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Development of a Six Sigma Double Sampling Plan using Kullback-Leibler Based Minimum sum of Risk

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Abstract - Six Sigma quality requirements are associated with extremely low levels of non-conformities, making the design of effective attribute sampling plans challenging under stringent quality requirements. This study develops a Kullback-Leibler (KL) information-based Double Sampling Plan (DSP) under a Poisson model for quality assessment at Six Sigma levels. The proposed methodology uses KL information between the specified acceptable quality level (AQL_{KL}) and limiting quality level (LQL_{KL}) to determine the required size of sample 'n', which is used equally at both stages of the double sampling plan, i.e., $n_1 = n_2 = n$. For the specified Six Sigma producer's and consumer's risk requirements, the acceptance numbers c_1 and c_2 are evaluated. Among the feasible plans, the minimum sum of producer's and consumer's risks are used as an additional criterion for selecting the sampling-plan parameters. The statistical performance of the proposed KL-information-based DSP is evaluated using its Operating Characteristic (OC) and Average Run Length (ARL) curves. Numerical illustrations and comparisons with an existing Six Sigma-based double sampling approach are presented to examine the discriminatory and detection performance of the proposed KL-information-based DSP. Tables are also developed for different combinations of AQL_{KL} and LQL_{KL} to facilitate practical implementation. The proposed approach provides an information-based framework for designing a Six Sigma-oriented Double Sampling Plan with equal size of samples at the two inspection stages and risk-based selection of the decision parameters.

Keywords - Six Sigma Sampling Plan, Six Sigma quality level, Double Sampling Plan, Kullback-Leibler Information, Poisson Distribution, Operating Characteristic curve, Minimum sum of risk, Average Run Length.

I. INTRODUCTION

Six Sigma is a structured quality improvement methodology aimed at reducing process variability and minimizing the occurrence of non-conforming units. At a Six Sigma quality level, the fraction defective is extremely small, which makes the design and implementation of conventional attribute acceptance sampling procedures challenging. Under such stringent quality requirements, a sampling plan should provide reliable lot decisions while maintaining specified producer's and consumer's risks. Therefore, the development of sampling procedures capable of discriminating effectively between very low levels of fraction defective is an important problem in statistical quality control.

Attribute acceptance sampling provides a statistical decision mechanism for classifying a submitted lot based on the number of non-conforming units observed in a sample. Among the available attribute sampling schemes, the Double Sampling Plan (DSP) provides greater flexibility than a Single Sampling Plan because the decision may be reached after the first sample when the observed evidence is sufficiently clear, while a second sample is examined when the first-stage result falls within an intermediate region. Guenther [1] presented a procedure for finding Double Sampling Plans for attributes based on specified operating characteristics. Subsequently, Soundararajan and Devaraj Arumainayagam [2] developed a generalized procedure for the selection of attribute Double Sampling Plans under specified quality and risk requirements. Further developments considered the construction and computational selection of Double Sampling Plans through appropriate operating-characteristic requirements [3], [4].

The selection of attribute Double Sampling Plans has also been studied in relation to specified acceptable and limiting quality levels. Govindaraju and Subramani [5] developed procedures for selecting Double Sampling Plans for specified acceptable quality level (AQL) and limiting quality level (LQL), incorporating producer's and consumer's risks into the plan-selection process. Kuralmani and Govindaraju [6] subsequently developed modified selection tables for Double Sampling Plans indexed by AQL and LQL under the Poisson model. These developments established the importance of specifying reference quality levels and corresponding risk requirements in the construction of attribute Double Sampling Plans.

Risk-based criteria have also been considered in the selection of sampling plans. Suresh and Sangeetha [7] investigated the selection of a special type of Double Sampling Plan using the minimum sum of risks as a criterion for identifying a suitable plan. This provides a useful basis for selecting among several feasible sampling-plan configurations when more than one combination of decision parameters satisfy the prescribed risk requirements. The Poisson distribution is commonly useful in attribute sampling situations involving counts of non-conformities, particularly when the fraction defective is small. Extensions of Double Sampling Plans have therefore also been investigated using modified P/;oisson-type models. Haridoss and Subramani [8] considered the design of optimal Double Sampling Plans using the weighted Poisson distribution, while Radhakrishnan and Sekkizhar [9] investigated sampling-plan

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construction using an intervened random-effect Poisson distribution. These studies demonstrate the applicability of Poisson-based and modified Poisson models in developing attribute sampling procedures for specialized quality-assurance situations.

