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Determination of Premium for Insurance Policies covering Long Outages in Cloud Services under the Exponential-Pareto Model

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***Abstract*— Today our existence is unimaginable without the cloud. The past couple of decades have witnessed an exponential growth in cloud adoption as a means to run businesses, health services, military operations, smooth functioning of the governments, educational endeavours, literary activities and so on. The list is indeed unending as almost every breath of our life relies on the cloud. This calls in for building full proof cloud service platforms with the help of systems that are robust and impregnable. The error margin is very little as even a single outage has the potential to jeopardize the entire set up and trigger financial turmoil to burn a hole in the finances of the cloud service provider (CSP) as every outage brings with itself payment of huge refunds to customers in accordance with the clauses of the accord called 'Service Level Agreement' (SLA) laid down between the CSP and the customer. As a result the CSP resorts to insurance for shielding himself from such a financial avalanche. The insurance companies offering cloud insurance have to be very tactful in pricing policies to cover outages. Particularly long outages can cause widespread mayhem and can lead to mammoth claims so much so that they have the ability to eradicate the insurance company from the insurance landscape. We therefore present some very significant methods to device premiums for policies offering indemnity for long outages by utilizing well-known premium principles for the very popular exponential-Pareto model of cloud outages. Magnificent numerical examples support and strengthen our research findings.**

***Keywords*—Cloud, Cloud Insurance, Service Level Agreement, Premium Principles, Exponential-Pareto model for outages**

I. INTRODUCTION

We are living in the world of 'Blink it'. Particularly in the Indian context, 'Blinkit' is not just seen as a service but a phenomenon that like the 'Aladdin Genie' just brings the ordered commodities to your doorstep in the blink of an eye. However the key takeaway here is that Blinkit and all other such e-commerce platforms thrive on a very strong cloud system to manage its operations and fulfill its commitment of delivering everything in 10 minutes. This example not only underscores the importance of cloud systems in our day to day lives but also rings a bell that cloud failures or outages have the ability to bring the life to a standstill. The reality is that although cloud has carved a niche in our busy lives at an unprecedented speed, with an even greater rate, cyber-attacks and cyber-threats have started to raise their head. Cloud outages may result from system failures caused by internet disruption, hacking events, ransomware attacks, espionage attempts, data breaches etc. and may lead to long outages causing head to toe topple situation for enterprises, governments, financial hubs, educational institutions, the travel industry, military operations and even the common man. The need of the hour is to build full proof systems that have the ability to withstand these havocs and resume operations at a lightning speed. The recent case of the quick commerce platform 'KiranaPro' being hacked and the cyber-attack leading to scrapping off the servers, app code and sensitive data on May 24, 2025 (c.f. [1]) is an alarm that cyber-attacks have become very frequent with each such incident threatening the smooth functioning of our day to day lives. KiranaPro was catering customers in 50 Indian cities with groceries from nearby kirana stores and supermarkets via a voice-based interface supporting four languages viz. Hindi, Tamil, Malayalam, and English. The startup had a huge volume of business with 55,000 registered users and 2000 daily orders prior to the attack being responsible for hampering its operations. To date the startup's app remains live but is unable to process any orders indicating the aftermath of the blow. In fact, all virtual machines running on Amazon Web Services' (AWS's) Elastic Compute Cloud (EC2) were reportedly deleted, along with user data consisting of names, mailing addresses, and payment information. This is the case of a long outage. While incidents like this call in for the making the cloud system technically robust, the financial aspect of each outage and in particular long outage is rather gloomy. For every outage the cloud service provider (CSP) needs to compensate its customers with a generous amount specified in the clauses of the 'Service Level Agreement' (SLA) that the two parties chalk out when they agree to carry out business among themselves. The amount obviously inflates for long outages according to a formula that is proportional to the duration of the outage. The problem here is that the CSP may not be able to do justice to his commitment on his level and so foreseeing such situations, he resorts to buying reliable insurance policies that will help him withstand such financial blows. The insurer on the other hand has to be vigilant enough to scan the cyber terrain to build the nitty-gritty of the insurance contract to determine a fair premium for the policy. In particular policies covering long outages are liable to plunge the insurance companies into deep down losses if the insurer is unaware of the gravity of the situation and misjudges possible disasters that can lead to huge claims. This research paper is therefore an attempt to help the insurer to decide a fair premium for policies covering long outages so that he can hold his ground in the event of catastrophic cyber failures. The

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research in this direction is rather thin and this paper is the maiden attempt to bridge the gap between the actuarial literature and premium pricing for long cloud outages. Prior to this, [2] have suggested premium pricing for usual cloud outages using the well-known Poisson model of outages but they have not addressed the case of long outages. We consider the Exponential-Pareto model of cloud outages and build in the premium formulae for policies offering coverage for long outages utilizing well-known premium principles available in the actuarial theory.

The structure of the paper is as follows. Section II deciphers the basic definitions and notations utilized in the paper. Section III presents the Exponential-Pareto model of outages for the cloud. Section IV throws light on long outages with particular reference to Exponential-Pareto model thereby deriving their distribution and assessing the economic loss incurred due to these outages. Section V portrays six well known premium principles and utilizes them to determine the premium formulae for the insurer who offers coverage for long outages assuming an Exponential-Pareto model for the cloud functioning. Section VI then exhibits the outlined theory by presenting impressive numerical results. Section VII concludes the paper and highlights directions for future work.

II. DEFINITIONS AND NOTATIONS

In this section, we throw light on the threads that will weave the tapestry for charging the optimum premium from the insurer's perspective for long cloud outages. We are considering a cloud insurer in the market who is willing to provide complete coverage to a CSP in the event of a long outage. A long outage is indeed a risk in the actuarial terminology and is generally denoted by X. While looking at the risk X, we are actually referring to the non-negative random variable that encompasses all possible claims that arise from a long outage and are to be settled by the cloud insurance company. Indeed the insurer designs a specific contract called a policy to cover such long outages and sells this policy at a stated price to the CSP. This price is referred to by the word premium in the actuarial jargon.

The aim of this research paper is to determine the correct premium for such insurance policies covering long outages. It is clear from the above definitions that the premium designated by the insurance company for policies covering long outages must be some function of X (c.f. [3]). In fact, we denote the premium as π_X and a rule or methodology that designates a numeric value to π_X is referred to as a premium calculation principle. A premium principle is thus presented as $\pi_X = \eta(x)$; where $\eta(x)$ denotes some function of x. A variety of functions can be employed to depict this dependence and this leads to a plethora of premium principles from which the insurer can pick the most suitable one according to his requirements. We undertake the detailed discussion on premium principles in Section 4.

III. THE EXPONENTIAL-PARETO MODEL FOR CLOUD OUTAGES

We now turn our attention to the most plausible models available in the literature that describe the functioning of the cloud. In general the cloud will either be 'working' or would be down, i.e. experience an 'outage'; the two situations that are commonly referred to as 'ON' and 'OFF' states respectively (c.f. [2], [4]-[8]). These states are depicted in Figure 3. The case of 'Graceful Degradation', which means a design strategy that will let the cloud to continue to operate even when some components fail or are functioning under suboptimal conditions, is not considered here. The 'ON' state signals that the system is surviving and so we denote it by the letter S while the `OFF' state signifies 'Death' or the cloud being 'Down' and is therefore denoted by the letter D.

