

# Deterministic and Probabilistic Design of Pressure Vessels

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**Abstract**—The pressure vessel is one of the large number of plant components for which stress analyses must be performed. A pressure vessel experiences stresses because of pressure inside pressure vessel. Analytically, pressure vessels are designed using ASME codes known as deterministic approach. These conventional (or deterministic) analysis techniques involve the use of safety factors as a way of accounting for variation in analysis input parameters. This often results in overly conservative designs. In some situations, accounting for this variability within analysis can be critical, or at least more cost-effective, than over-designing products with expensive materials or manufacturing processes. Recently, reliability and structural safety have been given highest priority in plant products as there is direct threat to human life. Hence, it is required to design critical components such as pressure vessels for wide range of input variations in geometry, material, loads and other operating parameters without oversizing these critical components. Design based on this approach gives information in advance about impact of input variations and risk associated. Because of the stochastic nature of many of the uncertainties, probabilistic approach as opposed to a deterministic approach is better suited. Probabilistic designs are widely adopted in civil structural design, aircraft, and aerospace design. In this work, the probabilistic design of pressure vessel is carried out to know the effect of uncertainties. Probabilistic designing is performed by using FEM package (ANSYS). Probabilistic design uses Gaussian distribution for various input (geometric, material and load) parameters. 100 samples have been generated using Monte Carlo simulation technique.

**Keywords** — Pressure vessel; Probabilistic design; FEA;

## I. INTRODUCTION

A pressure vessel is a closed container designed to hold gases or liquids at a pressure substantially different from the ambient pressure [1]. Pressure vessels are used in a variety of applications in both industry and the private sector. They appear in these sectors as industrial compressed air receivers and domestic hot water storage tanks. Other examples of pressure vessels are diving cylinder, recompression chamber, distillation towers, autoclaves, and many other vessels in mining or oil refineries and petrochemical plants, nuclear reactor vessel, habitat of a space ship, habitat of a submarine, pneumatic reservoir, hydraulic reservoir under pressure, rail vehicle airbrake reservoir, road vehicle airbrake reservoir and storage vessels for liquefied gases such as ammonia, chlorine, propane, butane, and LPG.

The legal definition of pressure vessel varies from country to country, but often involves the maximum safe pressure that the vessel is designed for and the pressure – volume product, particularly of the gaseous part. In the United States, the rules for pressure vessels are contained in the American Society of Mechanical Engineers Boiler and Pressure Vessel Code.

**Classification of Pressure vessels:** Typically pressure vessels can be classified as mentioned below [2]:

- Basis on wall thickness
  - Thin walled pressure vessels
  - Thick Walled pressure vessels
- Basis on Heads [3]
  - Hemispherical head
  - Ellipsoidal Head
  - Torispherical Head
  - Flat Head
  - Diffuser Head

Analytically, pressure vessels are designed using ASME codes [4] known as deterministic approach. These conventional (or deterministic) analysis techniques involve the use of safety factors as a way of accounting for variation in analysis input parameters. This often results in overly conservative designs. However, the validity or conservatism in the results from such analyses depends on the real-life variability or uncertainty of the input values. In some situations, accounting for this variability within analysis can be critical, or at least more cost-effective, than over-designing products with expensive materials or manufacturing processes [5]. Recently, reliability and structural safety have been given highest priority in plant products as there is direct threat to human life. Hence, it is required to design critical components such as pressure vessels for wide range of input variations in geometry, material, loads and other operating parameters without oversizing these critical components [6]. Design based on this approach gives information in advance about impact of input variations and risk associated.

The need to incorporate uncertainties in an engineering design has long been recognized. Because of the stochastic nature of many of the uncertainties, probabilistic approach as opposed to a deterministic approach is better suited. Thus, the probability of structural failure can be limited to a reasonable level maintained by a risk informed program. Today, risk informed technologies and probabilistic design are widely adopted in civil structural design, aircraft, and

aerospace design. In this work, the probabilistic design of pressure vessel will be carried out to know the effect of uncertainties.

In spite of number of investigations devoted to pressure vessel research and analysis, there still remains to be developed a general approach capable of predicting the effects of variations in geometry, material and loading conditions on the behavior of pressure vessels. As seen from literature review, deterministic design approaches do not take into account uncertainties. Hence, this work proposes use of probabilistic design approach. Probabilistic design is an analysis technique for assessing the effect of uncertainties in input parameters and assumptions in the design. A probabilistic analysis allows determining the extent to which uncertainties in the model affect the results. The objective of the work presented here is design of pressure vessel considering uncertainties in input parameters within framework of FEA tool ANSYS. In view of this, following objectives are set for present study:

- To design pressure vessel using ASME codes and validate design using ANSYS. This is traditional deterministic approach for design.
- Design pressure vessel by using probabilistic design approach using ANSYS to study effect of input variations (geometric, material, loading and operating) on stress.
- To study sensitivity and probability of input variation on stresses of pressure vessel.

## II. DETERMINISTIC DESIGN

In deterministic design for one set of input one gets one design. Deterministic design of pressure vessel uses ASME codes. Deterministic design of pressure vessel can use either design by rule or design by analysis using analytical methods to calculate following stresses [7-9]:

- Hoop (circumferential) Stress
- Axial (longitudinal) Stress
- Thermal stresses, if thermal loading is present.

### A. Application of pressure vessels

Present work selects Propane (LPG) tanks which are used for vapor and/or liquid service. These propane storage tanks are built and tested in accordance with the ASME boiler and pressure vessel code section VIII division 1 for unfired pressure vessels. Present work selects propane cylinders used for industrial use built to 250 psi working pressure and they are typically vertical as shown in Fig. 1.



Figure 1: Vertical propane tanks

### B. Input Specifications

All input specifications are converted to SI unit. Henceforth, this work follows SI units for all calculations. Table 1 shows inputs in both units.