The concept of Six Sigma has subsequently been incorporated into the construction of acceptance sampling plans. A significant development in this direction was initiated by R. Radhakrishnan and P.K. Sivakumaran, who pioneered the incorporation of Six Sigma quality levels into acceptance sampling plan construction. Radhakrishnan and Glorypersial [10] developed mixed sampling plans indexed through Six Sigma quality levels with a Double Sampling Plan as the attribute component. Radhakrishnan and Sivakumaran [11] specifically developed Double Sampling Plans indexed through Six Sigma quality levels, thereby establishing a direct connection between Six Sigma quality requirements and Double Sampling methodology. Further developments extended the Double Sampling framework to other quality-oriented sampling structures, including three-class attribute Double Sampling Plans indexed through maximum allowable average outgoing quality [12]. These studies demonstrate the continued development of attribute sampling procedures for increasingly stringent quality-performance requirements.

The theoretical and practical foundations of statistical quality control and acceptance sampling, including producer's risk, consumer's risk, operating characteristic curves, and sampling-plan selection, have also been discussed comprehensively by Montgomery [13]. Schilling and Neubauer [14] provide a detailed treatment of acceptance sampling, including the construction and evaluation of attribute sampling plans and their associated operating characteristics. These references provide the broader statistical foundation for evaluating the performance of the sampling procedure developed in the present study.

Although these developments have provided several approaches for constructing Double Sampling Plans, the determination of an appropriate sample size remains particularly challenging when the acceptable and limiting quality levels are extremely small. Conventional approaches generally determine the sampling-plan parameters through tabulated procedures, operating-characteristic requirements, or direct search over feasible combinations of sample sizes and acceptance numbers. The present study considers an information-based approach in which the distinction between two specified quality levels is quantified using Kullback–Leibler (KL) information.

Let the acceptable quality level be denoted by AQL_{KL} and the limiting quality level by LQL_{KL}, with $AQL_{KL} < LQL_{KL}$. The proposed methodology uses these two quality levels as the reference points for determining the information required to discriminate between acceptable and undesirable lots. Under the Poisson model, KL information is used to determine the required common sample size, which is subsequently incorporated into a Double Sampling structure with equal sample sizes at the two inspection stages, $n_1 = n_2 = n$. Thus, KL information determines the amount of inspection required, while the Double Sampling mechanism provides the staged decision structure.

After determining the common sample size, feasible combinations of the first-stage acceptance number c_1 and final acceptance number c_2 are examined under the specified Six Sigma producer's and consumer's risk requirements. Among the feasible combinations, the minimum sum of producer's and consumer's risks is used as an additional criterion for selecting the preferred sampling plan. The proposed framework therefore integrates three components: KL information for sample-size determination, Double Sampling for staged inspection, and minimum sum of risk for selection of the decision parameters.

In the present study, the stringent Six Sigma producer's risk is specified as $\alpha = 3.4 \text{ X } 10^{-6}$ and the consumer's risk is taken as $\beta = 2*\alpha$.

The statistical performance of the resulting KL-information-based Double Sampling Plan is examined through its Operating Characteristic (OC) and Average Run Length (ARL) behaviour. The proposed plan is also compared with an existing Six Sigma-based Double Sampling Plan to assess its discriminatory and detection performance. In addition, selection tables are developed for different combinations of AQL_{KL} and LQL_{KL}, to facilitate practical implementation of the proposed procedure.

The main objective of this study is to develop a Six Sigma-based Double Sampling Plan in which Kullback–Leibler information is used to determine the required sample size and the minimum sum of producer's and consumer's risks is used to select the decision parameters. The study further evaluates the OC and ARL performance of the proposed plan, compares its performance with an existing Six Sigma-based Double Sampling Plan, and provides selection tables for practical application under different acceptable and limiting quality-level combinations.

Glossary of Symbols

N - Lot size
n_1, n_2 - size of samples at the first and second stages respectively; $n_1 = n_2 = n$.
c_1 - First-stage acceptance number.
c_2 - Second-stage acceptance number.
d_1 - Number of non-conformities in the first sample
d_2 - Number of non-conformities in the second sample
D -Combined number of non - conformities, where $D = d_1 + d_2$.
AQL_{KL} -Six Sigma quality level - 1 under the proposed KL-based sampling plan.
LQL_{KL} - Six Sigma quality level - 2 under the proposed KL-based sampling plan.
α -Producer's risk
β -Consumer's risk
$P_a(p)$ - probability of acceptance
MSR - minimum sum of risk
p^* - KL discrimination point
OC - Operating Characteristic

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ARL - Average Run Length.