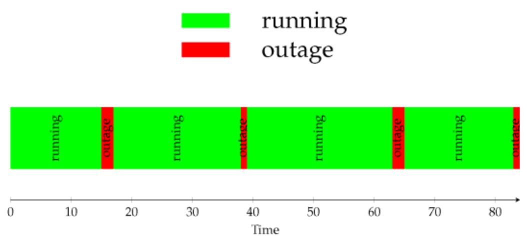

Figure 1. Cloud State Sequence

On scrutinizing the available literature on cloud functioning minutely, one come across only a bunch of models that portray the Uptime or ON period and the downtime or OFF period (c.f. [6]). These are listed below:

- the Poisson Model (the Markov Model or Exponential-Exponential Model)
- the Exponential-Pareto Model
- the Pareto-lognormal Model

These three models are distinguished by the probability distributions that describe the duration of the ON and OFF states. Out of these the focus of this paper is the exponential-Pareto model which we describe below.

The Exponential-Pareto Model is described by the duration of the ON state following an exponential distribution while the duration of the OFF state being assumed to be a Generalized Pareto distribution with parameters 0, σ and ξ abbreviated as $\mathrm{GPD}(0,\sigma,\xi)$. In this case the c.d.f.s are given by:

$$F_S(x) = 1 - e^{-x\lambda_S}\ ;\ x \geq 0 \qquad (1)$$

and

$$F_D(x) = \begin{cases} 1 - \left(1 + \dfrac{\xi}{\sigma}x\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - e^{-\frac{x}{\sigma}} & \text{if } \xi = 0 \end{cases} \qquad (2)$$

where the support is $x \geq 0$ for $\xi \geq 0$ and $0 \leq x \leq -\sigma/\xi$ for $\xi < 0$. In fact σ is the scale parameter and ξ is the shape parameter and the location parameter μ is taken be to 0. As the duration of the operating state is described by an exponential(λ_S) distribution, the number of outages say N_T over a time interval say T follow Poisson distribution with parameter $\lambda_S T$. The Poisson distribution for number of outages

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has also been employed in service level agreement decision making (c.f. [9]). This means that

$$P[N_T = n] = e^{-\lambda_S T}\frac{(\lambda_S T)^n}{n!}\ ;\ n = 0,1,2,\dots, \quad (3)$$

which is the probability law for the Poisson Distribution. For brevity, we denote λ_S by λ.

The Exponential-Pareto model was designed specifically for cloud services by Maurizio Naldi (c.f. [10]) based on a best-fit procedure on the database of customer-reported outages for five cloud providing titans viz. Google, Amazon, Rackspace, Salesforce, and Windows Azure. The data was extracted through Cloutage (cloutage.org), which was coined by the Open Security Foundation in April 2010 and the International Working Group on Cloud Computing Resiliency (IWGCR, hosted on http://iwgcr.org/). This working group specializes in scrutinizing cloud computing resiliency. Of late, Cloutage has been unavailable. The Exponential-Pareto model has been utilized in research papers to estimate the plausibility of refunds connected to insurance contracts for cloud services (c.f. [7]) as well as to understand the role of the network for the overall unavailability (c.f. [11]).

IV. CONFIGURING LONG OUTAGES UNDER THE EXPONENTIAL-PARETO MODEL

In this section we formally define long outages and derive their distribution in the backdrop of Exponential-Pareto model. We also determine the economic loss that arises due to long outages thereby assessing the amount of claims that can arise for the insurer that provides coverage for such risks by deciphering the distribution of the economic loss. As hinted in the introduction the 'Service Level Agreement' is a pact between the CSP and its customers that demands that the CSP compensates its customers for each outage or long outage in terms of a monetary amount. Outages and long outages are in fact referred to as 'Quality of Service' metrics in the cloud literature (c.f. [7]-[8]). In the present research paper our interest lies in long outages where the insurer assumes an exponential-Pareto model for durations of alternating On-Off sequence of states respectively. In order to get the in-depth understanding of how this entire set up works, we portray the performance of the cloud during surveillance period T by two sequences of alternating active mode periods and outages as follows:

$$\mathcal{S} = \{S_1, S_2, \dots, S_i, \dots\} \quad (4)$$

and

$$\mathcal{D} = \{D_1, D_2, \dots, D_i, \dots\} \quad (5)$$

where, as is clear from equations (1) and (2) all the $S_i{}'$s are independently and identically distributed (i.i.d.) random variables following an exponential distribution while all the $D_i{}'$s are i.i.d. random variables following a Generalized Pareto distribution. Sequences of the type $S_1S_2D_3S_4S_5\dots$ may arise with all possible types of permutations and combinations. Further we denote the threshold value to count an outage to be a long outage by 'w'. This number could vary according to the kind of businesses or operations relying on cloud being used. It could be 2 hour for a normal Google application and so on. If we denote the number of long outages by N_{LO}, the following equation defines the relationship between N_{LO}, the number of outages N_T over the observation time T and the cut-off point w, viz.

$$N_{LO} = \sum_{i=1}^{N_T} \mathbb{1}_{[D_i > w]} \quad (6)$$

where $\mathbb{1}_{[\xi]}$ is the indicator function which assumes the value 1 if the condition ξ is fulfilled and the value 0 if the condition is negated. In fact the indicator function is a Bernoulli random variable taking the value 1 with probability

$$\mathbb{P}\left[\mathbb{1}_{[D_i > w]} = 1\right] = \left(1 + \frac{\xi}{\sigma}x\right)^{-\frac{1}{\xi}} = p\ (\text{say}) \quad (7)$$

Further let

$$q = 1 - p = 1 - \left(1 + \frac{\xi}{\sigma}x\right)^{-\frac{1}{\xi}} \quad (8)$$

We now try to obtain the distribution of N_{LO} which is a random Poisson sum of independent Bernoulli $\left(\left(1 + \frac{\xi}{\sigma}x\right)^{-\frac{1}{\xi}}\right)$ random variables (abbreviated Ber (p)) since N_T terms are involved in the sum and N_T is a Poisson random variable as noted in Section III (c.f. equation (3)). For this we try to obtain the moment generating function of N_{LO} and then try to identify its distribution using the uniqueness theorem of m.g.f. Let $M_{N_{LO}}(t)$ denote the m.g.f of N_{LO} assuming that it exists. Then using the definition of m.g.f. and the fact that the indicator functions are i.i.d. Ber (p) random variables, we have

$$\begin{aligned} M_{N_{LO}}(t) &= \mathbb{E}\left[e^{tN_{LO}}\right] \\ &= \mathbb{E}\left[e^{t\sum_{i=1}^{N_T}\mathbb{1}_{[D_i>w]}}\right] \\ &= \mathbb{E}\left[\left(\mathbb{E}\left[\prod_{i=1}^{n} e^{t\mathbb{1}_{[D_i>w]}}\right] \middle| N_T = n\right)\right] \\ &= \mathbb{E}\left[\prod_{i=1}^{n} \mathbb{E}\left[e^{t\mathbb{1}_{[D_i>w]}}\right]\right] \\ &= \mathbb{E}[(q + pe^t)^n] \\ &= e^{-\lambda T\left(1-(q+pe^t)\right)} \\ &= e^{p\lambda T(e^t - 1)} \end{aligned} \quad (9)$$

which is the m.g.f. of Poisson ($p\lambda T$) distribution. Hence by uniqueness theorem of m.g.f., we have that:

$$N_{LO} \sim \text{Poisson}\ (p\lambda T) \quad (10)$$

with p given by (7).

