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# Determination of Multiplication Factor for Gabor Wavelet Transform: An Application to Denoising of Digital Signals

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## Abstract

*The main objective of the paper is to find the multiplication factor 'k' for all frequencies ranging from 1 Hz to 1 GHz and subsequently show it's application to denoising of digital signals of ultra high frequency range. The multiplication factor 'k' is used to obtain the true contribution of any frequency component in the original signal when Gabor wavelet is used to find the time domain contribution of that signal. In this paper, the author considered an UHF signal generated from an experimental set up used to simulate partial discharge signals in high voltage power transformer and added a 20 MHz sinusoidal component with very low magnitude level. The sine component has been treated as noise and Gabor wavelet transform is applied to extract the same component out and reconstructed the main signal. Finally, a comparison is made between original and reconstructed signal to find the efficacy of Gabor wavelet transform.*

Key Words: - Gabor wavelet transform, FFT, partial discharges, UHF signals

## 1. Introduction

The Fourier analysis decomposes a signal into constituent sinusoids of different frequencies (fundamental and harmonics). The FFT of a measured partial discharge signal recorded through oscilloscope gives the presence of all frequencies in the signal. Such a transformation of the signal from time domain to frequency domain, though useful, causes it's time information to be lost. This is undesirable, particularly when the signal is non-stationary and its transitory characteristics are important. To overcome the above problem, wavelet analysis has been found to be helpful [1-5].

It is observed that Gabor wavelet is most localized in time – frequency domain compared with other wavelets. It is easy to discuss the frequency components of the signal because Gabor wavelet is based on exponential function like the Fourier transform. Gabor wavelet can be used for de-noising of recorded signal during experiment. The signal can be reconstructed using Gabor wavelet transform and compared with original signal. Since, there is a provision to remove any frequency, the reconstruction is possible with selected frequencies[6]. This method can be adopted to remove the noise from any given signal corrupted with random noise, as this is a very difficult process otherwise.

## 2 Present Work

The present work deals with model formulation, partial discharge simulation, high frequency measurement, and signal processing using Wavelet Transform. In order to carry out the work, the time domain signal of partial discharge is recorded on a digital oscilloscope. The experimental set up used and generation of partial discharge signals is reported in Ramprasad et al. [7,8]. A typical signal is considered for Fourier Transform analysis. The time domain signals are analyzed for their contribution in the main signal using Gabor Wavelet. The actual contributions are calculated by multiplying with factor 'k'. When a Gabor wavelet is used to extract any frequency component, initially it will not give the true contribution. Instead the signal is attenuated due to normalization factor, which is used to maintain the same energy for the wavelets dilated and translated. The reconstructed signal is compared with the original signal after removal of noise present to evaluate the performance of Gabor Wavelet in obtaining component signals. The work also aims at reconstructing the signal to determine the effectiveness of Wavelet Transform in breaking in terms of accurate time domain signal [9,10].

## 3 GABOR WAVELET MODEL

In order to analyze the signal using wavelet the following mother wavelet integral and wavelet equation has been chosen [11].

$$W(b,a) = 1/\sqrt{a} \int_{-\infty}^{\infty} I(t)\, \Psi((t-b)/a)\, dt \qquad (1)$$

$$a = 1/2\pi f \qquad (2)$$

where, $a$ = scale parameter
$b$ = translation parameter
$f$ = frequency

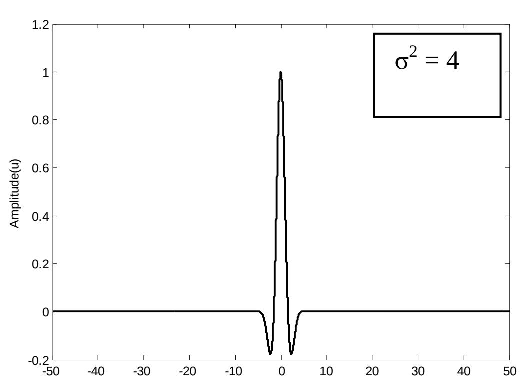
In the present study, the Gabor wavelet equation has been used and is given by the following equation