II. DEVELOPMENT OF THE PROPOSED KL-INFORMATION-BASED DOUBLE SAMPLING PLAN

A Double Sampling Plan (DSP) is an attribute inspection procedure in which the decision concerning a submitted lot may be reached using information obtained from one or two samples. In the first stage, a sample is inspected and its observed number of non-conformities is used to determine whether the lot can be accepted or rejected immediately. When the first-stage observation does not provide sufficient evidence for either decision, an additional sample is inspected and the information from both stages is combined to obtain the final decision.

In the proposed approach, the Double Sampling structure is integrated with KL information for determining the size of sample under specified Six Sigma quality requirements. The proposed plan is based on Poisson distribution and uses Kullback-Leibler information to determine the required size of sample. Equal size of samples are considered at the two stages of inspection. Hence, [$n_1 = n_2 = n$] where 'n' denotes the common size of sample used at each stage. Let AQL_{KL} denote the specified acceptable quality level and LQL_{KL} denote the specified limiting quality level, where $AQL_{KL} < LQL_{KL}$. The plan is characterized by the first-stage size of sample n_1, second-stage size of sample n_2, first-stage acceptance value c_1 and final acceptance value c_2. The values of these parameters [n_1, n_2, c_1, c_2] determine whether a lot is accepted at the first stage, rejected at the first stage, or subjected to further inspection.

The KL-information criterion provides the basis for determining the amount of inspection required to distinguish between the specified acceptable and limiting quality levels. After obtaining the required size of sample, feasible combinations of c_1 and c_2 are examined under the prescribed producer's and consumer's risk constraints. The feasible combination having the minimum sum of these risks is selected as the preferred design.

The resulting framework therefore combines three elements: KL information for sample-size determination, the Double Sampling mechanism for staged inspection, and the minimum sum of risk criterion for selecting the decision parameters. This provides the basis for constructing the proposed Six Sigma-based sampling plan.

2.1 Conditions for applications:

i) The proposed procedure is intended for attribute inspection under stringent Six Sigma quality requirements.

ii) The acceptable and limiting quality levels satisfy $AQL_{KL} < LQL_{KL}$.

iii) The producer's risk is specified as $\alpha = 3.4 \times 10^{-6}$ and the consumer's risk is taken as $\beta = 2*\alpha$.

iv) Equal size of samples are used at the two stages, so that $n_1 = n_2 = n$.

2.2 Design procedure of the Proposed Double Sampling Plan:

Consider a finite lot of size N, from which samples of sizes n_1 and n_2 are drawn at the first and second stages, respectively. Let d_1 and d_2 denote the observed numbers of non-conformities in the two samples.

The operating procedure of Double Sampling Plan is as follows:

Step 1: A first random sample of size (n_1) is selected from the lot N under inspection, and the number of non-conformities observed in this sample is recorded as (d_1).

Step 2: If the observed number of non-conformities satisfies ($d_1 \leq c_1$), the lot is accepted without further inspection.

Step 3: If the number of non-conformities in the first sample is greater than (c_2), i.e., ($d_1 > c_2$), the lot is rejected immediately.

Step 4: When the first-sample result falls in the intermediate region, ($c_1 < d_1 \leq c_2$), a second random sample of size (n_2) is drawn from the remaining portion of the lot. The number of non-conformities in the second sample is denoted by (d_2).

Step 5: The numbers of non-conformities obtained from both samples are combined (i.e,) $D = d_1 + d_2$ that satisfies ($d_1 + d_2 \leq c_2$), the lot is accepted.

Step 6: If the combined number of non-conformities exceeds the second-stage acceptance number, i.e., ($d_1 + d_2 > c_2$), the lot is rejected

The equal-sample-size structure simplifies the implementation of the two-stage inspection procedure while retaining the principal advantage of a DSP, namely that a conclusive first-stage result avoids unnecessary second-stage inspection.

2.3 Operating Characteristics Function:

Under the Poisson model, the probability of acceptance is obtained by considering acceptance after the first stage and acceptance following the second-stage inspection. It can be expressed as

$$P_a(p) = \sum_{x=0}^{c_1} p_i + \sum_{j=c_1+1}^{c_2} P_j \sum_{i=0}^{c_2-j} q_i \qquad (1)$$

Where,

$$p_i = \frac{e^{-n_1 p} (n_1 p)^i}{i!} \qquad (2)$$

$$q_i = \frac{e^{-n_2 p} (n_2 p)^i}{i!} \qquad (3)$$

The OC curve is obtained by evaluating Pa(p) over a range of fraction defective values. It provides a graphical representation of the ability of the proposed plan to discriminate between different levels of lot quality.