Table 1: Input specifications of propane tank [10]

	Parameters	BTU	SI units
<b>Geometric: Shell</b>	Inner Diameter (Di)	35.6 in	0.90424 m
	Outer Diameter (D0)	36 in	0.91440 m
	Length of the vessel (L)	121 in	2.36220 m
<b>Ellipsoidal Head</b>	Ratio	2:1	2:1
<b>Material – Steel SA 455</b>	Youngs Elasticity (E)	30x106 psi	206.8x10 <sup>3</sup> MPa
	Density	0.289 lb/in <sup>3</sup>	7700 Kg/m <sup>3</sup>
	Yield Strength		520 MPa
	Allowable Stress		160 MPa
<b>Operating</b>	Operating Temp.	200 F	775.3 K
	Environmental Temp.	98 F	423.15 K
	Operating pressure (P)	250 psi	1.724 MPa
	Factor of Safety (FOS)	2.5	2.5
Others	Joint Efficiency	1	1

**Design of Shell:** Minimum shell thickness is calculated using Eq. 1 given by ASME codes.

$$t = \frac{PR}{SE - 0.6P} \quad (1)$$

$$t = \frac{1.72368932 \times 452.12}{160 \times 1 - 0.6 \times 1.72368932} \quad (2)$$

$$t = 0.005 \text{ m}$$

$$t = 5 \text{ mm}$$

**Design of Semi-Ellipsoidal Head:** Minimum head thickness is calculated using Eq. 3 given by ASME codes.

$$t = \frac{PD}{2SE - 0.2P} \quad (3)$$

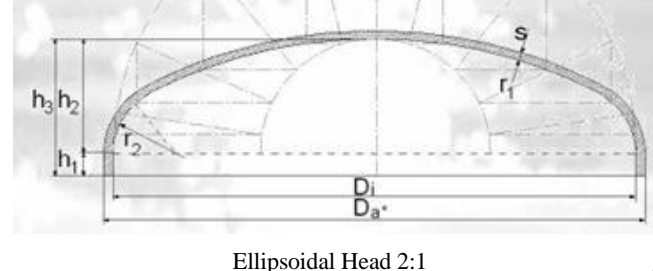
$$t = \frac{1.72368932 \times 904.24}{2 \times 160 \times 1 - 0.6 \times 1.72368932} \quad (4)$$

$$t = 0.00488\text{m}$$

$$t \approx 4.88 \text{ mm (rounded off)}$$

For manufacturing simplicity, it is recommended to use same thickness for shell as well as head i.e. 5.00 mm. Other dimensions of Semi-Ellipsoidal Head are calculated using ASME code and formulation. Flanged and dished elliptical 2:1 ratio ASME code type heads are used extensively in the construction of tanks for liquefied petroleum gas, air receivers, and other unfired pressure vessels. Present study uses same material as shell for head. Table 2 gives detailed design of ellipsoidal head using ASME codes.

Table 2: Geometrical design of ellipsoidal head



Ellipsoidal Head 2:1

ASME Code	Section VIII, Div. 1, Art. UG-32	
$D_i = D_o - 2S$	$S = 5 \text{ mm}$	(5)
$r_1 = 0.9D_i$	$r_1 = 0.9 * 0.90424 * 1000 = 813.816 \text{ mm}$	(6)
$r_2 = 0.17D_i$	$r_2 = 0.17 * 0.90424 * 1000 = 153.721 \text{ mm}$	(7)
$h_1 = 3s$	$h_1 = 3 * 5 = 15 \text{ mm}$	(8)
$h_2 = 0.25D_i$	$h_2 = 0.25 * 0.90424 * 1000 = 226.06 \text{ mm}$	(9)
$h_3 = h_1 + h_2$	$h_3 = 15 + 226.06 = 241.06 \text{ mm}$	(10)

#### Circumferential Stress calculations for cylindrical shell:

After calculating minimum thickness for shell and head, stresses are back calculated and checked the design for safety. Circumferential Stress in Shell:

$$\sigma = \frac{PR}{t} \quad (11)$$

$$\sigma = \frac{1.72368932 \times 0.45212 * 1000}{5}$$

$$\sigma = 155.86 \text{ MPa} < \text{allowable limit } 160 \text{ MPa}$$

**Stress calculations for semi ellipsoidal head:** Stress in head may not be same as in shell. Hence, check for stresses in head is separately required. Stress in head is calculated by Eq. 12. Meridional stress (Stress in Head):

$$\sigma = \frac{PR^2}{2th_2} \quad (12)$$

$$\sigma = \frac{1.72368932 \times (0.45212 \times 1000)^2}{2 \times 5 \times 226.06}$$

$$\sigma = 155.86 \text{ MPa} < \text{allowable limit } 160 \text{ MPa}$$

In present case stresses in shell and head are same.

### III. FINITE ELEMENT ANALYSIS AND ITS BENCHMARKING USING ANSYS

Before performing probabilistic design, benchmarking analysis is carried out using baseline inputs within FEA framework. The FEA is a numerical procedure for obtaining approximate solutions to many problems encountered in engineering analysis [11]. During the last two decades considerable advances have been made in the applications of FEM techniques to analyze pressure vessel problems [12-18].

Table 3 lists all inputs used in this analysis. In this section parametric modeling approach, element selection, meshing strategy and loading options have been benchmarked and finalized.

Table 3: Parameters used for pressure vessel analysis

Parameter Type	Parameter	Baseline Input
Geometric	Inner Radius	452.12 mm
	Thickness	5.0 mm
	Length	2362.0 mm
Material	Youngs Modulus	206000 MPa
	Poissions Ratio	0.33
Load	Pressure	1.724 MPa

Pressure vessel FE analysis can be performed using different approaches as mentioned below:

- Axi-symmetric approach
- Analysis on a quarter section
- Analysis by drawing the complete vessel

This work uses axi-symmetry approach which simplifies the model and also reduces the computational time. This approach can be used if the geometry is revolved about a particular axis. In ANSYS axi-symmetry is used about Y-axis [19].