$$\Psi(t) = \exp(-t^2/\sigma^2) * \cos(t) \qquad (3)$$

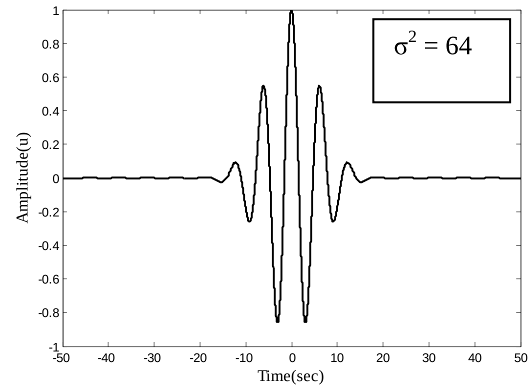
where, $\sigma^2$ is a constant and controls the band of frequencies to be identified. The necessary and sufficient condition for a function to become wavelet is discussed in detail by Rao et al. [12].

In order to evaluate the time varying current waveform at a particular frequency, it is essential to determine the multiplication factor (k). This factor converts the wavelet transform of the time waveform at a particular scale parameter 'a' into the transient waveform for the corresponding frequency. The true contribution of any signal component in original signals can be obtained by multiplying with this factor [13].

Figure 1 shows the Gabor mother wavelet transform for different values of $\sigma^2$ evaluated at 4 and 64.




(a)



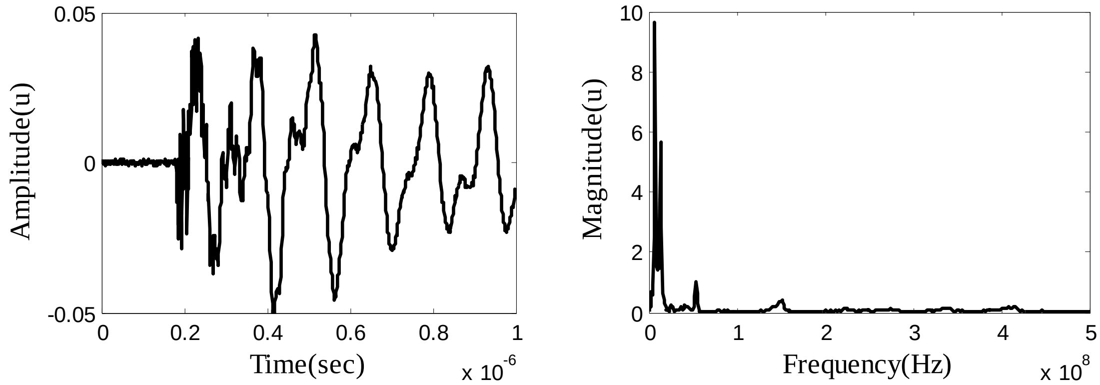

(b)

**Figure 1: Gabor Mother Wavelet for (a) $\sigma^2 = 4$ & (b) $\sigma^2 = 64$**

## 4 DE-NOISING OF SIGNALS USING GABOR WAVELET

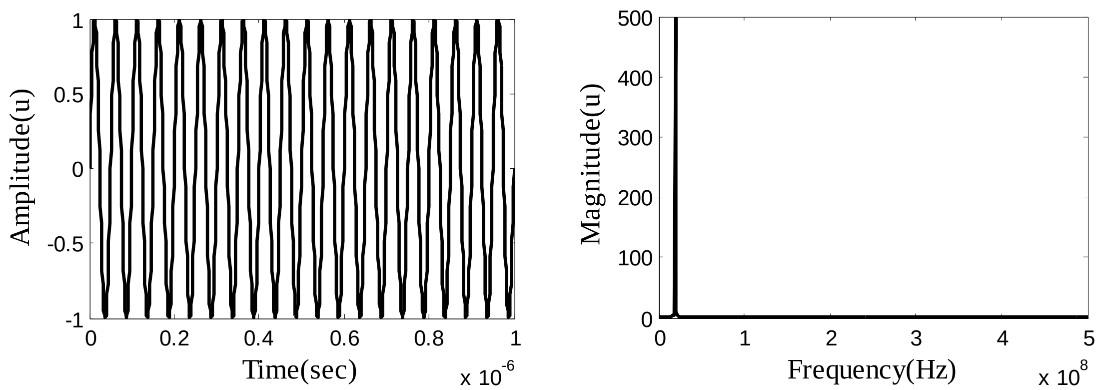
For the sake of explanation that how any noise component is removed from the given signal, one of the signal generated due to float type partial discharge during the experimental simulation process is taken into consideration and which is shown in Figure 2(a) and it's corresponding FFT is shown in Figure 2(b) [14]. A sinusoidal signal of 20 MHz is generated with small amplitude of 0.005 p.u., which is shown in Figure 3(a). Figure 3(b) is it's FFT signal. The author intentionally added this 20 MHz signal to float signal as it is treated as some noise with small amplitude. Such a new float type discharge signal corrupted with noise signal is shown in Figure 4(a). The 20 MHz peak is clearly seen in the corresponding FFT shown in Figure 4(b), which is not present in FFT of original float signal shown in Figure 2(b).

