

# Development of Mathematical Model for Breakdown Voltage of Positive and Negative Point Electrodes in Small Air-gap Separations

Priye Kenneth Ainah, John Tarilayon Afa, Charles Godspwer Idongha  
Department of Electrical and Electronic Engineering  
Niger Delta University  
Amassoma, Wilberforce Island, Bayelsa State, Nigeria

**Abstract**—the paper aims at presenting a mathematical relationship between the fifty percent voltage breakdown and the gap length for positive and negative point electrodes for small airgap distances. Sharp non-uniform electrodes such as sharp, needle, blunt, and cone were used in this paper. Gap distances of 1cm to 12.5cm were maintained for the four electrodes and the breakdown test was carried out. The negative point electrode test was also carried out using the needle electrode. The experiment captured also capture the pressure, wet and dry mercury bulb temperature. Using the needle electrode (positive point) a mathematical relationship between the 50 percent probability and the gap distance was found. A generalize expression for positive and negative point electrodes, two constants  $K_e$  (Electrode coefficient), and  $K_p$  (polarity co-efficient) were found. With the appropriate application of these constants, the general mathematical model can be used to predict the breakdown voltage for small air gap distances for both positive and negative point electrodes

**Keywords**— Electrode co-efficient, non-uniform field, point electrode, Streamer mechanism, small air gap.

## I. INTRODUCTION

Electrical discharges between electrodes have long been a critical area of research in high-voltage engineering, plasma physics, and related fields [1][2][3][4]. When an electric field across a gap gets strong enough to ionize the air, electricity can pass through an insulating material and cause an electric discharge. Understanding this phenomenon is critical to comprehending the behavior of electrical systems, particularly those that operate at high voltages where insulation integrity and dependability are critical. Particularly interesting are point electrodes, which have a sharp tip and produce extremely concentrated electric fields [5]. Strong electric fields produced by positive point electrodes which have a higher potential than their surroundings have the ability to ionize the air around them, causing a corona discharge and eventual breakage. On the other hand, because of the attraction of positive ions and the repulsion of electrons, negative point electrodes display distinct field distributions and discharge patterns. These electrodes are commonly used in a variety of applications, including electrical insulation research, high-voltage testing, and lightning arrester design [6][7]. The fundamental processes governing electrical breakdown, the dynamics of ionized gases, and the behavior of electric fields near sharp, localized points is

essential for designing and optimizing electrical devices and systems. Insights into these phenomena can be obtained by studying the behavior of discharges between point electrodes and either a flat surface or another point electrode. To attain the intended electrical properties, the distance between electrodes is often kept to a minimum in high-voltage applications [8]. Particularly essential in situations requiring precise control of the electric field are small air gap separations. In these kinds of arrangements, the behavior of the positive and negative point electrodes has a big influence on the system's overall dependability and performance. The properties of the discharge are mostly determined by the air gap spacing between the electrodes. Especially, small air gap separations bring special difficulties and behaviors because of the strong electric fields produced and the quick ionization processes that take place. Gaining an understanding of these phenomena is essential for boosting electrical insulation system performance, creating safer and better-designed high-voltage equipment, and creating stronger safeguards against electrical discharges. The research seeks to develop a general mathematical model that can be used for positive and negative breakdown voltage for small air gap separation. By employing both experimental and analytical approach, a thorough mathematical model for positive and negative point electrode breakdown voltage in small air gap separations was developed in this paper.

## II. CLASSIC THEORIES OF BREAKDOWN

Electrical breakdown in dielectrics, particularly gases, has been extensively studied, leading to the development of several classic theories [9][10][11]. These theories explain the mechanisms by which a material, usually an insulator, becomes conductive under high electric fields. The main classic theories of breakdown include Townsend's theory, Streamer theory, and Paschen's law. Each theory offers insights into different aspects of the breakdown process and is applicable under various conditions. The two primary theories explaining the collapse uniform field with small gaps are the streamer theory proposed by Loeb and Meek [12][13], and the well-known avalanche theory, also referred to as the Townsend and Raether mechanisms. The Townsend criterion for spark formation at low pressure can be expressed as follows:

$$\gamma \left\{ \exp \left( \int_0^d \alpha dx \right) - 1 \right\} = 1 \quad (1)$$

This could be rewritten as  $\int_0^d \alpha dx = \log_e (1 + 1/\gamma)$

$\alpha$  and  $\gamma$  depicts the first and second coefficients of Townsend Criterion.