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2.4 KL-information Framework for Sample-Size determination:

The principal feature of the proposed procedure is the use of KL information for determining the common size of sample 'n'. Let AQL_{KL} denotes the specified acceptable quality level and LQL_{KL} represents the limiting quality level. For the development of the proposed plan, these two quality levels are denoted by p_1 and p_2 respectively. Thus, $p_1 = AQL_{KL}$ and $p_2 = LQL_{KL}$.

Kullback-Leibler (KL) information, also referred to as KL divergence, is employed to quantify the distinction between the probability distributions corresponding to the two specified quality levels. For a fraction defective p, the KL information provides a measure of the evidence for discriminating the distribution associated with p_1 and p_2. For the Poisson model, KL information between the quality levels p_1 and p_2 is given by

$$D(p_1 \,||\, p_2) = p_1 \log\left((p_1)/(p_2)\right) + p_2 - p_1 \tag{4}$$

The discrimination point p* is determined by equating the KL information measured with respect to the two reference quality levels. Hence,

$$I(p*:p_1) = I(p*:p_2) \tag{5}$$

The resulting p* represents the quality level at which the information for distinguishing the acceptable and limiting quality levels is balanced. The KL information quantifies the separation between the specified acceptable and limiting quality levels. The required size of sample is determined using KL-information criterion together with the specified producer's risk α and consumer's risk β. Let the resulting theoretical size of sample be n*. The integer size of sample is obtained as

$$n = [n*] \tag{6}$$

The same value is used at both stages:

$$n_1 = n_2 = n$$

The resulting size of sample is subsequently incorporated into the proposed Double Sampling Plan.

2.5 Determination of Acceptance numbers:

After determining 'n', suitable acceptance numbers c_1 and c_2 are obtained for the Double Sampling Plan. The acceptance parameters satisfy, $0 \le c_1 \le c_2$. For each pair (c_1, c_2), the probability of acceptance P_a (p) is evaluated at the specified quality levels. The corresponding producer's and consumer's risks are

$$\alpha = 1 - P_a(p_1) \tag{7}$$
$$\beta = P_a(p_2) \tag{8}$$

respectively. Only those combinations satisfying the prescribed Six Sigma requirements are considered feasible.

2.6 Selection by Minimum Sum of Risk:

Several feasible combinations of c_1 and c_2 may satisfy the specified risk requirements. Therefore, the minimum sum of producer's and consumer's risks are used as an additional criterion for selecting the preferred plan. The minimum sum of risk is defined as

$$MSR = \alpha + \beta \tag{9}$$

Where,

$$\alpha = 1 - P_a(p_1)$$
$$\beta = P_a(p_2)$$

Among the feasible (c_1, c_2) combinations, the combination having the minimum value of MSR is selected.

2.7 Construction Procedure:

The complete construction of the proposed KL-information-based Six Sigma Double Sampling Plan can therefore be summarized as follows:

Step 1: Specify the acceptable quality level (AQL_{KL}) and limiting quality level (LQL_{KL}) with $AQL_{KL} < LQL_{KL}$, α and β.

Step 2: Calculate the KL information between the two specified quality levels.

Step 3: Determine the discrimination point p* using the equal-information criterion.

Step 4: Determine the required common size of sample n and set $n_1 = n_2 = n$.

Step 5: Search feasible combinations of c_1 and c_2 satisfying the risk constraints.

Step 6: Calculate MSR = α + β and select the feasible combination with the minimum MSR.

The proposed procedure therefore integrates information-based sample-size determination, two-stage attribute inspection, Six Sigma risk requirements and minimum-sum-of-risk selection.

III. STATISTICAL PERFORMANCE ASSESSMENT

The statistical performance of the proposed KL-information-based Double Sampling Plan is assessed using the Operating Characteristic (OC) and Average Run Length (ARL) measures. The OC curve is used to describe discriminatory ability of the proposed plan over different fraction-defective levels, while the ARL curve is used to assess the responsiveness of the plan to deterioration in lot quality. The proposed plan is also compared with the existing Six Sigma-based Double Sampling Plan under identical quality-level conditions.