Using the equations from 1 to 3, Gabor wavelet transform is applied to pick out the 20 MHz component. The selection of $\sigma^2$ value is based on the accuracy with which the output waveform (wavelet transform of the input waveform) at a particular scale parameter matches with the input/original waveform. The $\sigma^2$ value has been increased slowly from 2 to 1024 in steps of 2 powers and each time the output waveform is observed with the help of it's frequency response calculated using FFT algorithm. Figure 5 shows the 20 MHz component signals at different values of $\sigma^2$. The corresponding frequency response signals are also shown. Only few component signals for $\sigma^2$ at 2, 64 1024 are shown due to space constraint.



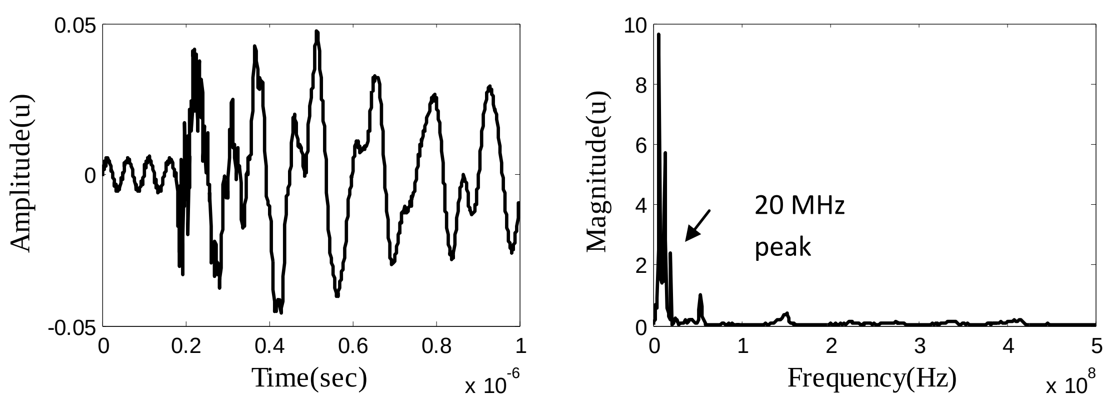

(a) (b)

**Figure 2: Float type PD signal and it's corresponding frequency response**




(a) (b)

**Figure 3: sine wave with frequency 20 MHz and it's corresponding frequency response**




(a) (b)

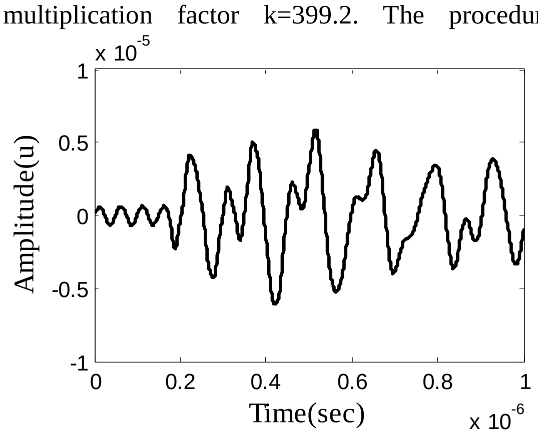
**Figure 4: Float signal mixed with sine wave shown in Fig. 2(a) and it's corresponding frequency response**

For 20 MHz and $\sigma^2$=2 the time amplitude is shown in Figure 5(a). This signal although sinusoidal has certain amplification, which varies with time and is not uniform. The FFT of the signal is calculated and is shown in Figure 5(b). It is clearly seen that there is no peak corresponding to 20 MHz component. Instead, it includes a few other frequencies like 4 and 13 MHz. So, the value of $\sigma^2$ is not sufficient to select the required band of frequencies, and therefore it has been increased from 2 to 1024 slowly and each time the FFT is being observed to have only single peak in the output frequency response. It is arrived first time only at $\sigma^2$ =256. But, the shape of the output waveform is not so uniform and hence the value of $\sigma^2$ is further increased and the corresponding waveforms are observed both in time and frequency domain for better choice. At $\sigma^2$ = 768 or 1024 there has been a single and clear 20 MHz peak and the sine wave also uniform on time axis, except there is small difference in the amplitude of few cycles in the begining and ending of the signal. Finally, $\sigma^2$ at 1024 was selected to obtain the contribution of 20 MHz compnonent in time domain in order to subtract it from the corrupted float signal.