When the smaller electrode is negatively charged at low pressures, the breakdown voltage is typically lower. At greater pressures, this also holds for non-attaching gases. Because of the stronger field at the cathode, or larger  $\gamma$ , a smaller value is required, which explains the effect of  $\gamma$  to satisfy the criterion by the equation (1).

Within the framework of streamer theory, Meek and Raether [12][13][14], have also addressed breakdown in a non-uniform field. In this scenario, the space charge field  $E$  at the head of the avalanche, after it has traveled a distance  $x$  in a non-uniform field, is expressed as follows [14]:

$$E_r = \frac{5.27 \times 10^{-1} \alpha \exp\left(\int_0^x \alpha dx\right)}{(x/p)^{1/2}} \text{ v/cm} \quad (2)$$

Where,  $p$  is the gas pressure.

When the space charge field is close to the applied field at the head of the avalanche in terms of magnitude, the criterion for streamer formation is reached [13][15] and can be used to determine the corona onset voltage in the non-uniform electrode system. At high pressures, breakdown typically occurs at a lower voltage when the smaller electrode is positive. This pronounced polarity effect at high pressures is generally attributed to the nature of the space charge in the pre-breakdown current. Corona forms at the small electrode as soon as the field gradient in its vicinity reaches a sufficient magnitude. With a positive point electrode, electrons are drawn into the anode, and this can lead to the formation of a positive ion space charge which subsequently results in positive streamers and complete breakdown [14][15]. Several researchers have proposed models or theories to explain the characteristics of the 50% breakdown voltage of long gaps. These models of gas spark over can be explained by one of the following mechanisms [16]:

When the space charge field  $E$  gets close to the applied field's magnitude at the avalanche's head, streamers form [13][15]. In non-uniform electrode systems, corona onset voltage has been successfully determined using this criterion. If the smaller electrode is positive, breakdown typically occurs at a lower voltage at high pressures. The pre-breakdown current's space charge properties are typically responsible for this strong polarity effect at high pressures. When the field gradient near the small electrode reaches a critical value, corona forms there. When utilizing a positive point electrode, the anode attracts electrons, producing a positive ion space charge that eventually causes breakdown and the development of positive streamers [14, 15]. The 50% breakdown voltage properties of lengthy gaps have been explained by a number of models and theories put forth by academics. There are currently no models that take into account every aspect of the leader corona size, number of densities, microscopic voltage gradient, distribution of charge, temperature, column radius, velocity, etc., in one comprehensive model. Some theories or models derive numerical estimates for the key governing elements from experimental data.

#### A. Breakdown Models for Long Air Gap in Non-uniform Field

Lemeke's model is among the significant models that were put out. This model takes into account the characteristics of two elements in the leader channel: a leader-corona discharge tip with a length of  $L_t$  and a potential gradient of  $E_t$ , and a leader length of  $L_l$ . The voltage at which sparkover occurs is...

$$V_t = E_l L_l + E_t L_t \quad (3)$$

With a 1-meter interval between measurements, the potential gradient  $V_{p\_t}$  in the leader corona's streamers is estimated to be 4.5 kV/cm, focusing on the tip of the corona. [17] is the crucial flashover voltage.

$$V_s = E_t [1 + (1 + E_0/E_t) \cdot \ln(d - \ln d)] \\ = 450[1 + 1.33 \ln(d - \ln d)] \text{ kV} \quad (4)$$

where Lemke picked 1.5 kV/cm for  $E_0$  and  $d$  is measured in centimeters.