Now, the economic loss is determined according to the covenants of the SLA, so that in general the reparation paid by the CSP to its clients for long outage episodes is taken to be proportional to the number of long outages in a specified interval say T. As a result

$$X = kN_{LO} \quad (11)$$

where X is total economic loss over the interval T and as defined in the Section II, this amount X is indeed presented by

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the CSP to the insurer as the claim amount. Further k is the compensation to be paid for each outage and N_{LO} is the number of long outages. As seen above, N_{LO} adheres to a Poisson law with parameter $p\lambda T$ with p given in (7). It is well known that for a Poisson distribution with parameter $p\lambda T$, the mean and variance coincide with the parameter. This means that:

$$\mathbb{E}[X] = kp\lambda T \tag{12}$$

and

$$\mathrm{V}[X] = k^2 p\lambda T \tag{13}$$

where $\mathbb{E}[X]$ and $\mathrm{V}[X]$ denote respectively the expectation and variance of the loss random variable X. In the next section, we list the various premium principles and capitalize on them for calculating the premium for long outages under the Exponential-Pareto model of cloud outages.

V. THE PREMIUM PRINCIPLES AND THEIR APPLICATION TO THE EXPONENTIAL-PARETO MODEL OF CLOUD OUTAGES

A variety of premium principles appear in the actuarial literature and the choice made by the insurer largely depends on the application at hand. We now consider six famous premium principles (c.f. [2], [3] and [12]) and then by utilizing them, compute the premium for long outages under the Exponential-Pareto model.

A. The Pure Premium Principle

Definition: The Pure premium principle postulates

$$\pi_X = \mathbb{E}[X] \tag{14}$$

As can be seen from its definition it is a perhaps the most straightforward principle but at the same time it is totally futile for the insurer as it does not allow for any profit margin and also has no allowance for an awkward situation where claims escalate, situations which arise frequently for cyber insurers.
Pure Premium for Long Outages under the Exponential-Pareto Model: Now direct application of (12) and (14) yields the pure premium for long outages under the Exponential-Pareto model as:

$$\pi_X = kp\lambda T \tag{15}$$

with p given in (7).

It is pretty evident from the formula (15) that the pure premium displays a direct proportionality to k, i.e., the reparation offered by the CSP to its customers for every episode of a long outage as well as the average number of long outages during time T viz. $p\lambda T$. Also if the average number of outages in the time interval T remains fixed but the probability of a long outage in the aforesaid interval viz. p increases, the premium is augmented, this is quite understandable. This relationship is pretty straightforward because as the amount offered by the CSP for each long outage is incremented or the average number of long outages tends to bulge, the claim amount would automatically increase and so the concerned insurer must do justice to selling such policies by levying a higher premium. An investigation of the dependence of the pure premium on k, p and $p\lambda T$ is performed in the next section by utilizing numerical examples.

B. The Expected Value Principle

Definition: This expected value premium principle sets

$$\pi_X = (1+\alpha)\mathbb{E}[X] \tag{16}$$

where $\alpha > 0$ is 'the Premium Loading Factor' and $\alpha\mathbb{E}[X]$ describes the loading.

Just like the pure premium principle, the expected value principle is also an uncomplicated one. The only limitation one can see is that does not give any importance to the variability of the risk distribution and therefore allocates equal premium to all risks having the same average values. However giving due importance to the dispersion of the risk distribution is vital when dealing with highly unpredictable events such as the long cloud outages.

Premium for the Exponential-Pareto Model: Laying hands on (12) and (16) implies that applying the expected value premium for long outages under the Exponential-Pareto model yields:

$$\pi_X = (1+\alpha)kp\lambda T \tag{17}$$

with p given in (7).

Equation (17) resonates the same ideas as the pure premium given in (15) with the loading factor adding the list of direct proportionality. We scrutinize the dependence of the premium on α, k, p and $p\lambda T$ numerically in the next section.

C. The Variance Principle

Definition: The variance principle proposes

$$\pi_X = \mathbb{E}[X] + \alpha V[X]. \tag{18}$$

Again 'the Premium Loading Factor' is given by $\alpha > 0$ with $\alpha V[X]$ representing the loading which is proportional to the variance $V[X]$.

It is evident from (18) that the variance principle is a step up from the expected value principle as it rectifies the shortcoming of the expected value principle by considering the variability of the risk distribution hence yielding a more precise premium. However, there is still scope for improvement as risks having equal first two moments may be strikingly different from the insurer's point of view particularly in cases where losses follow a highly skewed distribution. As a result it may be highly beneficial to consider premium principles that involve the entire probability distribution of the risk X rather than incorporating just a few statistical properties (c.f. [13] and [2]). The present situation where we are considering long cloud outages which is a new type of risk in the insurance domain champions the use of such principles. In fact on doing a little soul searching one can find many such principles in the actuarial literature such as the Orlicz principle (c.f. [14]), the Esscher Principle (c.f. [15]) and the risk adjusted premium principles (c.f. [16]). Amongst these, we consider the Esscher premium principle here.

Premium for the Exponential-Pareto Model: Using (12), (13) and (18), the variance principle produces the following premium for policies offering coverage of long outages in the case of Exponential-Pareto model of outages:

$$\pi_X = (1+\alpha k)kp\lambda T \tag{19}$$

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with p given in (7).

Equation (19) exhibits a quadratic relationship between the premium π_X and k which is the amount offered for each long outage by the CSP to its customers. This is indicative of the insurer's sensitivity to increases in the compensation offered by the CSP. We perform a thorough investigation of the relationship between the premium π_X and the involved quantities α, k and $p\lambda T$ quantitatively in the ensuing section. Since the relationship between π_X and $p\lambda T$ is linear and so is between π_X and p, we limit ourselves to displaying the former for the variance principle.

D. The Standard Deviation Principle

Definition: The standard deviation premium principle declares

$$\pi_X = \mathbb{E}[X] + \alpha\sqrt{V[X]} \qquad (20)$$

where $\alpha > 0$ is 'the Premium Loading Factor' and $\alpha\sqrt{V[X]}$ denotes the loading which is directly proportional to the standard deviation of the loss distribution.

The standard deviation principle reflects the variance principle and utilizes the variability of the risk distribution yielding a more precise premium value.

Premium for the Exponential-Pareto Model: Exploiting (12), (13) and (20), the standard deviation principle yields the following premium for policies offering coverage of long outages in the case of Exponential-Pareto model of outages:

$$\pi_X = (\sqrt{p\lambda T} + \alpha)k\sqrt{p\lambda T} \qquad (21)$$

with p given in (7).

We glance at the behaviour of the premium π_X and the involved quantities α, k and $p\lambda T$ numerically in the upcoming section. Just as for the variance principle since the relationship between π_X and $p\lambda T$ is and π_X and λT, is essentially the same, we investigate the former.