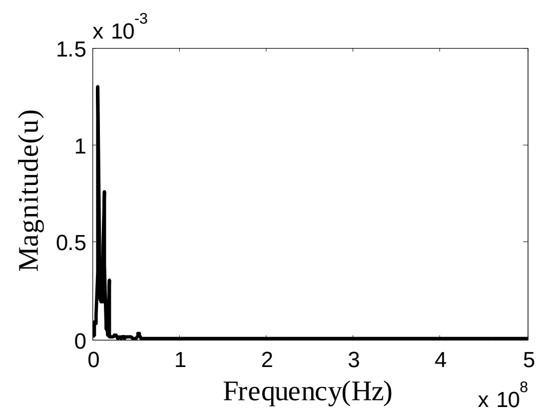
Before, it is subtracted, it's true contribution in the main signal is obtained by multiplying with multiplication factor k=399.2. The procedure to calculate multiplication factor 'k' is discussed in the next section. The reconstucted signal and it's FFT are shown in Figure 6(a) and 6(b).

Figure 2 and 6 shows the comparison of original and reconstructed waveforms. Both original and reconstructed waveforms are more or less identical except that there is a slight reduction in the peak amplitude of reconstructed waveform.

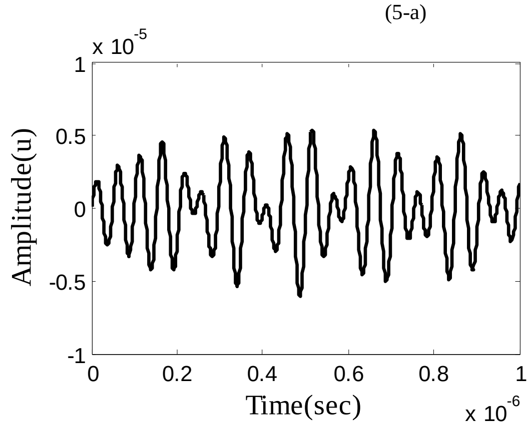
It is confirmed that the amplitude of the component signal with frequency 20 MHz taken out with Gabor wavelet model is closely matching with the original signal. The validity of the calculation is further verified by comparing the resultant waveform i.e., reconstructed waveform with the original waveform.



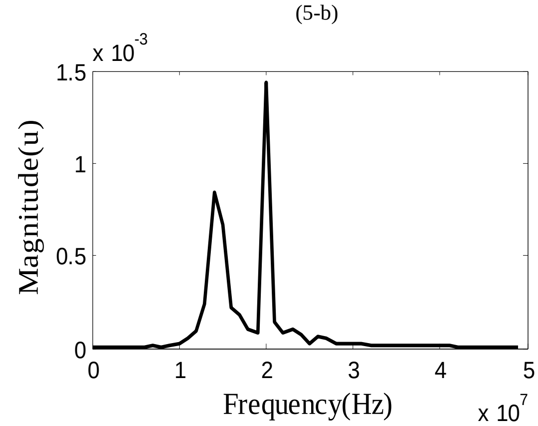

(5-a)



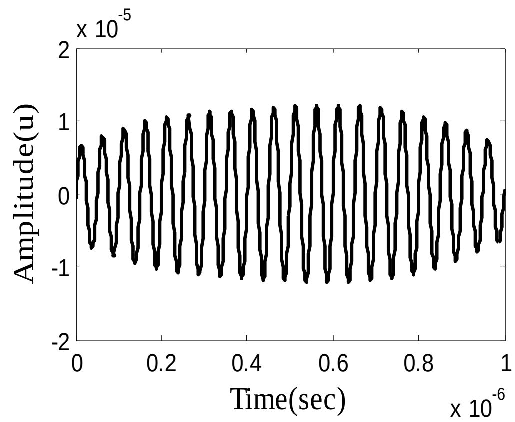

(5-b)