Three parameters are considered to control the breakdown in Alexandrou's model: the radius of curvature of the tip ( $r$ ), the electric field intensity at the head of the growing discharge ( $E_t$ ), and the leader tip potential ( $V_t$  s  $V_s$ ). Their relationship is denoted by [16].

$$V_t = E_t \cdot \sqrt{1 - \frac{r}{d}} \tanh^{-1} \sqrt{1 - r/d} \quad (5)$$

When the leader-tip potential is denoted by  $V_t$ , the radius of curvature is represented by  $r$ , which is estimated as 0.91 meters for a 2-meter gap, and the electric field gradient at the head of the growing discharge is denoted by  $E_t$ . The value of  $E_t$  under spark over circumstances is 12.6 kV/cm (1260 kV/m).

Thus, the spark over voltage equation is

$$V_s = 1260 r \sqrt{1 - r/d} \tanh^{-1} \sqrt{1 - r/d} \text{ kV} \quad (6)$$

Water's model assumes that the current flow in the positive leader column  $I$  relies parabolically on the tip potential  $V$ , as determined by Townsend. It is believed that the unbridged gap between the plane cathode and the leader tip will collapse when the leader-tip potential equals  $(0.5 V + 175)$  (0.5V+175) kV, where  $V$  is the anode potential. The sparkover voltage was determined from additional experimental observations using a 20 cm leader length in a 2-meter gap at 1000 kV [12].

$$V_s = [1.5 \times 10^6 + 3.5 \times 10^5 d]^{1/2} - 350 \text{ kV} \quad (7)$$

Based on findings made by the Electricite de France team, Waters also generated the following formula, where  $d$  is measured in meters [18].

$$V_s = 563 \cdot d^{0.5} \text{ kV} \quad (8)$$

The variation of the critical flashover (CFO) voltage is roughly represented by  $d^{0.5-0.6}$  ( $V_{50} \propto d^{(0.5-0.6)}$ ). A significant amount of experimental data, primarily collected by Paris and his colleagues, was used to determine this link.

The clearance needed for a rod-plane gap of the same length is proportional to the air gap necessary from a conductor to a tower. The Paris formula for V50V50, with  $d$  in meters, is provided for the rod-plane gap as.

$$V_{50} = 500d^{0.6} \text{ kV} \quad (9)$$

Many nations have started using this technique for creating air gap clearances. Long air gaps are the foundation of most established work because of their importance in high voltage engineering. Many attempts have been made to relate rod-plane gap data to data for other structures. Researchers have demonstrated [19][20][21] that the expression corresponds to the positive 50% switching surge voltage of various airborne constructions between 2 and 8 meters.

$$V_{50} = K_g 500d^{0.6} \text{ kV} \quad (10)$$

where  $G$  is the gap factor, or the proportionality ratio of the 50% flashover voltage of any gap geometry to that of a rod-plane gap, and  $d$  is the gap distance in meters. A more inclusive expression that provides the minimal strength and pertains to a longer time to crest was put out by Gallet and Leroy [22],[23]. It is as follows.

$$V_{50} = K \frac{3450}{1 + \frac{8}{d}} \text{ (kV)} \quad (11)$$

Where  $K_g$  and  $d$  have the same meaning as in equation (10).

Because of the influence of strong secondary phenomena, calculating the field distribution in lengthy gaps does not yield as clear an insight into breakdown features as it does in very tiny gaps. As a result, the research concentrated on tiny gaps and contrasted them with larger gaps. [22][24]

### III. MATERIALS AND METHODS

#### A. Materials used

Materials for positive and negative point electrodes for tiny gaps that were utilized in the formulation of the mathematical model are;

1. Single-stage high voltage kit (Figure 1)
2. Electrodes and configuration (such as needle, sharp, blunt, and cone)

#### B. Method of Analysis

The methods used for the formulation of the mathematical model for positive and negative electrodes for small air gaps took the following steps;

- Experimental Test to determine the breakdown at 50% with different air gaps using different electrodes such as needle, sharp, blunt, and cone
- Development of the mathematical model for breakdown voltages for needle, sharp, blunt, and cone electrodes.