The first four principles consider here encompass the moments of the loss distribution. The next principle banks upon 'Expected Utility Theory' which is a pivotal concept in actuarial science and insurance.

E. The Principle of Zero Utility

Daniel Bernoulli (1738) (c.f. [17]) founded the utility concept which sparked the development of 'Expected Utility Theory'. A utility function (u.f.) $u(x)$, is a function that quantifies the value or utility that an individual or an organization allocates to the monetary amount x (c.f. [18]). In fact, utility is a function $u: \mathbb{R}_0^+ \to \mathbb{R}_0^+$ that computes a utility for each monetary amount of the domain. The utility function abides by the principle of non-satiation, i.e. $u'(x) > 0$. This means that $u(x)$ is indeed an increasing function of the wealth x such that individuals will like more wealth to less. In insurance, finance and actuarial sector, the investor preferences depend on their attitude towards risk, which take shape in the form of properties of utility functions. There are three cases: risk-averse, risk-neutral or risk-loving investors (c.f. [2] and [3]). A risk-averse (risk-loving) investor is one for whom an incremental increase (decrease) in wealth is of less interest than an incremental decrease (increase). This means that risk averse investors are more concerned about potential losses than potential gains. However, this statement flips around for risk-loving investors. As a result, the utility function $u(x)$ is strictly concave (convex), that is, $u"(x) < (>) 0$ for a risk averse (risk loving) investor. On the other hand, a risk- neutral investor is unfazed by risk and thus he is characterized by $u'(x) > 0$ and $u"(x) = 0$. The utility function is constructed to reflect an individual's choices as to whether or not, he prefers, hates or is indifferent to risk. The more marked the curvature of $u(x)$, the greater will be the risk aversion which is manifested by the coefficient of risk aversion defined as:

$$r(x) = -\frac{u"(x)}{u'(x)}. \qquad (22)$$

Another name for $r(x)$ is the Arrow-Pratt measure of absolute risk-aversion (ARA) (c.f. [19] and [20]). $r(x)$ can either be increasing or decreasing function of the wealth x, the respective cases being abbreviated as IARA and DARA. Finally if $r(x)$ does not show any variation with respect to change in wealth x, then the utility function exhibits what is called 'Constant Absolute Risk Aversion (CARA)'.

We list below some of the most popular utility functions employed in the insurance sector as well as in the fields of actuaries, finance and economics (c.f. [3] and [12]):

1. Exponential with parameter β:
$$u(x) = -exp\,(-\beta x); \beta > 0.$$

2. Quadratic with parameter β :
$$u(x) = x - \beta x^2; x < \frac{1}{2\beta}, \beta > 0.$$

3. Logarithmic with parameter β :
$$u(x) = \beta \log x; x > 0, \beta > 0.$$

4. Fractional with parameter β:
$$u(x) = x^\beta; x > 0, 0 < \beta < 1.$$

5. Linear:
$$u(x) = x.$$

The CARA property that we detailed above is indeed a characteristic of the exponential utility function and this has led to the widespread use of exponential utility function in the cyber and cloud insurance literature (c.f. [2], [4]-[6], [21]-[23]). The stage is now set to furnish the formal statement of the principle of zero utility.

Definition: The principle of zero utility prescribes that the minimum premium π_X that an insurer with initial wealth, w, should set to provide full insurance coverage against a risk X should abide by the following equation.

$$u(w) = \mathrm{E}[u(w + \pi_X - X)] \qquad (23)$$

This clearly means that the premium in general hinges upon the insurer's surplus or wealth w. However an exclusion is the case of the exponential utility function furnished in the list of u.f.s for which

$$\pi_X = \frac{1}{\beta}\ln M_X(\beta) \tag{24}$$

where $M_X(\beta)$ is the moment generating function (m.g.f.) of the loss random variable X with parameter β such that:

$$M_X(\beta) = \mathrm{E}\left[e^{\beta X}\right] \tag{25}$$

provided the expectation exists. Significantly, here β is also the coefficient of risk aversion defined earlier. For the present case the principle of zero utility is nomenclated as the exponential principle. This principle being based on the m.g.f. of the loss distribution, encompasses greater information about X than any principle discussed up till now. In [4] the exponential principle is utilized to compute the maximum premium that the CSP is prepared to pay sought complete protection under outages using the Poisson model of outages.
To calculate the premium for the Exponential-Pareto model of cloud outages we will consider the exponential u.f. and therefore consider the exponential principle.

Premium for the Exponential-Pareto Model: Exploiting (24), and (11), the following premium is obtained for long outages under the exponential premium principle in the case of Exponential-Pareto model of outages:

$$\pi_X = \frac{p\lambda T(e^{k\beta} - 1)}{\beta} \tag{26}$$

where we lay hands on the fundamental properties of m.g.f. and utilize the fact that the m.g.f. of a Poisson random variable say Y having a Poisson distribution with parameter λ is given by:

$$M_Y(t) = e^{\lambda(e^t - 1)} \tag{27}$$

where t happens to be the parameter of the m.g.f. In our case $t = c\beta$.

Equation (26) portrays an encapsulating relationship between the premium π_X and the average number of long outages during time T viz. $p\lambda T$, k, i.e., the reparation specified by the CSP to the customer for each long outage and the parameter β of the exponential u.f. It is clear that as k and β increase, an exponential escalation is experienced in the premium. We scrutinize the dependence of the premium on these quantities numerically and graphically in the next section.
Lastly, we consider the Esscher premium principle which once again banks upon the m.g.f. of the loss distribution to obtain the premium for the insurer.

F. The Esscher Principle

Postulated by Bühlmann, H. (c.f [24]), the Esscher premium principle contingent upon the Esscher transform formulated by the famous Swedish actuary F. Esscher in 1932 (c.f. [15]) contains a transform that modifies the probability law of the risk distribution and then declares the premium to be the pure premium for the transformed distribution (c.f. [3]).
Assume that the risk random variable X is a continuous random variable on $(0,\infty)$ with probability density function (p.d.f.) f. We then declare the p.d.f. for a new transformed random variable $\tilde{X}$ as g by applying the following modification:

$$g(x) = \frac{e^{hx} f(x)}{\int_0^\infty e^{hy} f(y)dy}, \qquad h > 0 \tag{28}$$

where the denominator is indeed the m.g.f. of the original random variable X, with parameter h, i.e., $M_X(h)$, with m.g.f defined in (25). The density g is in fact a weighted version of density f. This is clear by writing (28) as:

$$g(x) = w(x)f(x) \tag{29}$$

where

$$w(x) = \frac{e^{hx}}{M_X(h)}. \tag{30}$$

Further, it is evident that $h > 0$ guarantees $w'(x) > 0$ and hence increasing weights are allocated with increase in x. Next from equation (28), the distribution function (c.d.f.) of the transformed random variable $\tilde{X}$ becomes:

$$G(x) = \frac{\int_0^x e^{hy} f(y)dy}{M_X(h)}, \qquad h > 0. \tag{31}$$

The distribution function G defines the Esscher transform of the original distribution function F with parameter h and this gives the premium principle its name. Further the m.g.f. of the transformed random variable $\tilde{X}$ takes the following form:

$$M_{\tilde{X}}(t) = \frac{M_X(t+h)}{M_X(h)}. \tag{32}$$

Finally the following expression gives the expectation of the transformed random variable $\tilde{X}$:

$$\mathrm{E}[\tilde{X}] = \frac{\int_0^\infty x e^{hx} f(x)dx}{\int_0^\infty e^{hy} f(y)dy} = \frac{E[Xe^{hx}]}{E[e^{hx}]} = \frac{E[Xe^{hx}]}{M_X(h)}. \tag{33}$$

This implies that identification of the distribution of the transformed random variable $\tilde{X}$, instantly means that by using the uniqueness theorem of m.g.f., one can easily lay hand on $\mathrm{E}[\tilde{X}]$. A formal definition of the Esscher premium principle follows.