The pressure vessel under consideration involves the parametric modeling using ANSYS APDL. The user needs to generate several models with different geometric properties as part of a probabilistic design requirement. Using this concept parametric CAD model can be generated in ANSYS using following set of APDL commands. CAD model is section of shell and head. Geometry is created using ASME code's parametric relations.

```
*SET,ri,452.12
*SET,ro,457.20
*SET,l,2362.0
t=(ro-ri)
r1=0.9*ri*2
r2=0.17*ri*2
```

$$h1=3*t$$

$$h2=0.25*ri*2$$

$$h3=h1+h2$$

Fig. 2 shows meshed baseline model with loads and boundary conditions. Internal pressure is applied as a load whereas symmetry boundary condition is applied on a line present on x-axis as analysis model is half the model of actual one. Fig. 3 shows meshed model of pressure vessel. Fig. 4 shows zoomed view of shell and head mesh. Fig. 5 shows different views of meshed model using quadrilateral 8-node PLANE element with axi-symmetric option. This element has a quadratic displacement behavior. The element is defined by 8 nodes having two degrees of freedom at each node: translations in the nodal x and y directions.

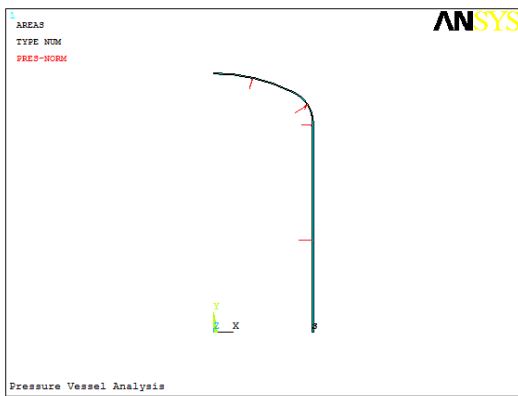


Figure 2: Baseline model with mesh and LBC

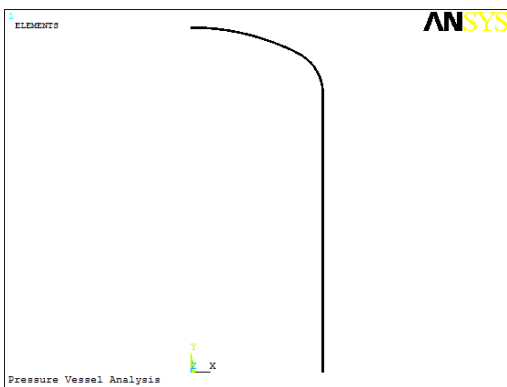


Figure 3: Meshed model of pressure vessel

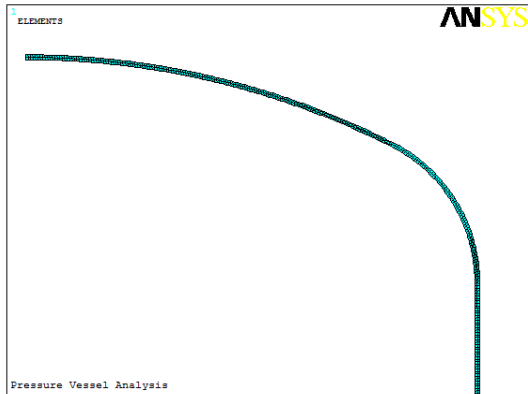
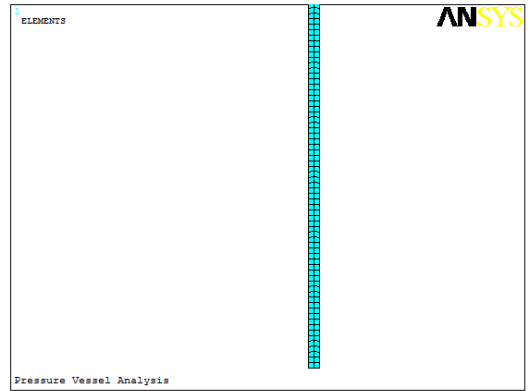
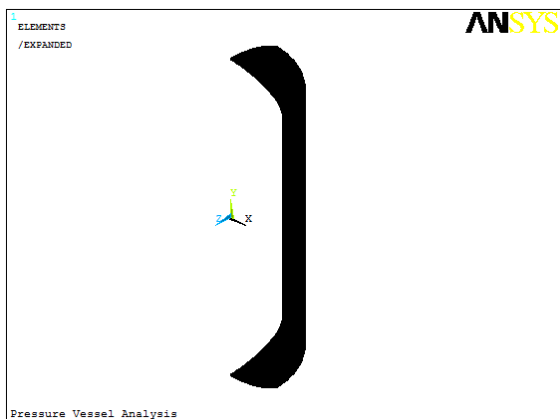
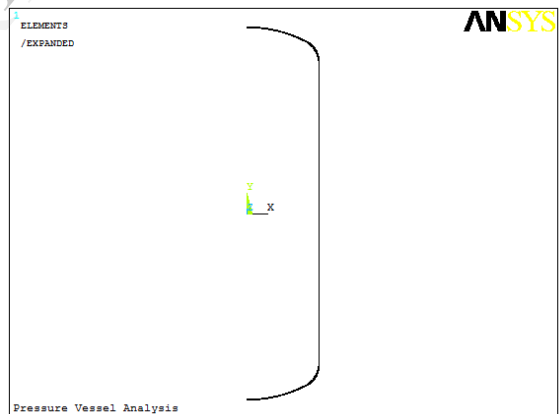