3.1 OC Curve Analysis

Example 1.1

Consider an acceptable quality level $AQL_{KL} = 0.00007$. Using the KL-information-based procedure, the required size of sample is obtained as $n_1 = n_2 = 4729$. Hence the resulting Double Sampling Plan is represented by $[(n_1, n_2, c_1, c_2) =$

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(4729, 4729, 1, 9)]. The selected parameters provide the basis for making the lot decision through the two-stage inspection procedure.

Example 1.2

For AQL_{KL} = 0.000115, the proposed KL-information-based procedure gives the sampling-plan parameters [(n_1, n_2, c_1, c_2) = (4300, 4300, 5, 9)].Thus, an increase in the specified acceptable quality level from (0.00007) to (0.000115) is accompanied by a reduction in the required size of sample from 4729 to 4300, while the acceptance value structure changes from ((1, 9)) to ((5, 9)). This demonstrates that the proposed design parameters are sensitive to the specified quality level.
The OC curves of the plans presented in Example 1.1 and Example 1.2 are shown in Figure 1.

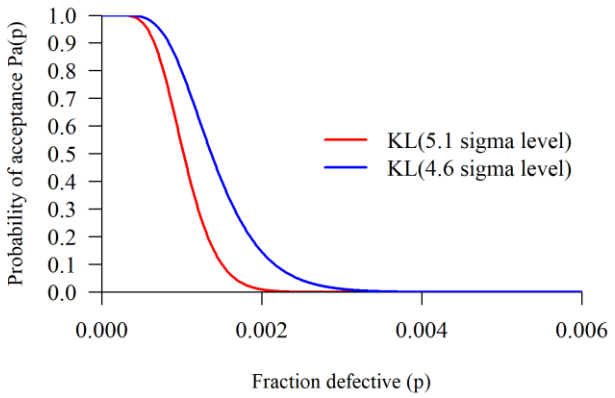

Fig 1: Operating Characteristic (OC) curves of the proposed KL-divergence-based sampling plan

(i): n_1= 4729, n_2 = 4729, c_1 = 1, c_2 = 9
(ii): n_1 = 4300, n_2 = 4300, c_1 = 5, c_2 = 9

Interpretation:

The OC curves show the probability of acceptance of the proposed KL-information-based Double Sampling Plans over a range for fraction defectives. For both plans, the probability of acceptance decreases as the fraction defective increases, demonstrating the expected discriminatory behaviour of the sampling plans. The differences between the curves arise from the different size of samples and acceptance values of combinations associated with specified AQL_{KL} and LQL_{KL} values. The curves therefore demonstrate that the proposed procedure provides different acceptance characteristics according to the quality levels used in the KL-information-based design.

3.2 Comparative Analysis of OC Curves:

For the specified quality levels AQL = 0.0003, LQL = 0.002 the parameters obtained from the existing Six Sigma-based Double Sampling Plan developed by R. Radhakrishnan and P.K. Sivakumaran are [(n_1, n_2, c_1, c_2) = (7800, 7800, 3, 5)] Whereas the proposed KL-information-based procedure developed by P.K. Sivakumaran and N.S. Durga Nandhini gives [(n_1, n_2, c_1, c_2) = (14802, 14802, 4, 9)]. Thus, the two plans have different size of samples and decision parameters for the same specified quality levels. The corresponding Operating Characteristic curves are used to examine their acceptance behaviour over a range of fraction-defective values.

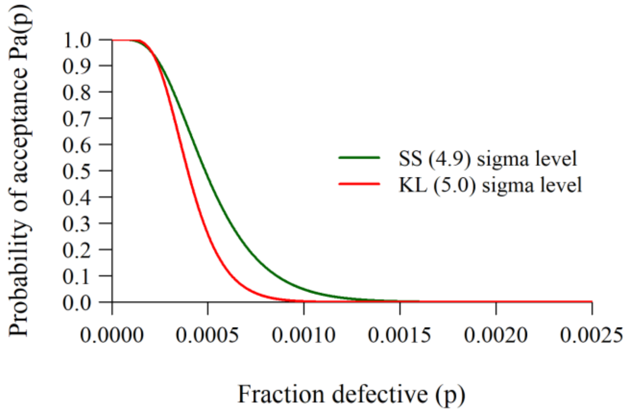

Fig 2: Comparison of OC curves between existing Six Sigma-based and proposed KL-information-based Double Sampling Plans

(i): n_1, n_2 = 7800, c_1 = 3. c_2 = 5
(ii): n_1, n_2 =14802, c_1 = 4, c_2 = 9

Interpretation:

The comparative OC curves demonstrate the difference in acceptance behaviour between the existing Six Sigma-based DSP and the proposed KL-information-based DSP for the same specified quality levels. Both curves exhibit the expected decreasing pattern as the fraction defective increases. The comparison indicates that KL information provides an alternative basis for determining the size of sample of the DSP. The proposed plan offers an information-based construction rather than merely reproducing the parameter selection of the existing Six Sigma procedure.