Definition: The Esscher premium principle postulates

$$\pi_X = \frac{\mathrm{E}[Xe^{hx}]}{\mathrm{E}[e^{hx}]} = \frac{\mathrm{E}[Xe^{hx}]}{M_X(h)}, \tag{34}$$

which is in fact $\mathrm{E}[\tilde{X}]$, as can be seen from (33). It is clear that the Esscher principle increases the probabilities of large values and diminishes the probabilities of small values.

Premium for the Exponential-Pareto Model: First of all we compute the m.g.f. of the transformed random variable $\tilde{X}$ in the Exponential-Pareto case. Utilizing equations (32) and (11), and banking upon the basic properties of m.g.f., we have:

$$M_{\tilde{X}}(t) = \frac{M_{kN}(t+h)}{M_{kN}(h)} = \frac{M_N(k(t+h))}{M_N(kh)}$$
$$= e^{\lambda T e^{kh}(e^{kt}-1)} = M_{N'}(kt) = M_{kN'}(t) \quad (35)$$

with $M_{N'}(kt)$ being the m.g.f. with parameter kt of a Poisson random variable with parameter $p\lambda T e^{kh}$. Next one can easily see

$$\tilde{X} = kN' \quad (36)$$

and as a result

$$\mathrm{E}[\tilde{X}] = kp\lambda T e^{kh} \quad (37)$$

Exploiting equations (33), (34) and (37), the Esscher premium principle generates the following premium for long outages in the case of Exponential-Pareto model of outages:

$$\pi_X = kp\lambda T e^{kh}, h > 0. \quad (38)$$

Equation (38) offers an engrossing relationship between the premium π_X and the average number of long outages during time T viz. λT, k, i.e., the compensation given by the CSP to the customer for each long outage and the parameter h of the Esscher transform. It is clear that as k and h increase, an exponential escalation would be seen in the premium. We inspect the dependence of the premium on these quantities numerically and graphically in the next section.

VI. THE PREMIUM PRINCIPLES AND THEIR APPLICATION TO THE EXPONENTIAL-PARETO MODEL OF CLOUD OUTAGES

We now undertake suitable examples to highlight the methodology of premium determination by utilizing the six premium calculation principles detailed in the previous section. We assume that the premium loading factor is $\alpha = 0.1$. The variation in premium is closely monitored with variation in the values of the quantities involved. We scrutinize the behavior of the premium with $p\lambda T$, p and k as stimuli for the pure premium and expected value principles together.

A. The Pure Premium and the Premium based on the Expected Value Principle

We first display the pure premium and the premium based on the expected value principle by keeping the reparation per outage constant, i.e., we set $k = 100$ and vary the average number of long outages during the time T, i.e. $p\lambda T$ from 0 to 50 in Figure 2 below. This figure clearly brings out that for a fixed α and k, as the value of $p\lambda T$ increases, the gap between pure premium and the premium calculated by the expected value principle starts widening. The gap will be more pronounced provided the premium loading factor α increases.
Next in Figure 3 we embark on exhibiting the aforesaid premiums by fixing $k = 100$, $\lambda T = 10$ and varying p from 0 to 1, where the value of $p = 0$ signifies that there is no chance of a long outage while $p = 0$ declares the occurrence of a long outage with certainty. A similar trend as in Figure 2 appears.
Finally in Figure 4 we portray the aforesaid premiums by fixing $p\lambda T = 10$ and varying k from 100 to 1000 and again it is visible that an increase in compensation per long outage would result in greater distance between the pure and expected value premiums.

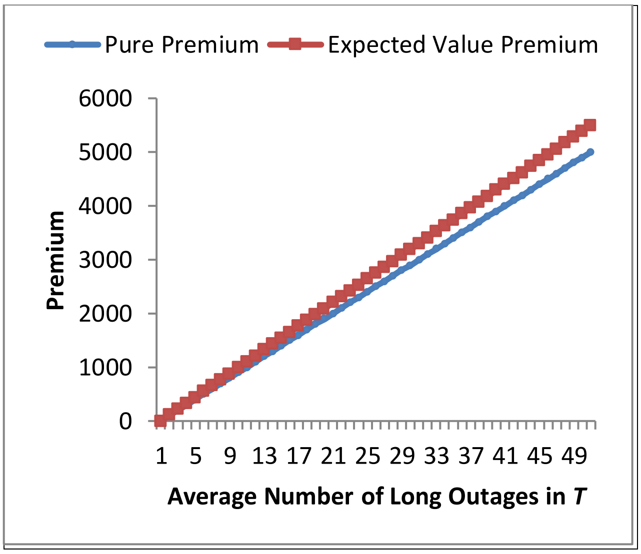

Figure 2: Pure and Expected Premium for $k = 100$ & $\alpha = 0.1$

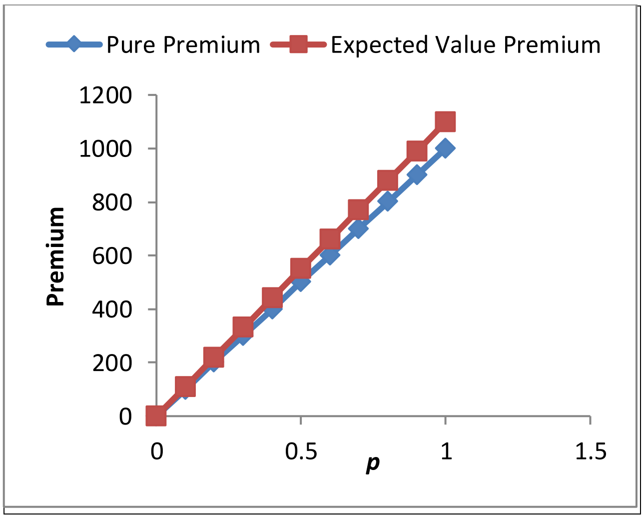

Figure 3: Pure and Expected Value Premium for $k = 100$, $\lambda T = 10$ and $\alpha = 0.1$