Figure 4: Meshed model – zoomed viewed of shell and head region



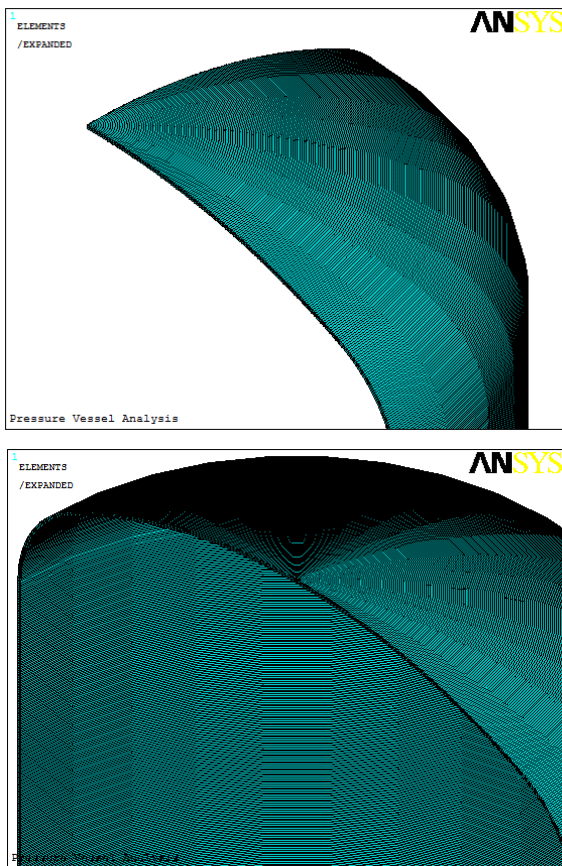


Figure 5: Meshed model – different views

Stress and deflections results are typically studied for pressure vessels. Fig. 6 shows stress distribution in baseline model of pressure vessel and maximum stress is 183 MPa. Fig. 7 shows deflection in baseline model of pressure vessel and maximum deflection is 0.7 mm. This figure shows deformed (colored) as well as undeformed pressure vessel (black). Stress results are very close to theoretical result hence proposed analysis method can be used for further probabilistic design and analysis.

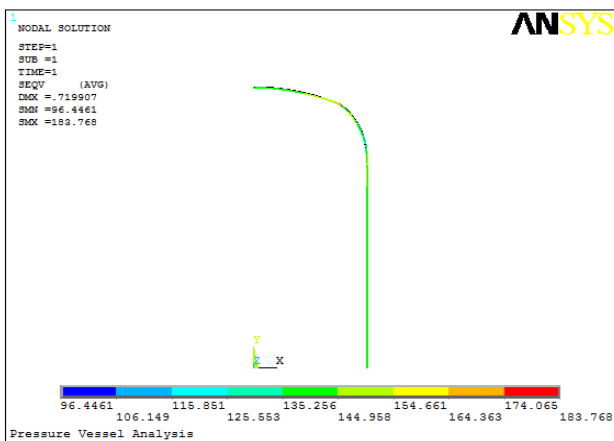


Figure 6: Von-Mises Stress distribution in baseline model of pressure vessel



Figure 7: Deflection in baseline model of pressure vessel

#### IV. PROBABILISTIC DESIGN OF PRESSURE VESSEL

Probabilistic design is an analysis technique for assessing the effect of uncertain input parameters in the model. A probabilistic analysis allows you to determine the extent to which uncertainties in the model affect the results of a finite element analysis. An uncertainty (or random quantity) is a parameter whose value is impossible to determine at a given point [20].

In deterministic approach, pressure vessel models are expressed and described with specific numerical and deterministic values; material properties are entered using certain values, the geometry of the component is assigned a certain length or width, etc. Naturally, the results of a deterministic analysis are only as good as the assumptions and input values used for the analysis. The validity of those results depend on how correct the values are for the component under real life conditions.

In reality, every aspect of an analysis model is subjected to scatter (in other words, is uncertain in some way). Material property values are different if one specimen is compared to the next. This kind of scatter is inherent for materials and varies among different material types and material properties. Likewise, the geometric properties of components can only be reproduced within certain manufacturing tolerances. The same variation holds true for the loads that are applied to a finite element model. This means that almost all input parameters used in a finite element analysis are inexact, each associated with some degree of uncertainty.

It is neither physically possible nor financially feasible to eliminate the scatter of input parameters completely. The reduction of scatter is typically associated with higher costs either through better and more precise manufacturing methods and processes or increased efforts in quality control; hence, accepting the existence of scatter and dealing with it rather than trying to eliminate it makes products more affordable and production of those products more cost-effective. To deal with uncertainties and scatter, one can use the ANSYS Probabilistic Design System (PDS) to know effect of input scatter on output:

- If the input variables of a finite element model are subjected to scatter, how large is the scatter of the output parameters? How robust are the output parameters? Here, output parameters can be any parameter that ANSYS can calculate.
- If the output is subjected to scatter due to the variation of the input variables, then what is the failure probability?
- Which input variables contribute the most to the scatter of an output parameter and to the failure probability? What are the sensitivities of the output parameter with respect to the input variables?

Probabilistic design can be used to determine the effect of one or more variables on the outcome of the analysis. Fig. 8 shows various parameters used for probabilistic design and analysis of pressure vessel. Present work considers:

- Geometric parameters: Inner radius, Outer radius, Length
- Material parameters: Youngs modulus and poisson ratio
- Load parameters: Pressure

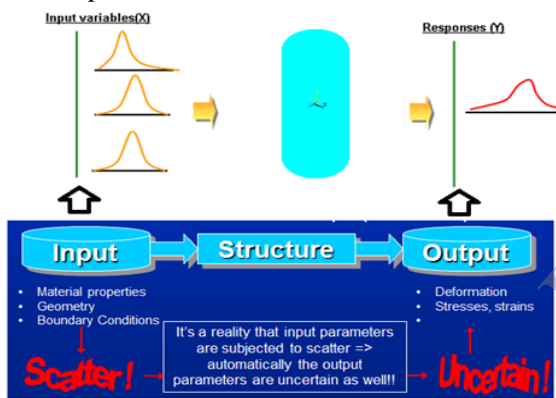


Figure 8: Parameter distributions in pressure vessel during probabilistic design

All the parameters are considered as varying with Gaussian ( or Normal ) distribution as shown in Fig. 8. Baseline model inputs for spring are used. Using uncertainties in above stated three parameters (see Table 4), probabilistic design is performed using ANSYS to know sensitivity of each parameter on stress and deflection. PDS within ansys uses Monte Carlo Simulation approach and analysis was looped through 100 sample points considering the variations defined in the input variables and the corresponding statistical analysis of the output parameters.