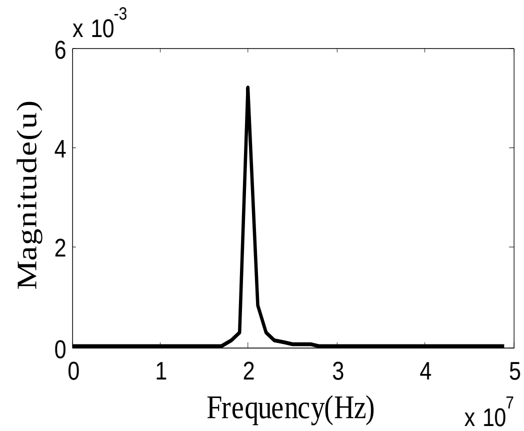

(5-c)




(5-d)

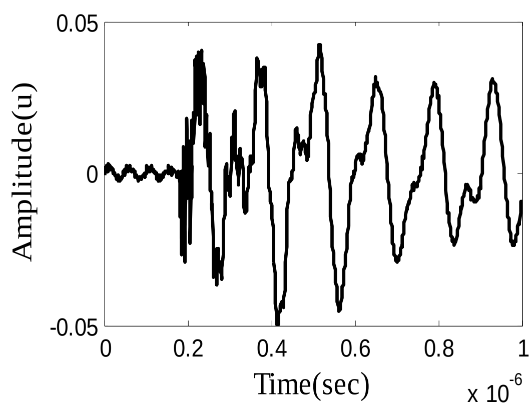



(5-e)

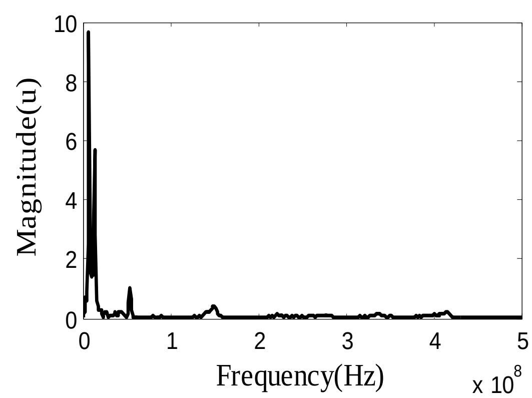



(5-f)

**Figure 5: 20 MHz component signal**
**(a) $\sigma^2$=2 (c) $\sigma^2$=64 (e) $\sigma^2$=1024**




(a)




(b)

**Figure 6: Reconstructed signal and it's frequency response**

## 5 VALIDITY OF THE MODEL

The selection of $\sigma^2$ value is based on the accuracy with which the output signal (Wavelet transform of the input signal) at a particular scale parameter matches with the corresponding transient signal in the input waveform. To confirm the validity of $\sigma^2$ value and the multiplication factors at different frequencies, a sinusoidal signal with amplitude of 1 p.u for various frequencies ranging from 1 Hz to 1 GHz are considered.

### 5.1 CALCULATION OF MULTIPLICATION FACTOR 'k'

In order to calculate the multiplication factor 'k', the signal shown in Figure 7, an arbitrary sinusoidal signal of 20 MHz and amplitude 1 p.u. is considered.

Figure 8 shows the wavelet transform of the arbitrary signal (Frequency: 20 MHz, amplitude: 1p.u) considered in Figure 7 using the above equations from (1) to (3) at different $\sigma^2$ values. Only few of them are shown here due to space constraint. These waveforms suggest that the number of time cycles required for the wavelet transform to match the original signal increase with the increase of '$\sigma^2$' value. Thus, for reproducing as it is at a particular frequency, it is necessary to keep

$\sigma^2$ value less than or equal to 4. But, the minimum value of $\sigma^2$ has been identified as 53.5 to satisfy the condition that the average value of the mother wavelet function at any frequency is zero. Nevertheless, it is found that there is a possibility of evaluating a signal comprising a band of frequencies with reasonable accuracy for the value of $\sigma^2$ less than the cut off value. The Figures shown in 8(b), 8(d) and 8(f) give the actual contributions of these component signals in the original signal. These waveforms are obtained by multiplying the wavelets with a constant value called multiplication factor ‘k’. The multiplication factor ‘k’ is calculated in such a way that when a wavelet is multiplied with such a multiplication factor ‘k’ then the amplitude raises to it’s original value (in this case it is 1 p.u., as this is the initial amplitude of sine wave).