3.3 Comparative ARL Analysis:

The Average Run Length (ARL) values obtained for the existing Six Sigma-based Double Sampling Plan and the proposed KL-information-based plan are presented in Table 1 for selected fraction-defective levels. The comparison is used to examine the faster detection of lots having deteriorating quality.

Interpretation:

The results in Table 1 show that a smaller ARL indicates that fewer lots are expected to be inspected before a lot is rejected at the corresponding fraction-defective level.

Example 1.3

For a given AQL = 0.00003 and LQL = 0.002, the parameters of Six Sigma based DSP developed by R. Radhakrishnan and P.K. Sivakumaran is ($n_1 = n_2$, c_1, c_2) = (7800, 3, 5). Also, for the same quality levels the parameters of Six Sigma based

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DSP using Kullback-Leibler information developed by P.K. Sivakumaran and N.S. Durga Nandhini is ($n_1 = n_2$, c_1, c_2) = (14802, 4, 9).

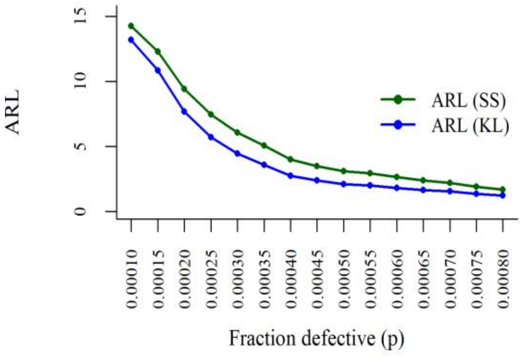

Fig 3: Comparison of ARL curves between the existing Six Sigma-based and proposed KL-information-based Double Sampling Plans

(i): n_1, n_2 = 14802, c_1 = 4, c_2 = 9
(ii): n_1, n_2 = 7800, c_1 = 3, c_2 = 5

Interpretation:

For the fraction-defective values considered, the proposed KL-information-based DSP exhibits lower ARL values than the existing Six Sigma-based DSP.

IV. NUMERICAL INVESTIGATION AND DISCUSSION

4.1 Comparative Risk and Minimum Sum of Risk Analysis:

The risk performance of the proposed KL-information-based Double Sampling Plan is examined by comparing it with the existing Six Sigma-based procedure, as presented in Table 2. The comparison is carried out using the producer's risk and consumer's risk corresponding to the specified acceptable and limiting quality levels. The minimum sum of risk (MSR) is then used to summarize the combined magnitude of these two risks.

Interpretation:

The numerical results presented in Table 2 demonstrate that the KL-information-based construction can satisfy the stringent Six Sigma risk requirements for the quality-level combinations investigated. The resulting values of MSR are very small, indicating that the selected decision parameters provide a low combined risk under the specified conditions. The KL-based design satisfies the prescribed risk constraints while producing a very small combined risk.

Practical Implementation Scenario:

To illustrate the practical use of the proposed procedure, if the manufacturer of disposable medical syringes fixes the quality at AQL_{KL} = 0.00007 (7 non-conforming syringes out of 1 lakh syringes), then take a first sample of 4729 syringes from the manufactured lot of a particular day/week and count the number of non-conforming syringes (d_1). If $d_1 \leq 1$ accept the items manufactured during that production period. If $d_1 > 9$, reject the lot and recommend an improvement in the manufacturing quality. If $d_1 = 9$, take a second sample of 4729 syringes from the remaining lot and count the number of non-conforming syringes (d_2). If $d_1 + d_2 \leq 9$, accept the lot manufactured in that day/week, otherwise reject the lot manufactured and recommend corrective action to improve the process quality.

4.2 Table for the proposed KL-based Double Sampling Plan:

A table is developed to provide the sampling-plan parameters corresponding to different combinations of acceptable and limiting quality levels. The table is constructed using the proposed KL-information-based procedure, with equal size of samples at the two inspection stages, i.e., $n_1 = n_2$. For each specified combination of AQL_{KL} and LQL_{KL}, Table 3 provides the required size of sample together with the associated acceptance numbers c_1 and c_2.