Table 4: Parameters used in probabilistic design of pressure vessel

Parameter Type	Parameter	Distribution Type	Mean (or Baseline)	Standard Deviation or Limits
Geometric	Inner Radius	Normal	452.12 mm	9.0
	Thickness	Normal	5.0 mm	0.70
	Length	Normal	2362.0 mm	23.62
Material	Youngs Modulus	Normal	206000 MPa	4120.0
	Poissions Ratio	Normal	0.33	0.033
Load	Force	Normal	1.724 MPa	0.085

The ANSYS APDL commands can be used to perform the entire probabilistic design analysis by creating input file and submitting it as a batch job. The usual process for probabilistic design consists of the following general steps. The items in parentheses indicate which ANSYS processor is necessary to perform the given task.

1. Create an analysis file for use during looping. The file should represent a complete analysis sequence and must do the following:
  - Build the model parametrically (PREP7).
  - Obtain the solution(s) (SOLUTION).
  - Retrieve and assign to parameters the quantities that will be used as random input variables and random output parameters (POST1/POST26).
2. Establish parameters in the ANSYS database which correspond to those used in the analysis file.
3. Enter PDS and specify the analysis file (PDS).
4. Declare random input variables (PDS).
5. Visualize random input variables (PDS). Optional.
6. Specify any correlations between the RVs (PDS).
7. Specify random output parameters (PDS).
8. Choose the probabilistic design tool or method (PDS).
9. Execute the loops required for the probabilistic design analysis (PDS).
10. Fit the response surfaces (if you did not use a Monte Carlo Simulation method) (PDS).
11. Review the results of the probabilistic analysis (PDS).

## V. RESULTS AND DISCUSSION

One of the objective of this research is to study probabilistic response analysis of pressure vessel. Hence, this section present results of probabilistic design results. An overview on the data is provided by several graphics in the next few pages.

Fig. 9 to 14 shows sample history for various input parameters. Each figure shows following data:

- Mean – This is as given in input Table 4
- Standard deviation – This is given in input Table 4
- Skewness – A measure of the asymmetry of the probability distribution
- Kurtosis – A measure of the peakedness of the probability distribution
- Minimum – Minimum value within sample history
- Maximum - Maximum value of within sample history