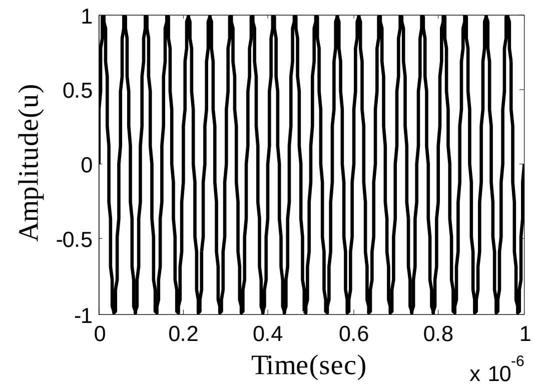
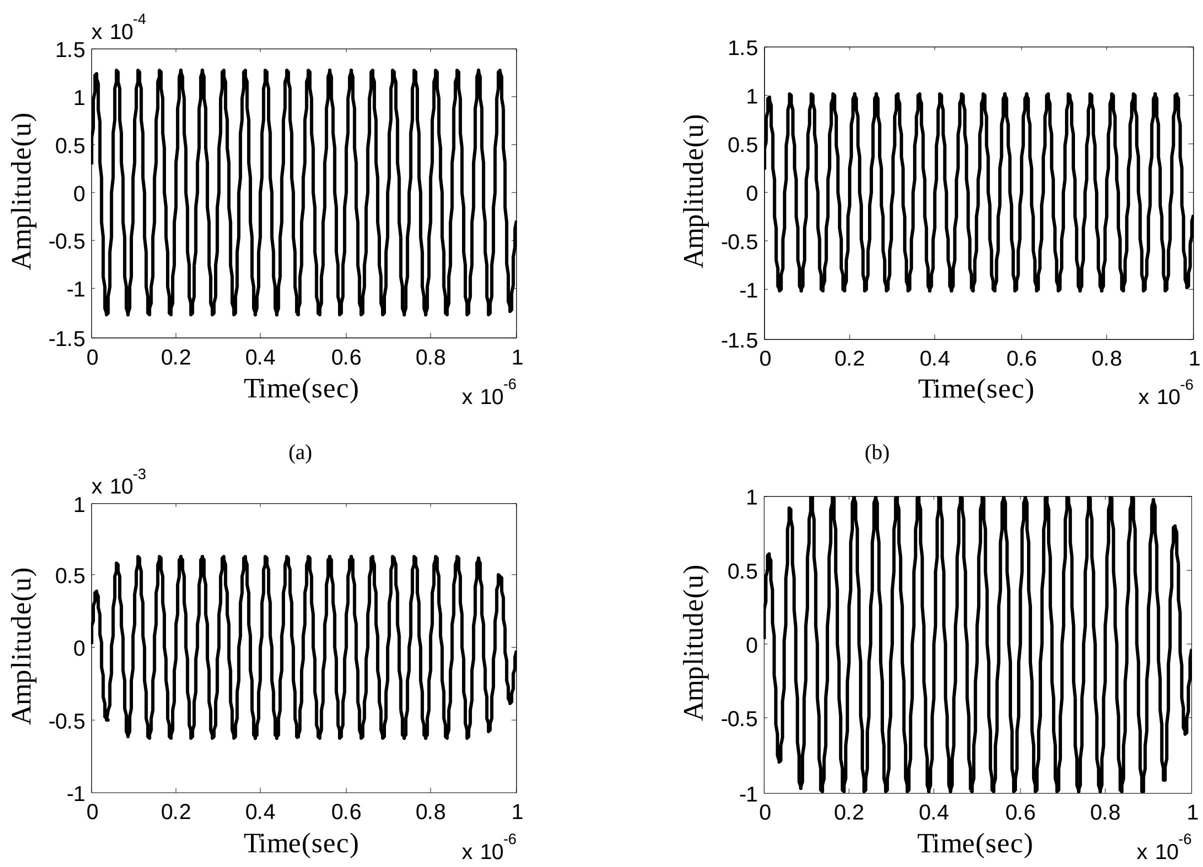