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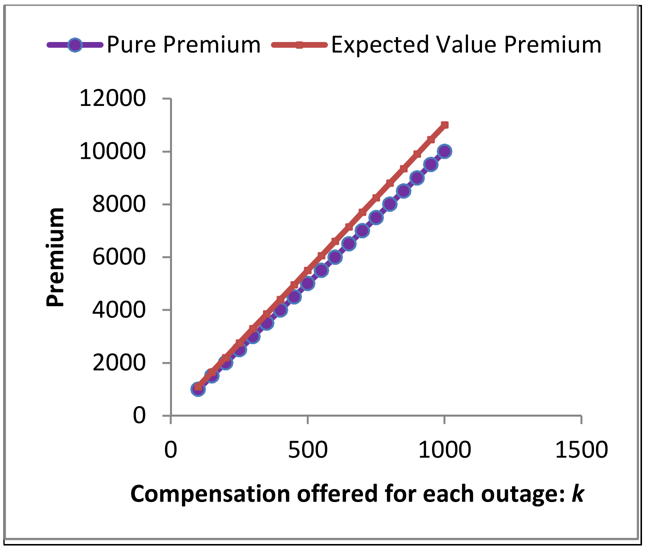

Figure 4: Pure and Expected Value Premiums for $p\lambda T = 10$ and $\alpha = 0.1$

B. The Premium based on the Variance Principle

We adopt a similar approach as above for the premium derived from the variance principle. Figure 5 displays the behaviour of this premium for a constant compensation of 100 for a long outage, i.e. $k = 100$ by increasing $p\lambda T$ from 0 to 50. A clear increasing linear trend can be perceived. Further in Figure 6 we portray the variance premiums by fixing the average number of long outages $p\lambda T = 10$ and varying k from 100 to 1000 and a parabolic curve emerges which is an obvious consequence of (19) which shows that the variance premium is indeed a second degree polynomial in k.

C. The Premium based on the Standard Deviation Principle

We follow the same pattern for the premium based on the standard deviation (S.D.) principle. In Figure 7 the premium is depicted by varying $p\lambda T$ for a constant remedial amount of 100 for an outage, i.e. $k = 100$. One can see a clear increasing trend. Further in Figure 8 we demonstrate the premiums based on standard deviation principle for varying k and a constant $p\lambda T = 10$ and observe an increasing linear trend which is an outcome of (21).

D. The Premium based on the Exponential Principle

Here we take the case of an insurer who adopts an exponential utility function with parameter β defined above. Banking on (26), we first inspect the response of the premium to alterations in the average number of long outages during the time T, i.e. $p\lambda T$ by fixing the other two quantities viz. the compensation per long outage, k and the parameter of the exponential utility function β so that we set $k = 100$ and $\beta = 0.01$. This leads to a clear increasing linear trend in Figure 9 which is a consequence of (26).

Further we depict the pulse of the premium to the changes in the values of the parameter β of the exponential utility function keeping $p\lambda T$ and k constant. As documented earlier, β is also the risk aversion coefficient of the insurer. As β appears in the exponent in (26), the premium starts to explode exponentially with an escalation in its values. Figure 10 supports this observation.

Finally Figure 11 displays the conduct of the exponential premium to the variation in the remedial amount offered for each long outage for face saving by the CSP. As in the previous case, an exponential leap in premium is evident as the values of k vary from 100 to 650 which is an obvious fallout of the fact that as an insurer would study the service level agreement minutely to gauge the magnitude of claims that can arise and only then will decide the premium. A greater reparation amount would mean higher claims and therefore higher premiums.

E. The Premium based on the Esscher Principle

A similar methodology as above is adopted for the premium based on the Esscher principle with β replaced by the parameter h of the Esscher transform, i.e. $h = 0.01$. Invoking (38), we discover similar behavioural pattern as above as can be seen from Figures 12-14. This is indeed attributed to a striking similarity in equations (26) and (38).