##### B.1 Experimental Test Setup and Procedure

The test was done within the months of October and November therefore the humidity variation was not pronounced. The equipment has the A.C., D.C., and impulse stages. To use the equipment for the negative polarity test, the rectifier G.S (figure 1) was reversed. The layout of the equipment is in Figure 1. The four types of electrode configurations (needle, sharp, blunt, and cone) were used for the test and their geometries and tip dimensions are shown in Table 1.

The gap distances maintained in the experiments were 1cm, 2cm, 3cm, 5cm, 8cm, 10cm, 12cm, and 12.5cm (for all positive point arrangements). Using the electrode stand TG, needle, blunt, sharp and cone electrodes were fixed different with the

specified gap distance, and the test were carried out. Before each test, the wet bulb thermometer, barometer pressure and the dry bulb thermometer were read. With the given test gap, the impulse voltage and the spark gap (Kf) were adjusted to give a percentage of breakdown probability, having allowed 10 shots in each test. At least three breakdown probability tests were carried out for each gap distance. The test was also carried out for the negative point electrode with a maximum test gap distance of 10cm.

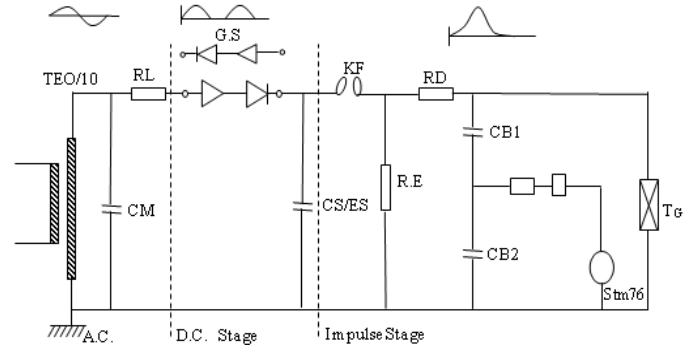


Fig. 1: Circuit Layout of Different Stages of H.V Equipment

Table 1: Different Electrode Configuration

S/N	Type of Electrode	Configuration	Tip Dimension
1	Needle		0.25 mm
2	Sharp		0.75 mm
3	Blunt		0.9 mm
4	Cone		1.3 mm

#### B.2 Development of the mathematical model for breakdown voltage

The mathematical model was developed by first experimenting with different electrodes to determine the breakdown voltage at 50% with different gap distances. The following steps were used to determine the relationship between fifty percent breakdown and the gap length for positive and negative point electrodes for small air gap distances;

- Determination of Breakdown probability, temperature, and pressure
- Determination of fifty percent breakdown voltages for all tested electrodes
- Derive the voltage breakdown for the needle electrode
- Determination of polarity co-efficient ( $K_p$ ) and electrode configuration co-efficient ( $K_E$ )
- Formulation of the mathematical expression for breakdown voltage for both polarities (negative and positive)

### IV. RESULTS AND DISCUSSION

#### A. Results

Results from the experiments for needle electrodes are depicted in Table 2. The  $V_{50\%}$  was corrected with the humidity and temperature values are also shown in Table 2.

The test object flashover voltage is given as;

$$V_f = V_r \times h/d \text{ but } d = \frac{0.289b}{273 + t} \quad (12)$$

Where,  $V_f$  is voltage under test condition,  $V_r$  = voltage under references atmospheric conditions,  $h$  = humidity correction factors,  $d$  = air density correction factor,  $t$  = atmospheric temperature  $^{\circ}\text{C}$ , and  $b$  = atmospheric pressure in millibars. From the corrected values (humidity), the fifty percent breakdown voltages for all tested electrodes were tabulated as given in Table 2 for the gap distances. The graph of fifty percent breakdown voltage ( $V_{50\%}$ ) and gap distance was drawn as it is shown in Figure 2