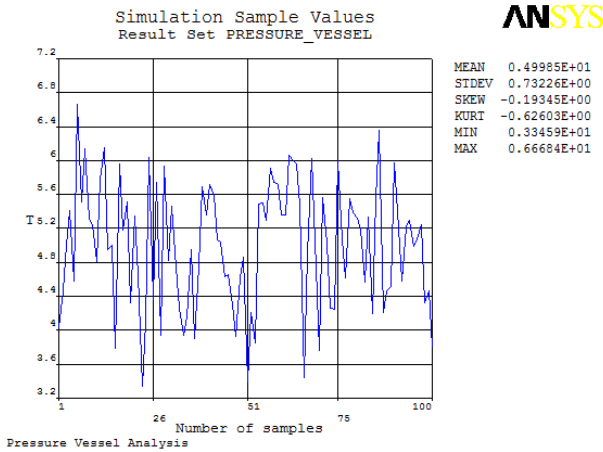


Figure 9: Sample history for thickness

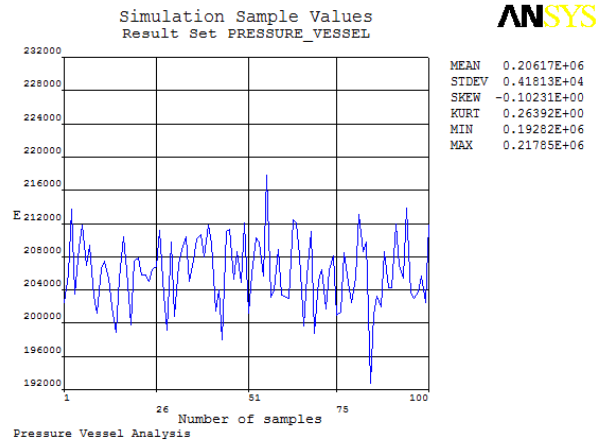


Figure 12: Sample history for modulus of elasticity

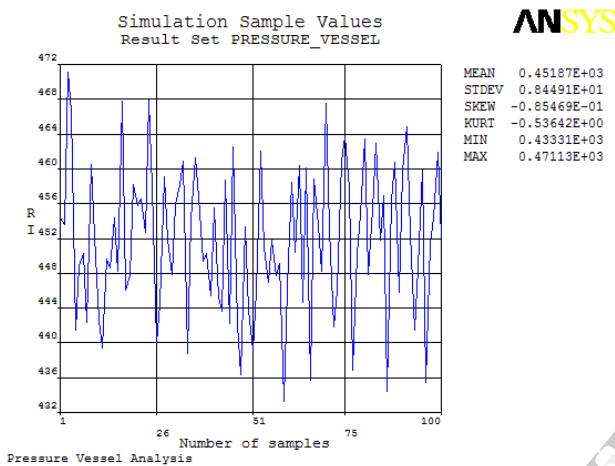


Figure 10: Sample history for inner radius

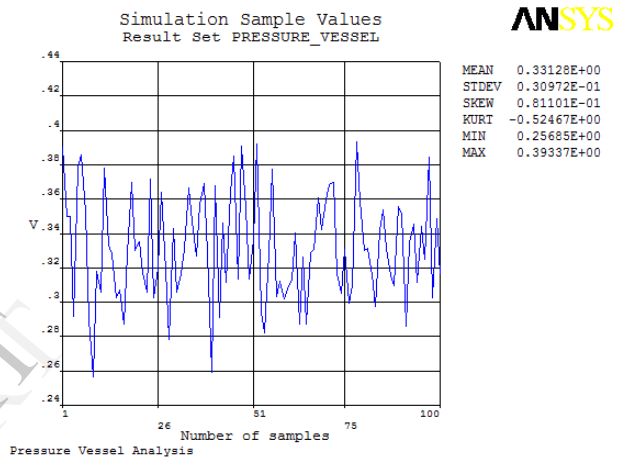


Figure 13: Sample history for poissions ratio

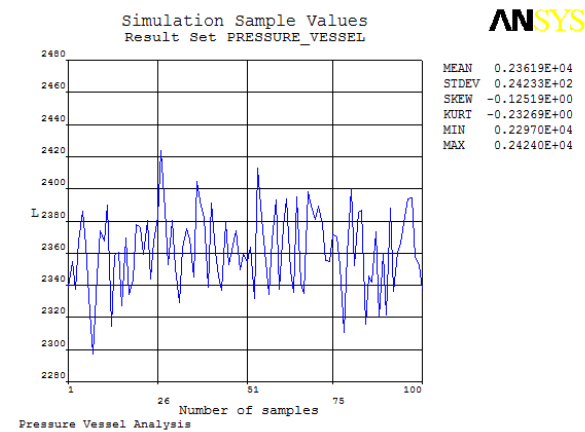


Figure 11: Sample history for length

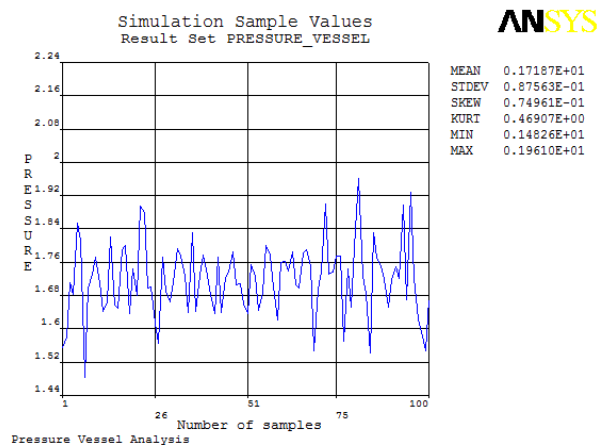


Figure 14: Sample history for internal pressure

Fig. 15 shows sample history for output parameters i.e. von-Mises stress.

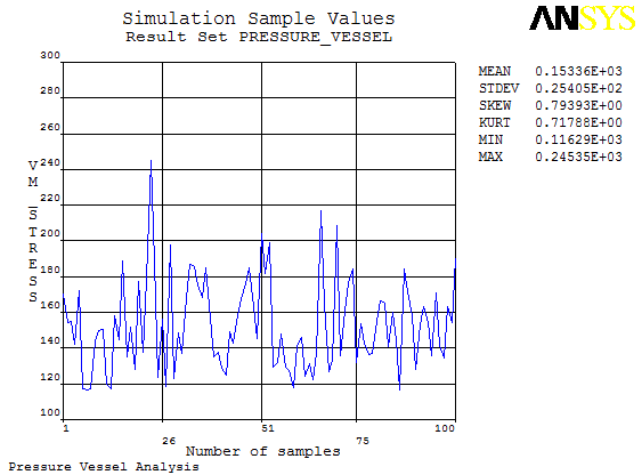


Figure 15: Sample history for von-Mises stress

Fig. 16 to 21 graphically depicts a histogram of each input and output parameters. The values given on each distribution plot were mean value, standard deviation, skewness, kurtosis, minimum value and maximum value, respectively. Values of skewness and kurtosis are very close to zero indicating validity of normal distribution. Red colored line shows distribution fitted within 100 samples. All the inputs confirm to the input distribution type.