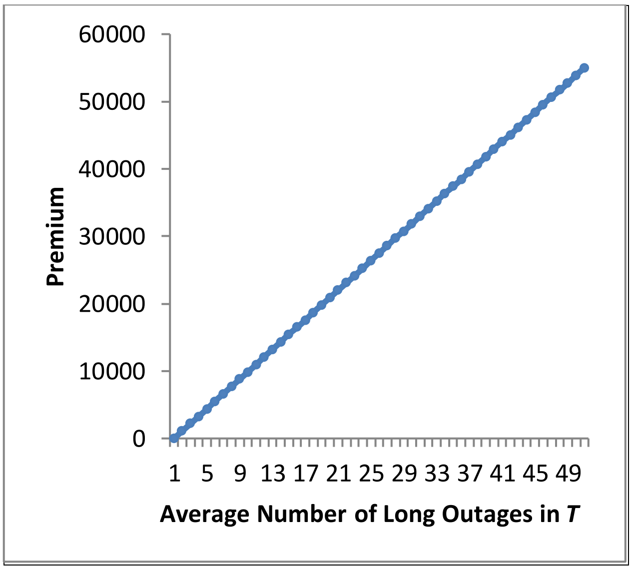

Figure 5: Variance Principle Premium for $k = 100$ & $\alpha = 0.1$

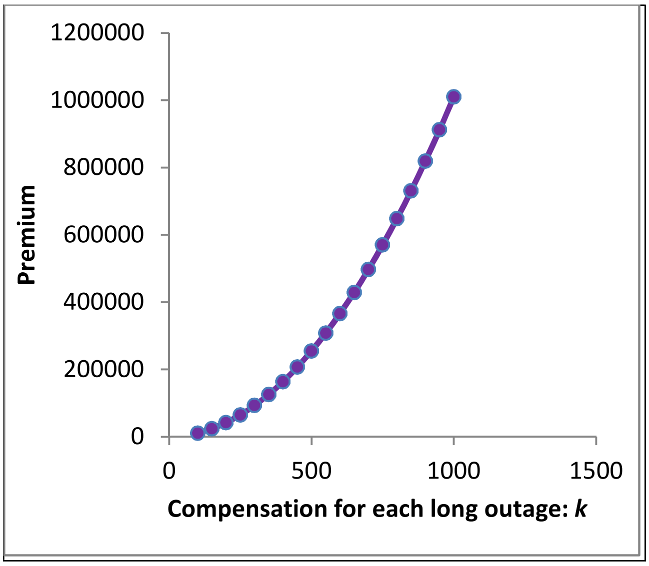

Figure 6: Variance Principle Premium for $p\lambda T = 10; \alpha = 0.1$

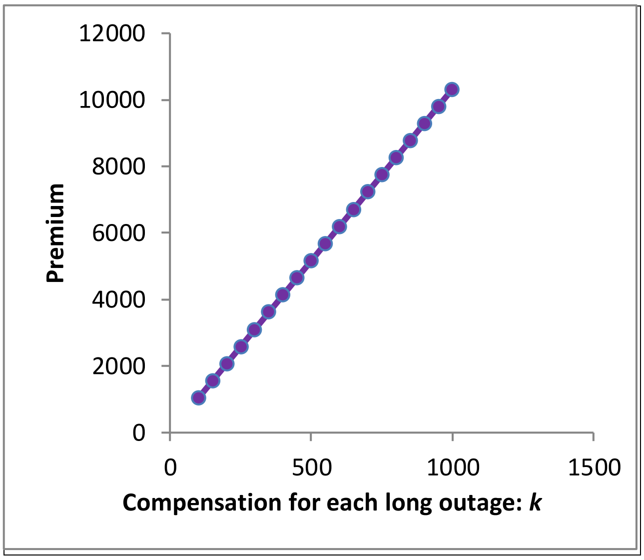

Figure 8: S.D. Principle Premium for $p\lambda T = 10; \alpha = 0.1$

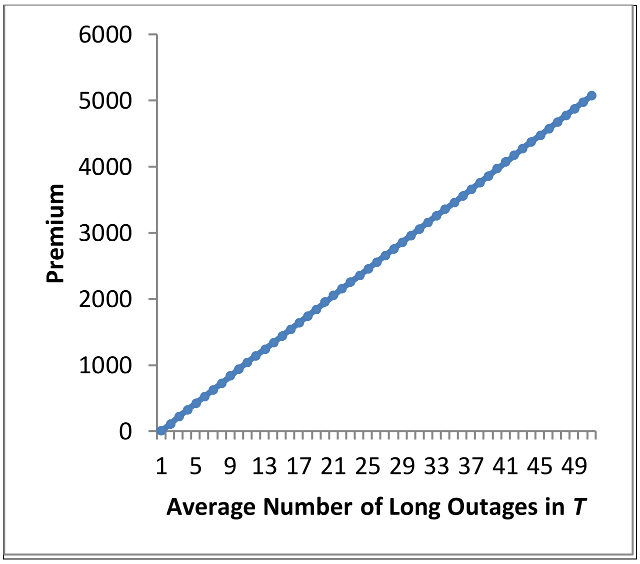

Figure 7: S.D. Principle Premium for $k = 100$ & $\alpha = 0.1$

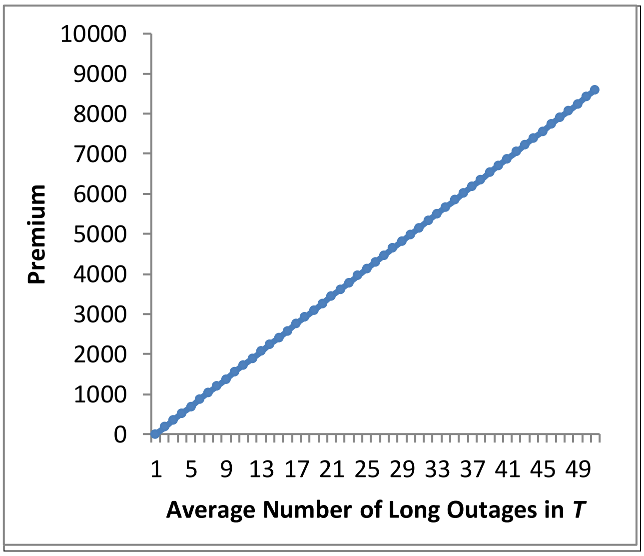

Figure 9: Exponential Principle Premium $k = 100; \beta = .01$

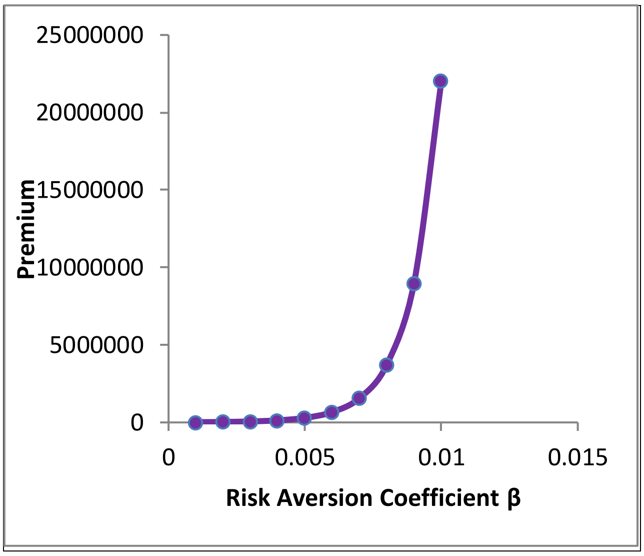

Figure 10: Expo. Principle Premium $p\lambda T = 10$ and $k = 1000$

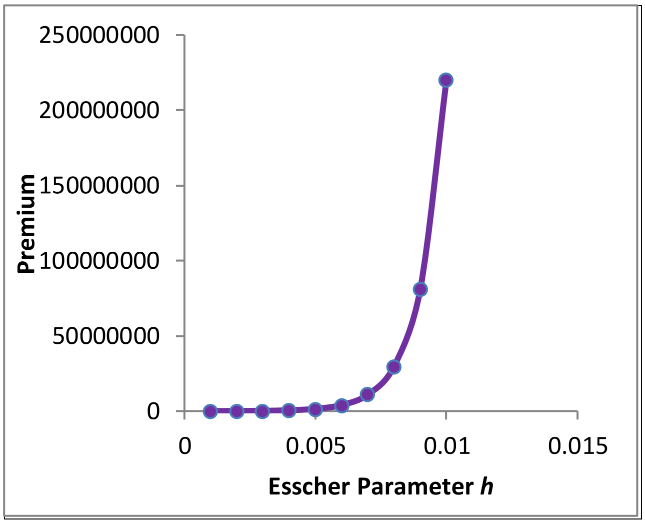

Figure 13: Esscher Principle Premium for $p\lambda T = 10$ and $k = 1000$

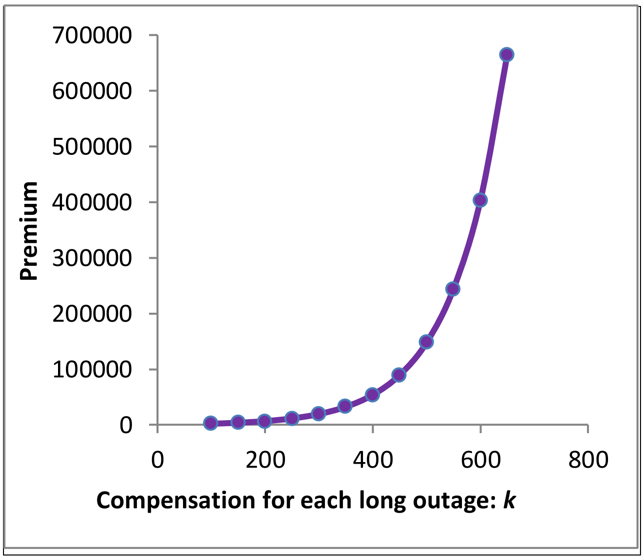

Figure 11: Exponential Principle Premium $p\lambda T = 10$; $\beta = .01$

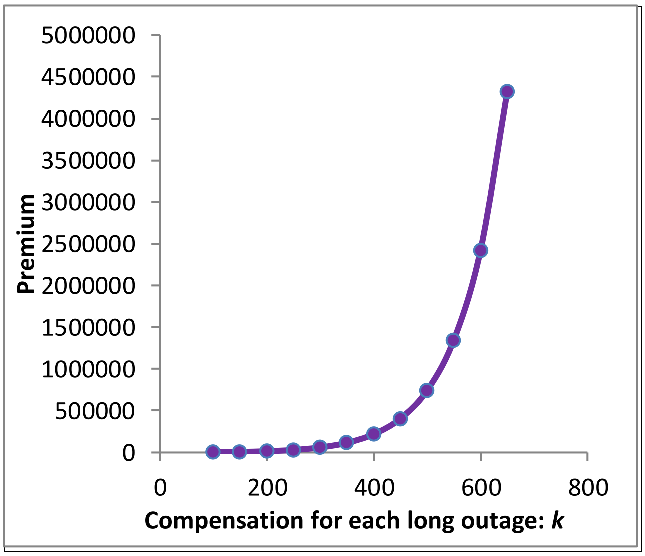

Figure 14: Esscher Principle Premium $p\lambda T = 10$; $h = 0.01$

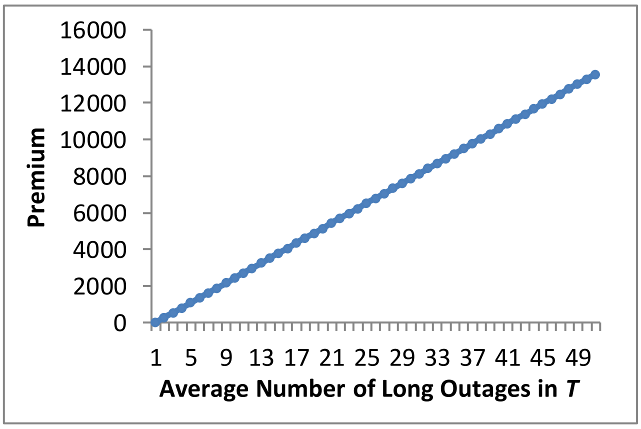

Figure 12: Esscher Principle Premium $k = 100$; $h = 0.01$

VII. CONCLUSION

In this paper we have presented an extensive methodology for determination of premium for insurance policies covering long outages in the cloud by utilizing six popular premium principles for the Exponential-Pareto model of cloud outages. This paper presents ground breaking research in deriving the distribution of number of long outages under the Exponential-Pareto model. Cloud insurance is in its embryonic stages and this paper presents remarkable strategies to help insurers determine competent policies with optimum premiums for long outages safeguarding them against a possible financial avalanche in case of catastrophic claims arising due to multiple cloud outages. Exploiting the information about past history of the cloud service provider (CSP), the insurer can cleverly prescribe the premium by choosing the most appropriate

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premium principle. In this research we have exploited the Exponential-Pareto model of cloud outages. In our forthcoming research we are working on the Poisson and the Pareto-lognormal models of cloud outages for calculating the premium for long outages.

REFERENCES

[1] J. Vishwakarma, "Cyberattack wipes out KiranaPro's servers, app code and sensitive data" https://indianstartupnews.com/news/cyberattack-wipes-out-kiranapros-servers-app-code-and-sensitive-data-9338983, June 2025 Accessed : 06-06-2025.

[2] R. K. Bahl, " Computation of Premium for Cloud Insurance under the Poisson Model" International Journal of Adanced Research, vol. 13(2) pp. 1515-1531, 2025.

[3] D.C.M. Dickson, Insurance Risk and Ruin, International Series on Actuarial Science. Cambridge University Press, 2005.

[4] R. K. Bahl, " Computation of Exact Premium for Cloud Insurance under CARA" Goya Journal, vol. 18(3) pp. 441-454, 2025.

[5] A. Mazzoccoli and M. Naldi. The expected utility insurance premium principle with fourth-order statistics: Does it make a difference? Algorithms vol. 13 (5), 2020.

[6] M. Naldi and A. Mazzoccoli. Computation of the insurance premium for cloud services based on fourth-order statistics. Int. J. Simul. Syst. Sci. Technol. vol. 19 (5), 21.1-22.6, 2018.