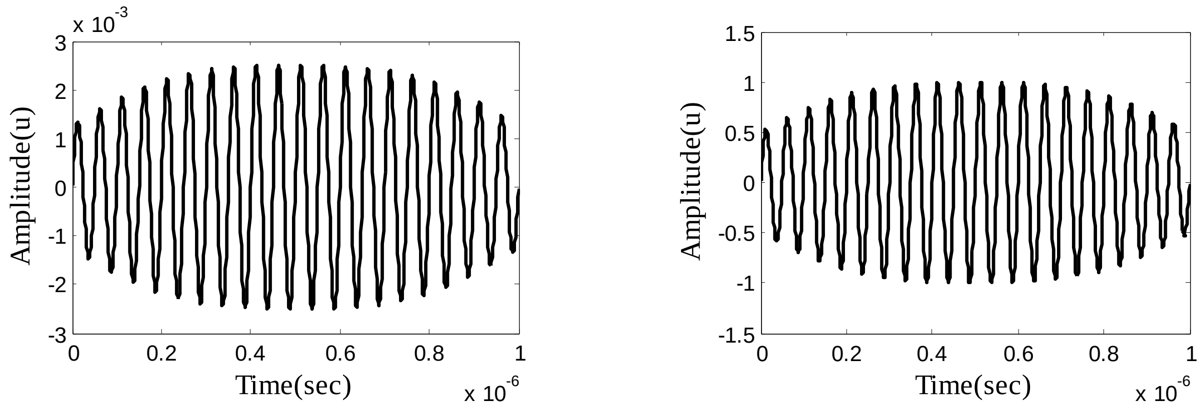

**Figure 7: sine wave with frequency 20 MHz and it’s FFT**




(a) (b)

(c) (d)

**Figure 8: 20 MHz component signal and their actual contribution in main signal**
**(a) $\sigma^2$=2 (c) $\sigma^2$=64**
**(b) & (d) actual contributions of LH side signals**




(e) (f)

**Figure 8: 20 MHz component signal and their actual contribution in main signal**
**(e) $\sigma^2$=1024**
**f) actual contributions of LH side signal**

Table 1 shows the variation of 'k' with $\sigma^2$ at a particular frequency 20 MHz. Similarly, 'k' values have been listed in Table 2 for different frequencies at a particular value of $\sigma^2$=256. From these results it is evident that 'k' value decreases with increase of $\sigma^2$ and the value of $k^2/f$ is constant for a particular value of $\sigma^2$. Table 3 can be used to find the multiplications factor for any given frequency and $\sigma^2$ value. Figure 9 gives how $k^2/f$ value for given $\sigma^2$ changing from 0 to 1024. For a given σ2, $k^2/f$ is constant, so for a given f, it is possible to find the multiplication factor.

**Table 1: Variation of k w.r.t. $\sigma^2$ for frequency 20 MHz**

| Frequency (MHz) | $\sigma^2$ | k |
|---|---|---|
| 20 | 2 | 7936.50 |
| | 4 | 6209.55 |
| | 8 | 4475.33 |
| | 16 | 3162.27 |
| | 32 | 2237.66 |
| | 64 | 1580.86 |
| | 128 | 1117.83 |
| | 256 | 792.-63 |
| | 512 | 559.10 |
| | 768 | 457.16 |
| | 1024 | 399.20 |

**Table 2: Variation of k w.r.t. Frequency for $\sigma^2$=256**

| $\sigma^2$ | Frequency (MHz) | k | $k^2/f$ |
|---|---|---|---|
| 256 | 5 | 395.60 | 0.0313 |
| | 20 | 792.06 | 0.0313 |
| | 50 | 1250.0 | 0.0312 |
| | 100 | 1767.65 | 0.0312 |
| | 200 | 2502.63 | 0.0313 |
| | 300 | 3067.74 | 0.0313 |
| | 400 | 3535.31 | 0.0312 |