Table 2: Breakdown of probability, temperature, and pressure

Gap length cm	Break down voltage (kV)	Breakdown probability	$V_{50\%}$ (kV)	Wet bulb thermo meter $t_w$ $^{\circ}\text{C}$	Dry bulb thermo meter $t_d$ $^{\circ}\text{C}$	Pressure mmHg
1	15 18 22	5:10 9:10 10:10	16	26.8	31	765
2	35 37 38	2:10 6:10 10:10	36	27	32	749
3	40 42 46	2:10 3:10 6:10	43	26	31	761
5	56 58 59 63	2:10 3:10 5:10 6:10	59	26.1	30.8	759
8	75 79 84	3:10 6:10 10:10	78	27.2	31	750
10	80 82 86	0:10 1:10 6:10	85.6	27	31	758
12	89 91 94	1:10 4:10 4:10	94	25.9	29.7	760
12.5	96 91 98	5:10 1:10 6:10	96	127.2	31.8	763

Table 3: Critical flashover voltage for the Different Electrode Arrangements

Distance	Positive Point Polarity $V_{50\%}$ (kV)				Negative Point Polarity $V_{50\%}$
	Needle $V_{50\%}$	Sharp $V_{50\%}$	Blunt $V_{50\%}$	Cone $V_{50\%}$	Needle $V_{50\%}$
1.0	16.3	17.0	17.3	19.0	24.0
2.0	36.0	36.5	37.0	39.0	49.3
3.0	42.6	46.3	51.0	51.0	68.0
5.0	59.4	63.8	66.8	69.0	90.0
8.0	75.8	82.0	88.0	90.0	141.0
10.0	79.0	88.0	106	108.0	149.0
12.0	92.8	101.2	112	114.5	----
12.5	97.0	103.0	114	123.5	----

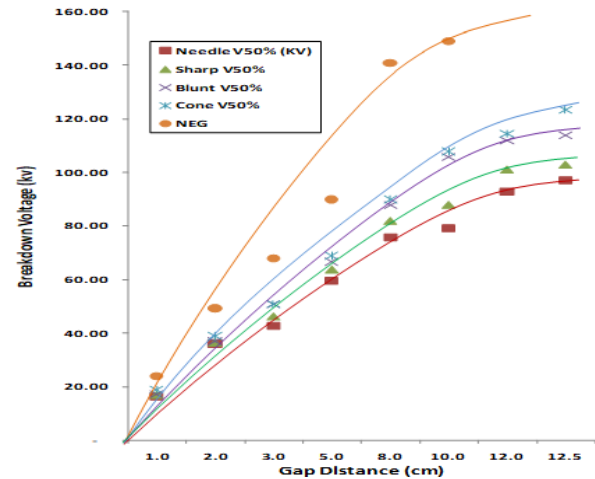


Fig. 2: Graph of Breakdown voltage ( $V_{50\%}$ ) and gap distance.

From the graph of Figure 3, a quadratic equation was derived to describe the voltage breakdown for the needle electrode as shown in equation 13. The quadratic equation was used to characterize the voltage breakdown for the needle electrode. Table 4 displays the precise needle electrode values that were obtained from Table 3 columns 1 and 2.

$$VB = A + Bd + Cd^2 \quad (13)$$

where,  $d$  stands for the electrode gap distance while  $A$ ,  $B$ , and  $C$  are constants

By applying equations 14(a), 14(b), and 14(c), and using a MATLAB program flowchart shown in figure 4. The values of the constants for curve fitting and the constants of the equations were calculated as shown in Table 4. The values of the constants  $A$ ,  $B$ , and  $C$  were ascertained as  $A = 5.32$ ,  $B = 12.85$ , and  $C$  is equal to  $-0.47$ .

$$\Sigma Y = nA + B\Sigma X + C\Sigma X^2 \quad (14a)$$

$$\Sigma XY = A\Sigma X + B\Sigma X^2 + C\Sigma X^3 \quad (14b)$$

$$\Sigma X^2 Y = A\Sigma X^2 + B\Sigma X^3 + C\Sigma X^4 \quad (14c)$$

Table 4:  $V_{50\%}$  Breakdown (bd) voltages and gap distances for positive point needle electrode.