Interpretation:

Table 3 demonstrates that the required size of sample and acceptance value structure vary according to the specified AQL_{KL} and LQL_{KL} values. In general, greater separation between the acceptable and limiting quality levels provides greater KL information per observation and therefore can reduce the required size of sample. The table provides practitioners with a direct means of selecting the proposed KL-based Double Sampling Plan for specified quality-level requirements.

V. CONCLUSION

A KL-information-based Double Sampling Plan has been developed for attribute inspection under stringent Six Sigma quality requirements. The proposed framework combines the Poisson model with KL information to determine the size of sample required for discriminating between an acceptable quality level and a limiting quality level. The resulting size of sample is incorporated into a two-stage inspection structure, allowing the lot decision to be reached either after the first sample or, when necessary, after examining a second sample.
The acceptance parameters c_1 and c_2 are selected from feasible combinations satisfying the specified producer's and consumer's risk requirements. The minimum sum of these risks are subsequently used to identify the preferred combination among the feasible alternatives. This provides an additional basis for selecting the decision parameters of the proposed Double Sampling Plan. The numerical results indicate that the required size of sample is influenced by the separation between the specified acceptable and limiting quality levels.
The OC analysis confirms that the proposed sampling procedure distinguishes between different levels of fraction

defective. The probability of acceptance decreases as the fraction defective increases, indicating that the plan is more likely to accept lots with better quality and reject lots with increasing levels of non-conformity. The ARL analysis similarly shows a reduction in the expected number of lots inspected before rejection as the fraction defective increases, reflecting a faster response to deterioration in lot quality.

The two-stage structure also provides an additional practical feature because the probability of proceeding to the second inspection stage can be examined along with the resulting inspection effort. Comparisons with the existing Six Sigma-based Double Sampling Plan demonstrates that the KL-information approach provides an alternative framework for size of sample determination and selection of the decision parameters.

The proposed methodology is particularly relevant to situations involving extremely small quality levels where strong discrimination between acceptable and undesirable lots is required. The tables developed for different combinations of acceptable and limiting quality levels further support practical implementation of the proposed Double Sampling plan.

As a possible extension, the proposed framework can be further investigated by considering unequal allocation of size of samples between first and second inspection stages.

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Table 1: ARL Values for the existing Six Sigma-based and Proposed KL-based DSP

P	ARL (SS)	ARL(KL)
0.00023	14.261	13.197
0.00024	12.308	10.847
0.00026	9.429	7.668
0.00028	7.462	5.709
0.0003	6.071	4.439
0.00032	5.058	3.580
0.00035	3.993	2.744
0.00037	3.483	2.372
0.00039	3.082	2.093
0.0004	2.914	1.979
0.00042	2.627	1.792
0.00044	2.393	1.645
0.00046	2.201	1.529
0.0005	1.907	1.360
0.00055	1.653	1.227
0.0006	1.479	1.144
0.00065	1.356	1.092
0.0007	1.267	1.058
0.00075	1.202	1.036
0.0008	1.153	1.023

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Table: 2 Comparison of existing Six Sigma-based and proposed KL-based Double Sampling Plans

Given values $\alpha = 0.0000034$ $\beta = 2\alpha$		Existing Six Sigma Based DSP						Six Sigma Based DSP using Kullback-Leibler information					
AQL	LQL	n_1	n_2	c_1, c_2	α	B	MSR ($\alpha + \beta$)	n_1	n_2	c_1, c_2	α	β	MSR ($\alpha + \beta$)
0.00002	0.002	10560	10560	4, 5	6.4e-07	6.79e-06	7.43e-06	13851	13851	0,9	4.5e-10	9.4e-13	4.54e-10
0.00003	0.002	7800	7800	3, 5	3.4e-06	1.2e-04	1.3e-04	14802	14802	4,9	2.3e-08	5.1e-09	2.8e-08
0.00006	0.015	1084	1084	2, 3	3.4e-06	1.3e-05	1.6e-05	1644	1644	0,9	1.0e-12	1.9e-11	2.0e-11

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Table 3: Parameters of the proposed KL-based Double Sampling Plan for specified AQL_{KL} and LQL_{KL}

