

# Determination of Effective Permittivity of Multilayers using Waveguide

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**Abstract**—In this paper we implement a technique of calculating of the effective permittivity of multi layered substances using a waveguide. This is done by increasing the number of layers while keeping the total volume of composition constant is done to obtain the effective permittivity. This analysis is done using the method of wave matrices in order to determine the S-parameters. The calculation of permittivity is then done by creating theoretical values for those S-parameters in terms of the dielectric constants.

**Keywords**—waveguide, layered medium, permittivity

## I. INTRODUCTION

New materials are developed as to meet the expectations of higher end technologies. New materials developed can be composite or homogeneous. Many techniques have been developed for calculating permittivity of homogeneous materials [1]. Of late some attention has focused on multilayers that are currently used for many practical applications that include microwave integrated circuits, monolithic microwave integrated circuits [2].

In this paper dielectric constant of composite layers is studied. The calculation of permittivity is done by creating theoretical values for those S-parameters in terms of the dielectric constants, which are solved for simultaneously in conjunction with the wave matrix S-parameters using a 2 dimensional Newton's root searching algorithm [3]. The wave matrix method is used for the analysis of the layered medium in this paper [4].

## II. FUNDAMENTAL THEORY

The wave matrix method is an exceptionally useful and simple tool for the analysis of layered dielectrics. For the case of an incident TEM wave from the left, each layer maybe described by its material properties and electrical thickness. When the wave arrives at each interface, a portion is transmitted and another portion is reflected.

In the present work the layered material is aligned inside a X-band waveguide. Assuming only the dominant  $TE_{10}$  mode propagating in waveguide, the S parameters can be expressed in terms of each layer's thickness ( $d_n$ ) and unknown permittivity ( $\epsilon\epsilon_n$ ) using ABCD parameters as [4].

$$\begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} = \begin{bmatrix} \cosh(\gamma_n * d_n) & Z_n * \sinh(\gamma_n * d_n) \\ \frac{\sinh(\gamma_n * d_n)}{Z_n} & \cosh(\gamma_n * d_n) \end{bmatrix}$$

For n-layers in the material if  $\epsilon_n$  and  $Z_n$  are propagation constant and wave impedance of  $n^{\text{th}}$  layer respectively and the combined T-matrix is given by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{i=1}^n \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}$$

$$Z_n = \frac{j\omega\mu}{\gamma_n} \text{ and } n = \sqrt{\left(\frac{\pi}{a}\right)^2 - (\omega^2\epsilon\epsilon_n)}$$

The ABCD parameter can be converted to s parameters by following equations:

$$S_{11} = \frac{(A + \frac{B}{Z_0} - CZ_0 - D)}{X}$$

$$S_{12} = \frac{2 * (AD - CB)}{X}$$

$$S_{21} = \frac{2}{X}$$

$$S_{22} = \frac{(-A + \frac{B}{Z_0} - CZ_0 + D)}{X}$$

$$X = A + \frac{B}{Z_0} + CZ_0 + D$$

The experimental analysis of the same is done and compared with the predicted mathematical result.. The results obtained are discussed in detail in the next section.

## III. ANALYSIS OF MULILAYERED MATERIAL

The effective permittivity of a composite mixture comprised of layered dielectrics is investigated. The composition consist of two different materials with different relative permittivity, also the first layer and last layer is same. So the system is symmetric. The total volume of material is held constant and the number of layers is increased in order to see the effective permittivity.

For experimental study layers of same thickness were stacked together and kept inside a rectangular waveguide with a matched termination which acted as the sample holder. Figure.1. shows the block diagram of the experimental set up. A solid sample or length  $l\epsilon$  is loaded in rectangular waveguide

against short circuit that touches it well.  $D$  &  $DR$  - the positions or first voltage minima of the standing wave pattern when waveguide is unloaded & loaded with the dielectric is noted. The respective distance from the short circuit will be  $(l + l_\epsilon)$  &  $(lR + l_\epsilon)$ . The impedance are equal so  $Z_0$  and  $Z_\epsilon$  are respectively the characteristic impedance of empty & dielectric filled waveguide  $\beta$  and  $\beta_\epsilon$  are respective propagation constant. Assuming dominant mode was propagating through the rectangular waveguide, the dielectric constant  $\epsilon_r$  is given by:

$$\frac{\tan \beta (D_R - D + l_\epsilon)}{\beta l_\epsilon} = \frac{\tan \beta_\epsilon l_\epsilon}{\beta_\epsilon l_\epsilon}$$

$$\epsilon_r = \frac{\left(\frac{a}{\pi}\right)^2 \left(\frac{\beta_\epsilon l_\epsilon}{l_\epsilon}\right)^2 + 1}{\left(\frac{2a}{\lambda_g}\right)^2 + 1}$$

where  $f$  is the volume fraction of material A, and  $1-f$  is the volume fraction of material B,  $\epsilon_{eff}$ ,  $\epsilon_A$ , and  $\epsilon_B$  are the effective permittivity, and permittivity of layers A and B, respectively. For convenience the thickness of both the materials are taken as same in the present study.

The measured results for the effective permittivity of the layered samples are found to be in good agreement. The experimental and calculated results are presented in Table.1.

Permittivity of layer A	Permittivity of layer B	Volume fraction of A and B	Effective permittivity	
			measured	calculated
1	2.1	1/3	1.4	1.3667
1	4.4		2	2.133
2.1	4.4		2.9	2.8667
4.4	2.1		3.7	3.633

Table.1.

#### IV. CONCLUSION

An effective way to find the dielectric constant of layered medium is presented. The experimental results obtained are in good agreement with the predicted results. The effect of adding more layers into the material and the volume fraction not being the same is a challenge being faced.

#### REFERENCES

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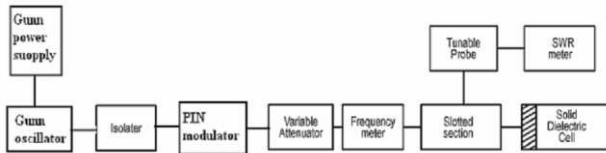


Figure.1. Experimental setup

The layered structure is as shown in Figure.2

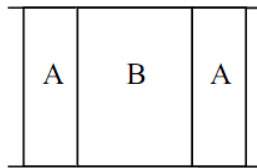


Figure. 2.

The measured permittivity is that of the layered material. The stacked three layered materials were studied. The effective permittivity of the material is calculated using the formula

$$\epsilon_{eff} = f\epsilon_A + (1-f)\epsilon_B$$