Gap distance (cm)	1.0	2.0	3.0	5.0	8.0	10.0	12.0	12.5
$V_{50\%}$ bd Voltage	16.3	36.0	42.6	59.4	75.8	79.0	92.8	97.0



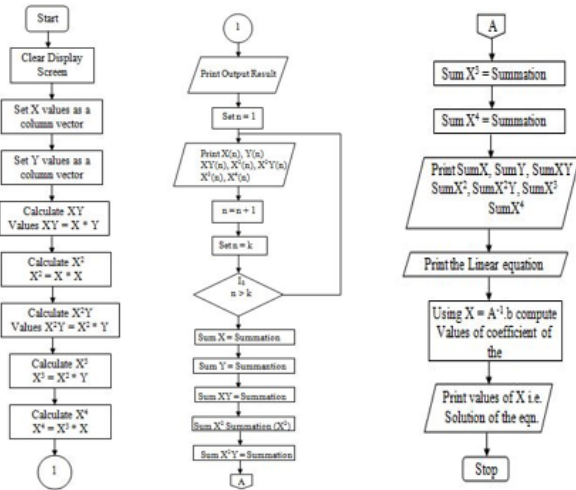


Fig 4. Flow chart for tabulating and the solution of simultaneous equations

Table 4: Curve Fitting Values

X	Y	XY	X <sup>2</sup>	X <sup>2</sup> Y	X <sup>3</sup>	X <sup>4</sup>
1.00	16.30	16.30	1.00	16.30	1.00	1.00
2.00	36.00	72.00	4.00	144.00	8.00	16.00
3.00	42.60	127.80	9.00	383.40	27.00	81.00
5.00	59.40	297.00	25.00	1485.00	125.00	625.00
8.00	75.80	606.40	64.00	4851.00	512.00	4096.00
10.00	79.00	790.00	100.00	7900.00	1000.00	10000.00
12.00	92.00	1113.60	144.00	13363.20	1728.00	20736.00
12.50	97.00	1212.50	156.25	15156.25	1953.12	24414.06
Σ53.50	Σ498.90	Σ4235.60	Σ503.25	Σ43299.35	Σ5354.12	Σ59969.06

Consequently, the needle electrode breakdown equation can be written as follows:

$$V_B = 5.32 + 12.85d - 0.47d^2 \quad (15)$$

To determine the mathematical model, electrode configuration co-efficient ( $K_e$ ) must be defined. The electrode configuration co-efficient ( $K_e$ ) is expressed as the ratio of the breakdown voltage between the reference electrodes to any other positive point electrodes as depicted in equation 16.

$$K_e = \frac{K_{A50\%}}{K_{N50\%}} \quad (16)$$

where  $K_{A50\%}$  and  $K_{N50\%}$  are  $V_{50\%}$  of any other electrode configuration, and  $V_{50\%}$  of needle electrode configuration respectively.

The link between the breakdown voltage of the reference electrode and that of any other positive point electrodes is represented by the electrode configuration coefficient,  $K_e$  depicted in Table 5 and the values were worked out from Table 3 and are tabulated in Table 5. The average electrode configuration co-efficient ( $K_e$ ) value for needle, sharp, Blunt and cone are 1.00, 1.06, 1.15 and 1.20 respectively

Table 5. Electrode configuration coefficient  $K_e$  values

Distance (cm)	1.0	2.0	3.0	5.0	8.0	10.0	12.0	12.5
Needle	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Sharp	1.06	1.014	1.09	1.06	1.06	1.05	1.07	1.06
Blunt	1.08	1.06	1.17	1.13	1.14	1.22	1.17	1.15
Cone	1.13	1.14	1.20	1.19	1.19	1.27	1.24	1.22

Also, the negative polarity of the needle electrode to positive (reference) polarity of needle electrode as depicted in equation 17.

$$K_p = \frac{V_{50\%} \text{ for Negative Polarity (needle)}}{V_{50\%} \text{ for positive (needle)}} \quad (17)$$

The values of the polarity coefficient and the gap distances were tabulated as shown in Table 6.

