f_{on}(x)$ are the window functions having dependency on state variable x .

The current voltage relationship is similar to the memristance, which linearly varies with x . Therefore

$$v(t) = \left[R_{ON} + \frac{R_{OFF} - R_{ON}}{x_{off} - x_{on}} (x - x_{on}) \right] i(t) \quad (11)$$

E. VTEAM Model

Some experiments on the memristor have expressed the existence of threshold voltage in spite of threshold current [2] [12] [13]. This results into the design of new memristor model. Hence, in 2014 Shahar Kvatinsky proposed a new model VTEAM (Voltage Threshold Adaptive Memristor Model) which contains threshold voltage [14]. Also threshold voltage is desirable to a greater extent than threshold current.

The VTEAM model is similar to TEAM model (both models has advantages like general, simple, designer friendly) but the only difference between the two is threshold. Therefore, the state variable derivative of VTEAM is same as that of TEAM model and is mathematically expressed as

$$\frac{dw(t)}{dt} = \begin{cases} k_{off} \cdot \left[\frac{v(t)}{v_{off}} - 1\right]^{\alpha_{off}} \cdot f_{off}(w), & 0 < v_{off} < v \\ 0, & v_{on} < v < v_{off} \\ k_{on} \cdot \left[\frac{v(t)}{v_{on}} - 1\right]^{\alpha_{on}} \cdot f_{on}(w), & v < v_{on} < 0 \end{cases} \quad (12)$$

Where $k_{off}, k_{on}, \alpha_{off}$ and α_{on} are constants. The parameters v_{off} and v_{on} are threshold voltage levels. Also, f_{on} and f_{off} describes the window functions which depends upon the state variable.

The current-voltage relationship for VTEAM can be written as

$$i(t) = \left[R_{ON} + \frac{R_{OFF} - R_{ON}}{w_{off} - w_{on}} (w - w_{off}) \right]^{-1} \cdot v(t) \quad (13)$$

where w_{off} and w_{on} defines the boundary of state variable w .

III. CONCLUSION

In this paper, different models of memristor- linear ion drift model, non-linear ion drift model, simmons tunnel barrier model, TEAM and VTEAM are explained. First three models does not contain any threshold which simply means their resistance varies for any voltage or current value. The TEAM and VTEAM models consist of threshold current and threshold voltage respectively. These two models are the most efficient with lesser computational complexity.

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