[7] L. Mastroeni and M. Naldi. Insurance Pricing and Refund Sustainability for Cloud Outages. Lect. Notes Comput. Sci. (including Subser. Lect. Notes Artif. Intell. Lect. Notes Bioinformatics), 10537 LNCS (January), 3–17, 2017.

[8] L. Mastroeni, A. Mazzoccoli and M. Naldi. Service level agreement violations in cloud storage: Insurance and compensation sustainability. Future Internet, vol. 11 (7), 2019.

[9] U. Franke, M. Buschle, M. Österlind. An experiment in SLA decision-making. Lect. Notes Comput. Sci. (including Subser. Lect. Notes Artif. Intell. Lect. Notes Bioinformatics), 8193 LNCS, 256–267, 2013

[10] M. Naldi. The availability of cloud-based services: Is it living up to its promise? 2013 9th Int. Conf. Des. Reliab. Commun. Networks, DRCN 2013 June, 282–289, 2013.

[11] M. Naldi. ICMP-based Third-Party Estimation of Cloud Availability. *Int. J. Adv. Telecommun. Electrotech. Signals Syst.* vol. 6 (1), 11, 2017.

[12] P. Lima Ramos. Premium calculation in insurance activity. Journal of Statistics and Management Systems, vol. 20(1), 39-65, 2017.

[13] M. Denuit, The exponential premium calculation principle revisited. ASTIN Bulletin: The Journal of the IAA, vol. 29(2), 215-226, 1999.

[14] J. Haezendonck and M. Goovaerts, A new premium calculation principle based on Orlicz norms. Insurance: Mathematics and Economics, vol. 1(1), 41-53, 1982.

[15] F. Esscher, On the Probability Function in the Collective Theory of Risk. Skandinavisk Aktuarietidskrift, vol. 15: 175-95, 1932.

[16] S. Wang, Premium calculation by transforming the layer premium density. ASTIN Bulletin vol. 26, 71-92, 1996.

[17] D. Bernoulli, Exposition of a New Theory on the Measurement of Risk. Econometrica, vol. 22(1), 23–36 (Translation of Bernoulli D 1738 Specimen theoriae novae de mensura sortis; Papers Imp. Acad.Sci. St. Petersburg 5 175–192), 1953.

[18] P. S. Kapoor, Harman and K. Jain, Premium evaluation for different loss distributions using utility theory. Discussiones Mathematicae Probability and Statistics, vol. 31(1-2), 41-58, 2011.

[19] K.J. Arrow, The Theory of Risk Aversion. In: Helsinki, Y.J.S., Ed., Aspects of the Theory of Risk Bearing, Reprinted in Essays in the Theory of Risk Bearing, Markham Publ. Co., Chicago, 90-109, 1971.

[20] J.W. Pratt, Risk Aversion in the Small and in the Large. Econometrica, Vol. 32, 122-136, 1964.

[21] F. Martinelli, A. Orlando, G. Uuganbayar and A. Yautsiukhin, Preventing the drop in security investments for non-competitive cyber-insurance market. In Proceedings of the International Conference on Risks and Security of Internet and Systems, Dinard, France, 19–21 September 2017; pp. 159–174, 2017.

[22] A. Marotta, F. Martinelli, S. Nanni, A. Orlando and A.Yautsiukhin Cyber-insurance survey. Computer Science Revolution, Vol. 24, 35–61, 2017.

[23] R. Böhme and G. Schwartz, Modeling Cyber-Insurance: Towards a Unifying Framework, 2010. Available online. https://www.semanticscholar.org/paper/Modeling-Cyber-Insurance%3A-Towards-a-Unifying-B%C3%B6hme-Schwartz/776884d844f406fbfd82ad67b85ebaabd2b0e360

[24] H. Bühlmann, An economic premium principle. ASTIN Bulletin: The Journal of the IAA, Vol. 11(1), 52-60, 1980.