Table 6: Electrode distances and coefficient  $K_p$

Electrode distance	1	2	3	5	8	10
Electrode coefficient $K_p$	1.49	1.42	1.67	1.86	1.90	1.95

From the coefficient  $K_p$  and the gap distance in Table 5, a straight-line graph is drawn as shown in figure 5.

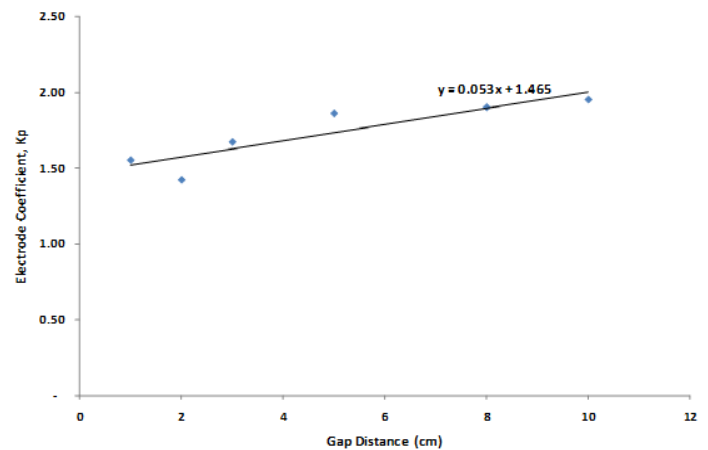


Fig 5: Graph of Polarity Constant and Gap Distant.

From Figure 5,  $K_p$  is given as;

$$K_p = mx + C \quad (18)$$

Where  $m$  = gradient of the line,  $c$  = constant (intercept), and  $x = d$  = electrode gap distance

Using the least square method, the solution of the equation is given as;  $a = 1.37$  and  $b = 0.067$

$K_p$  can be expressed as;

$$0.067d + 1.37 \quad (19)$$

Therefore, the general expression (mathematical model) for breakdown voltage for both polarities (negative and positive) is given by equation (18).

$$V = K_p K_e V_B \quad (20)$$

where  $V_B = 5.32 + 12.8d - 0.47d^2$

## B. Discussion

From the experiment, the critical flashover voltage  $V_{50\%}$  was found using the needle electrode that is, figuring out how spark-over voltage and gap length function together. The polynomial form of expression was used in this relationship. To apply to other gap geometries, a relationship  $K_e$  was involved. The values of  $K_e$  when compared to long gaps in practical engineering situations were found to be similar. This is because different experts have drawn different conclusions from the avalanche and streamer ideas. In his conversations, K. Berger [12][16] pointed out that there is still much to learn about the underlying characteristics of breakdown in a 60 cm gap as opposed to a 6-meter gap, even if the breakdown voltages  $V_S$  and gap distance have identical external characteristics. Therefore, as the gap length increases careful consideration is required for both the electrode coefficient and the polarity coefficient. The polarity effect increases with the gap length due to greater asymmetry in the field. In the experiment the value  $K_p$  increases with the gap length in a linear form [20].

## V. CONCLUSION

It has been established that some controlling factors affect the breakdown of air gap. Some of these factors governing the spark over are not easily controlled in long gaps and, therefore may not lead to a clear understanding of the breakdown characteristics of the gap. As a result, experimental methods are chosen with properly designed electrodes that represent as much as possible to that encountered in the field. No known models have represented a negative point polarity breakdown. From the mathematical model presented both polarities could be used. This was because the factors could easily be controlled. Humidity variation is a common parameter in the tropical region, for the same environment the humidity could vary from 55 percent to 85 percent (relative) depending on the dry and wet seasons. Therefore, this formula can generally be applied if the humidity variation is taken into consideration. For positive point electrodes, the expression can be used for medium and long gaps and outdoor situations but the negative point electrodes may have some deviations in certain conditions of humidity level.

For indoor conditions especially for small and medium gaps, if the parameters are properly coordinated the expression can be used for any rod-plane electrode configurations (negative and positive).

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