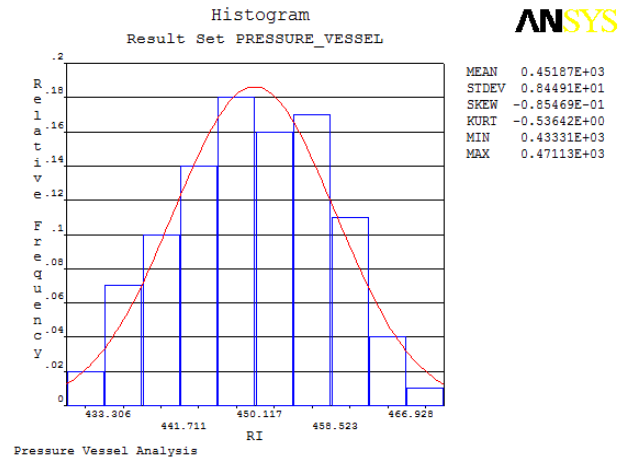


Figure 17: Histogram of inner radius

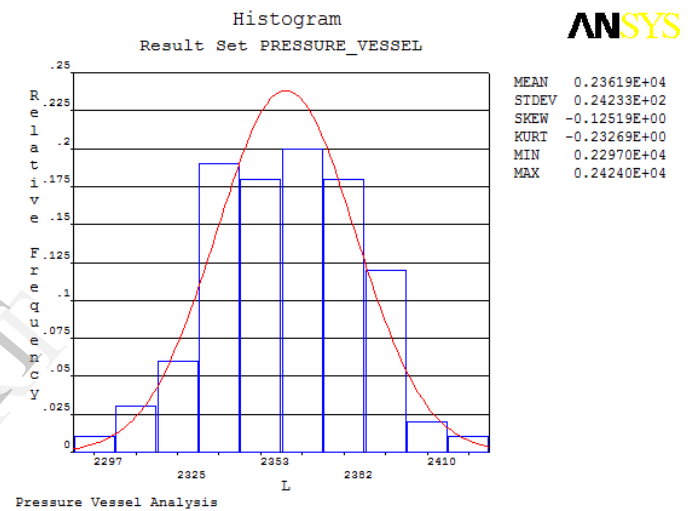


Figure 18: Histogram of length

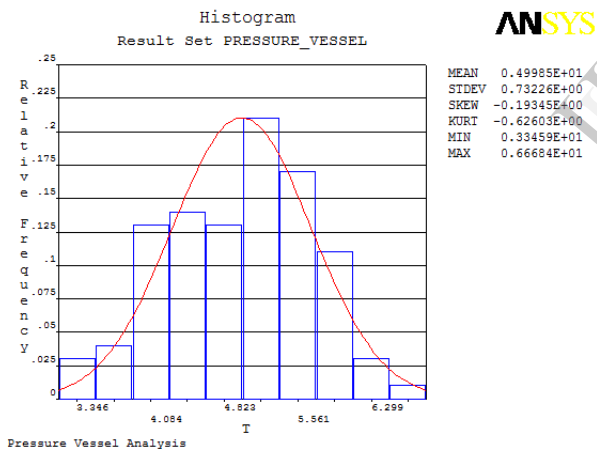


Figure 16: Histogram of thickness

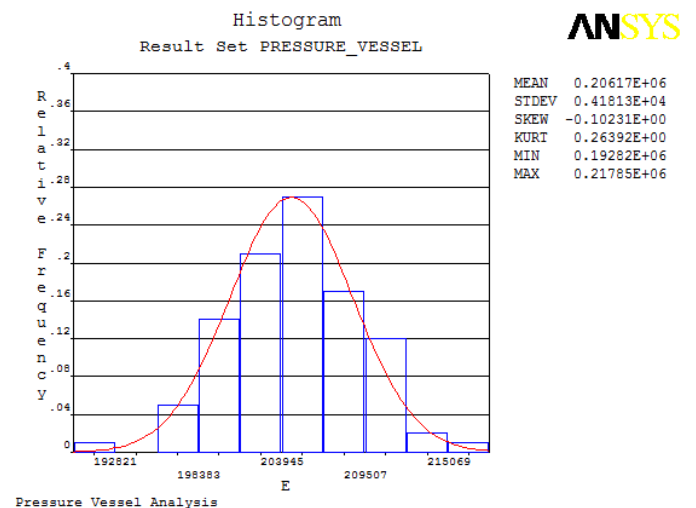


Figure 19: Histogram of Youngs Modulus



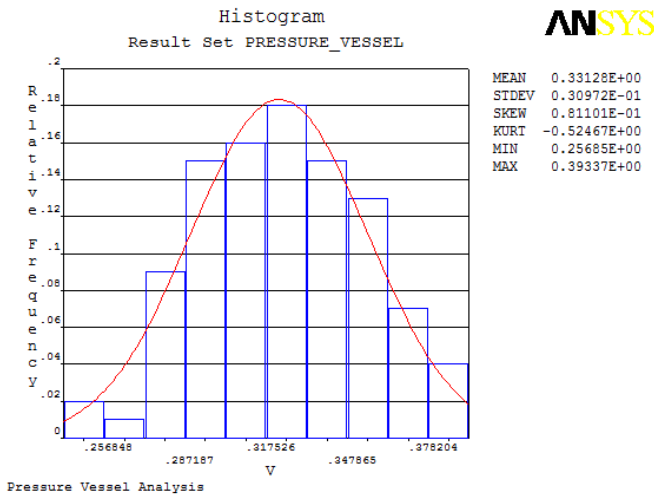


Figure 20: Histogram of Poissons ratio

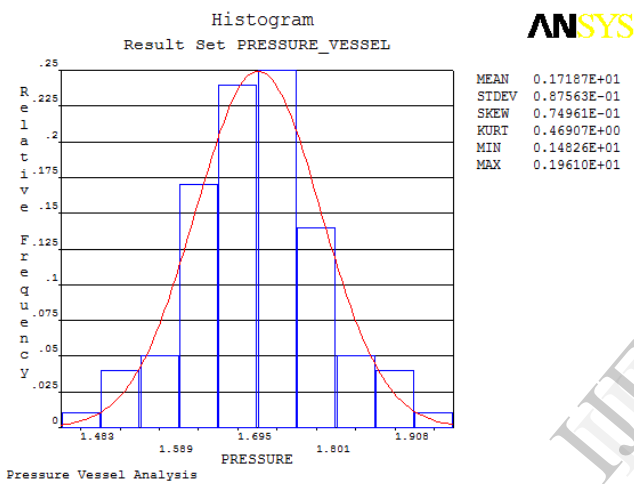


Figure 21: Histogram of internal pressure

Fig. 22 shows the value of mean, standard deviation, skewness, kurtosis, maximum and minimum value for von-Mises stress. Although all input parameters vary using normal distribution function but output stress don't follow same. It can be seen from values of Kurtosis and Skewness.

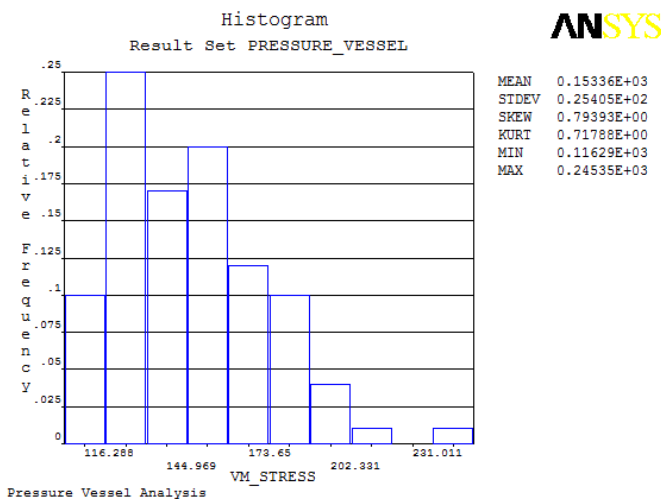


Figure 22: Histogram of von-Mises stress

Technical products are typically designed to fulfill certain design criteria based on the output parameters. For pressure vessel, a design criterion is that the stress should be within allowable limit. The cumulative distribution curve for von-Mises stress is shown in Fig. 23. The line in middle is the probability P. The upper and lower curves in Fig. 23 are the confidence interval using a 95% confidence level. The confidence interval quantifies the accuracy of the probability results.