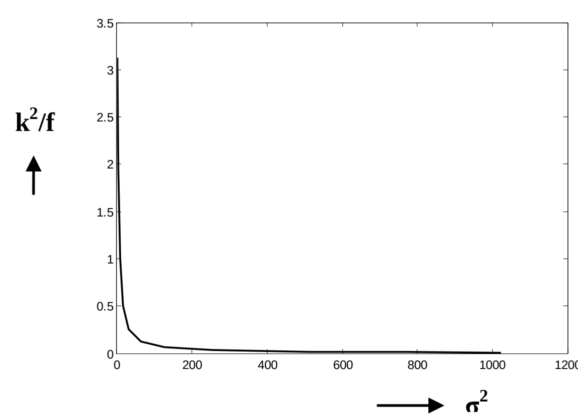



**Figure 9: graph $\sigma^2$ Vs $k^2/f$**

**Table 3: Variation of k w.r.to both frequency and $\sigma^2$**

| | $k^2/f$ | $\sigma^2$ value | | | | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | | $\sigma^2$=2 | $\sigma^2$=4 | $\sigma^2$=8 | $\sigma^2$=16 | $\sigma^2$=32 | $\sigma^2$=64 | $\sigma^2$=128 | $\sigma^2$=256 | $\sigma^2$=512 | $\sigma^2$=768 | $\sigma^2$=1024 |
| FREQUENCY VALUES | **5 Hz** | 3.252 | 1.929 | 1.001 | 0.499 | 0.249 | 0.127 | 0.069 | 0.032 | 0.015 | 0.01 | 0.009 |
| | **20 Hz** | 3.142 | 1.932 | 1.005 | 0.503 | 0.251 | 0.125 | 0.062 | 0.031 | 0.015 | 0.01 | 0.007 |
| | **50 Hz** | 3.109 | 1.961 | 1.005 | 0.501 | 0.249 | 0.125 | 0.062 | 0.031 | 0.015 | 0.01 | 0.007 |
| | **100 Hz** | 3.103 | 1.933 | 1.001 | 0.503 | 0.251 | 0.125 | 0.062 | 0.031 | 0.015 | 0.01 | 0.007 |
| | **5KHz** | 3.111 | 1.961 | 1.005 | 0.501 | 0.251 | 0.126 | 0.069 | 0.032 | 0.016 | 0.01 | 0.009 |
| | **20 KHz** | 3.125 | 1.937 | 1.003 | 0.503 | 0.251 | 0.126 | 0.063 | 0.031 | 0.015 | 0.01 | 0.008 |
| | **50 KHZ** | 3.124 | 1.928 | 1.001 | 0.501 | 0.249 | 0.125 | 0.062 | 0.031 | 0.015 | 0.01 | 0.007 |
| | **100 KHz** | 3.131 | 1.931 | 1.005 | 0.501 | 0.251 | 0.125 | 0.062 | 0.031 | 0.015 | 0.01 | 0.007 |
| | **5 MHz** | 3.124 | 1.928 | 1.004 | 0.499 | 0.251 | 0.126 | 0.069 | 0.031 | 0.016 | 0.01 | 0.008 |
| | **20 MHz** | 3.149 | 1.927 | 1.001 | 0.499 | 0.251 | 0.124 | 0.062 | 0.031 | 0.015 | 0.01 | 0.007 |
| | **50 MHz** | 3.109 | 1.929 | 1.005 | 0.499 | 0.249 | 0.125 | 0.062 | 0.031 | 0.015 | 0.01 | 0.007 |
| | **100MHz** | 3.111 | 1.933 | 1.001 | 0.499 | 0.250 | 0.124 | 0.062 | 0.031 | 0.015 | 0.01 | 0.007 |
| | **200MHz** | 3.011 | 1.927 | 1.001 | 0.499 | 0.249 | 0.125 | 0.062 | 0.031 | 0.015 | 0.01 | 0.007 |
| | **Finally Approximated to** | 3.123 | 1.935 | 1.002 | 0.500 | 0.250 | 0.125 | 0.062 | 0.031 | 0.015 | 0.010 | 0.007 |

## 5 CONCLUSIONS

The work reported in this paper deals with the determination of multiplication factor ‘k’ for the Gabor wavelet transform for all frequencies ranging from 1 Hz to 1 GHz and subsequently it’s application to denoising of digital signals. The partial discharges generated using the experimental simulation model are used to show the effectiveness of Gabor wavelet in eliminating any unwanted components present. The partial discharge, an UHF signal is initially subjected to FFT analysis to determine the various frequency components and the Gabor wavelet is used to find the time domain contribution of any selected frequency component. The selected frequencies are removed and signal is reconstructed with the remaining frequencies. The original and reconstructed signals are compared to prove the efficacy of Gabor wavelet in removing out any selected frequency. Finally, it is shown that the multiplication factor ‘k’ can be obtained for any given frequency and $\sigma^2$ as it proved that $k^2/f$ is constant for a given $\sigma^2$.

## 6 ACKNOWLEDGEMENTS

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