LQL_{KL} / AQL_{KL}	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010	0.011	0.012	0.013	0.014
0.00002	31267 5 9	13851 0 9	8728 0 9	6320 0 9	4932 0 9	4033 0 8	3405 0 8	2942 0 7	2588 0 7	2308 0 7	2081 0 7	1894 0 7	1738 0 6	1604 0 6
0.000025		14347 2 9	8993 0 9	6492 0 9	5056 0 9	4128 0 9	3481 0 8	3005 0 8	2641 0 8	2354 0 7	2122 0 7	1930 0 7	1770 0 7	1633 0 7
0.00003		14802 4 9	9234 0 9	6648 0 9	5168 0 9	4213 0 9	3549 0 9	3062 0 8	2689 0 8	2395 0 8	2157 0 8	1962 0 7	1798 0 7	1659 0 7
0.000035		15229 5 9	9458 1 9	6792 0 9	5270 0 9	4292 0 9	3612 0 9	3113 0 9	2732 0 8	2432 0 8	2190 0 8	1990 0 8	1823 0 7	1682 0 7
0.00004		15634 5 9	9668 3 9	6926 0 9	5365 0 9	4364 0 9	3669 0 9	3160 0 9	2772 0 9	2466 0 8	2220 0 8	2017 0 8	1847 0 8	1703 0 8
0.000045		16022 5 9	9868 4 9	7053 0 9	5455 0 9	4432 0 9	3723 0 9	3205 0 9	2809 0 9	2498 0 9	2247 0 8	2041 0 8	1869 0 8	1722 0 8
0.00005			10060 5 9	7174 2 9	5541 0 9	4497 0 9	3774 0 9	3246 0 9	2844 0 9	2528 0 9	2274 0 9	2064 0 9	1889 0 8	1741 0 8
0.000055			10244 5 9	7290 3 9	5622 0 9	4558 0 9	3823 0 9	3286 0 9	2878 0 9	2557 0 9	2298 0 9	2086 0 9	1909 0 9	1758 0 8
0.00006			10423 5 9	7401 4 9	5701 1 9	4617 0 9	3870 0 9	3324 0 9	2910 0 9	2584 0 9	2322 0 9	2107 0 9	1927 0 9	1775 0 9
0.000065			10596 5 9	7519 5 9	5777 2 9	4674 0 9	3915 0 9	3361 0 9	2940 0 9	2610 0 9	2345 0 9	2127 0 9	1945 0 9	1791 0 9
0.00007			10766 5 9	7615 5 9	5850 3 9	4729 1 9	3985 0 9	3396 0 9	2970 0 9	2635 0 9	2367 0 9	2146 0 9	1962 0 9	1806 0 9
0.000075				7717 5 9	5921 4 9	4783 2 9	4000 0 9	3430 0 9	2998 0 9	2660 0 9	2388 0 9	2164 0 9	1978 0 9	1821 0 9
0.00008				7817 5 9	5991 4 9	4834 3 9	4040 1 9	3463 0 9	3026 0 9	2683 0 9	2408 0 9	2183 0 9	1994 0 9	1835 0 9
0.000085				7915 5 9	6059 5 9	4885 3 9	4080 2 9	3495 0 9	3052 0 9	2706 0 9	2428 0 9	2200 0 9	2009 0 9	1849 0 9
0.00009				8011 5 9	6125 5 9	4934 4 9	4119 2 9	3527 0 9	3078 0 9	2728 0 9	2447 0 9	2216 0 9	2024 0 9	1862 0 9
0.000095	[$n_1=n_2$, c_1,c_2]			8106 5 9	6190 5 9	4983 4 9	4156 3 9	3557 1 9	3104 0 9	2750 0 9	2465 0 9	2233 0 9	2039 0 9	1875 0 9
0.000100					6254 5 9	5030 5 9	4193 3 9	3587 2 9	3129 0 9	2771 0 9	2484 0 9	2249 0 9	2053 0 9	1887 0 9

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LQL_{KL} / AQL_{KL}	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010	0.011	0.012	0.013	0.014
0.000105					6317 5 9	5077 5 9	4230 4 9	3616 2 9	3153 1 9	2791 0 9	2501 0 9	2264 0 9	2067 0 9	1900 0 9
0.000110					6378 5 9	5122 5 9	4265 4 9	3645 3 9	3177 2 9	2811 0 9	2519 0 9	2279 0 9	2080 0 9	1912 0 9
0.000115					6439 5 9	5167 5 9	4300 5 9	3673 3 9	3200 2 9	2831 1 9	2536 0 9	2294 0 9	2093 0 9	1924 0 9
0.000120						5255 5 9	4368 5 9	3728 4 9	3246 3 9	2870 2 9	2569 1 9	2323 0 9	2119 0 9	1935 0 9