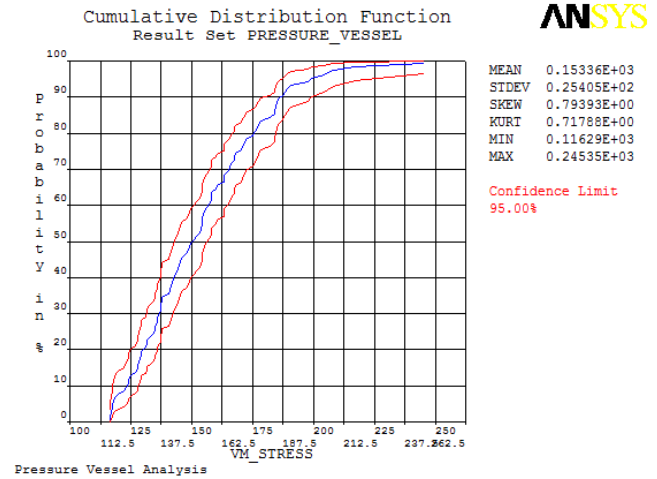


Figure 23 : 95% confidence interval for von-Mises stress

After the reliability of the pressure vessel has been quantified, it may happen that the resulting value is not sufficient. Then, probabilistic methods can be used to answer the following question: Which input variables should be addressed to achieve a robust design and improve the quality? The answer to that question can be derived from probabilistic sensitivity diagrams plot.

The result of the proposed method is Spearman rank-order correlation to determine which random parameters are most significant in affecting the uncertainty of the design. The sensitivity analysis results obtained are shown in Fig. 24. The sensitivities are given as relative values (bar chart) and relative to each other (pie chart). From Figures as shown below, the thickness and internal pressure have significant influence on the von-Mises stress. On the other hand, inner radius, length, modulus of elasticity and poissions ratio have insignificant influence on von-mises stresses.

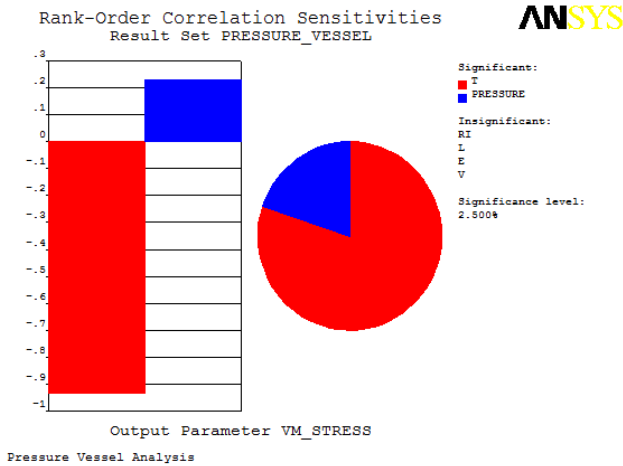


Figure 24: Sensitivity plot for von-Mises stress

While the sensitivities point indicate which probabilistic design parameters one need to modify to have an impact on the reliability probability, scatter plots give a better understanding of how and how far one should modify the sensitive input variables. Improving the reliability and quality of a product typically means that the scatter of the relevant random output parameters must be reduced. To reduce the scatter of the random output parameter to improve reliability and quality, one has two options:

- Reduce the width of the scatter of the most important random input variables
- Shift the range of the scatter of the most important random input variables

Fig. 25 and 26 shows scatter of sensitive input parameters on von-Mises stress. Scatter of sensitive parameters for stress is as expected and follows particular increasing or decreasing trend.

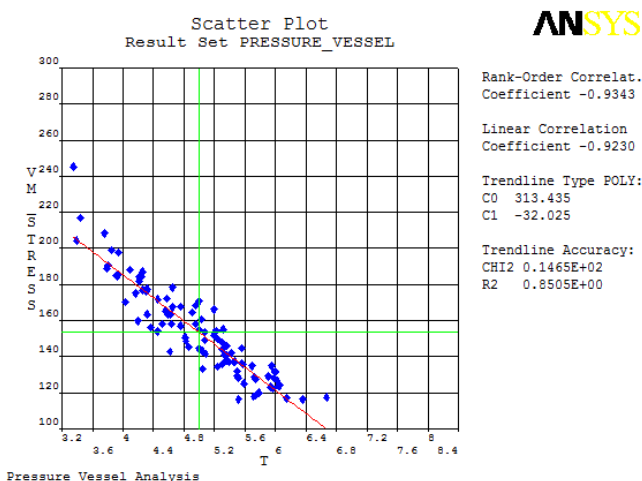


Figure 25: Scatter of thickness for von-Mises stress

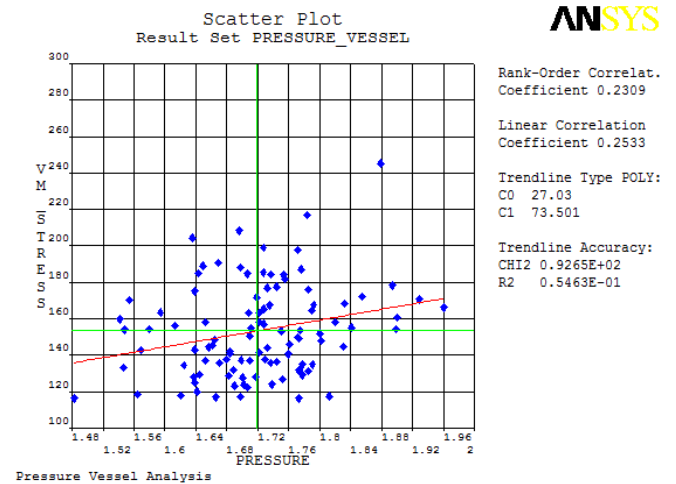


Figure 26: Scatter of internal pressure for von-Mises stress

## VI. CONCLUSIONS

Major conclusions for present study are listed as below:

- Successfully carried out probabilistic design of pressure vessels used using ANSYS PDS feature. Probabilistic design uses Gaussian distribution for various input parameters and simulation uses Monte Carlo simulation technique for sampling.
- From study it appears that the thickness and internal pressure have significant influence on the output parameter i.e. von-Mises stress. On the other hand, modulus of elasticity and poissions ratio, inner radius and length have a insignificant influence on von-Mises stress.
- Analytically calculated stress in pressure vessel (baseline model) is lower than allowable limit of material. This ensures safe design of pressure vessel based on ASME calculations. These results have been validated using commercial FEA tool ANSYS and error is within 10%.

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