Determination of Control Input/Output Signals for Damping Low Frequency Oscillations

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Abstract- This paper presents the two approaches of control feedback signal selection for wide area controller to damp the inter-area oscillations in two-area system namely geometric approach and residue approach. With the help of simulation studies, the best approach is selected. The result obtained show that from both the measures we get the same control signal but the magnitude is relatively larger in case of residue approach.

Keywords—Control loop selection, Geometric measures, inter-area oscillations, power system stabilizer, Residue measures, wide-area control.

I. INTRODUCTION

With the increase in use of new technologies, complex type of instability is occurring in power system. In particular, oscillatory instability has drawn concern. With the bulk power transfer, such system exhibits low frequency inter-area modes of oscillations. Their frequency range is 0.1-0.7 Hz.

Today, the damping of inter-area oscillations can be achieved using Power System Stabilizer(PSS), the static VAR compensator (SVC) modulation and FACTS devices with local control modes[3],[5]. With the increase in interconnections among the Power system and coupling between the machines, with the occurrence of fault in one area will tend to affect the larger area. Therefore, a multi controller must be designed that can minimize the interactions between the controllers and at the same time damp the low frequency oscillations. We have considered a two area system with two generators in each area. Modal analysis of the system is done to obtain the modes responsible for oscillations [1]. Two control loop selection measure is applied to the system in order to select the most effective feedback signal for damping inter-area oscillations. As the conventional PSS is only limited to damp the local area modes effectively, therefore, we require a wide area controller which requires a global control signal that can damp out the oscillations increases the reliability of the system [5]. This paper concentrates on two approaches for control signal selection which can be used to select the control signal [5]. However, the objective for control signal selection is to select pair of signals that can provide maximum controllability/observability or to minimize the interaction between the controllers [4]. Based on the results, the better method will be selected that is robust and efficient and provide better results for damping the oscillations.

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The paper is organized as follows: Section II is demonstrates the system modelling. Then the signal selection methodologies are presented in Section III. Section IV depicts the studied case with the results given in Section V, the conclusion is provided in Section VI.

II. SYSTEM MODELLING

Power system is a non-linear system which is described by ordinary non-linear differential of the following from:

\[ x = f(\chi, u) \quad (1) \]

\[ y = g(\chi, u) \quad (2) \]

where \( x \in R^{n \times n} \), \( u \in R^{p \times n} \) and \( y \in R^{p \times n} \) are the state, input and output vectors, respectively. \( n \) is the dimension of the system, \( m \) is the number of inputs, and \( p \) is the number of outputs.

For measurement and control signal selection, a linearized model is used which obtained by the modal analysis of the network. As the perturbations are small, a non-linear function \( f(\chi, u) \) can be expressed in terms of Taylor’s series. To obtain the small signal model around an operating point, several software such as MASS/PEALS, POSSIM/MANSTAB, and SDYN can be used. In this paper linearized is obtained using the MATLAB/simulink. The linearized form of the eq.(1) and (2) are given by:

\[ \Delta \dot{x} = \Delta A x + B \Delta u \]

\[ \Delta y = C \Delta x + D \Delta u \quad (3) \]

Where

\( \Delta x \) is the state vector of dimension \( n \)

\( \Delta u \) is the input vector of dimension \( r \)

\( \Delta y \) the output vector of dimension \( m \)

\( A \) is the state matrix of dimension \( n \times n \)

\( B \) is the input matrix of dimension \( n \times r \)

\( C \) is the output matrix of dimension \( m \times n \)

\( D \) is the feed-forward matrix of dimension \( m \times r \)

The electromechanical modes are calculated with eigenvalues of \( A \{ \lambda_i = \sigma_i \pm j \omega_i \} \). Frequency (Hz) and damping ratio of the \( i^{th} \) mode is calculated using \( f_i = \omega_i/2\pi \) and \( \zeta_i = -\sigma_i/\sqrt{\sigma_i^2 + \omega_i^2} \).

III. SIGNAL SELECTION METHODOLOGIES

In this section, two unique techniques are utilized to select the feedback control signal for wide area PSS in order to damp inter-area oscillations. The same analysis is done on a different system in [5]. We’ll discuss them one by one as follows:
The signal selection consists of selecting the pairs of control and measurement signal which can maximize the observability and controllability of inter-area modes of oscillations using geometric approach. Its specifically allow the signal selection with good observability and controllability while to minimize the interaction between the different control loops, the residue approach is considered.

Let us consider a linear model of the network given by (3). An eigenvalue for matrix A is given by \( \lambda_i \) (i=1,2,....,n) and corresponding matrix of right and left eigenvector are given by: 
\[ L=[l_1, l_2, ..., l_n] \] and \[ R=[r_1, r_2, ..., r_n] \] respectively. The eigenvector \( l_i \) and \( r_i \) corresponding to eigenvalue \( \lambda_i \) are orthogonal and normalized.

### A. Geometric approach

The geometric measure for controllability \( m_{cl} \) and observability \( m_{ol} \) associated with mode ‘i’ is given by:
\[
m_{cl}(k) = \cos(\theta(r_i, b_k)) = \frac{|b_k^T r_i|}{\|b_k\| \|r_i\|} \tag{4}
\]
\[
m_{ol}(q) = \cos(\theta(c_q, l_i)) = \frac{|c_q^T l_i|}{\|c_q\| \|l_i\|} \tag{5}
\]

In (4) and (5), \( b_k \) is the \( k^{th} \) column of B matrix and \( c_q \) is the \( q^{th} \) row of the C matrix, \( \theta(r_i, b_k) \) is the acute angle between the input vector \( b_k \) and the left eigenvector \( r_i \), \( \theta(c_q, l_i) \) is the acute angle between the output vector \( c_q \) and the right eigenvector \( l_i \) are, respectively, \( |z| \) and \( \|z\| \) are the modulus and the Euclidean norm of \( z \) respectively. Using (4) and (5), the joint controllability/observability measure is given by:
\[
m_{col}(k,q) = m_{cl}(k) \cdot m_{ol}(q) \tag{6}
\]
A large value of \( m_{col} \) indicates that control loop is effective in controlling \( i^{th} \) mode.

### B. Residue approach

The residue indicate the movement of the eigenvalues (poles) corresponding to small gain whereas for the large gain, it is determined by the location of the zeroes. This method uses a combination of A, B, C matrix and left and right eigenvector as well. The residue of the signal can be calculated using eq. (9). The transfer function associated with system (3) is as follows:
\[
R(S) = \frac{Y(S)}{U(S)} = C(SI - A)^{-1}B \tag{7}
\]

The above mathematical statement can be rewritten as utilizing the orthogonality between the left and right eigenvector as
\[
G(S) = \sum_{i=1}^{n} \frac{R_i}{S - \lambda_i} \tag{8}
\]
Where, \( R_i \) is the residue matrix associated with mode ‘i’ which is given by:
\[
R_i = C l_i r_i^H B \tag{9}
\]
Where, \( R_i \) is Residue matrix for eigenvalue \( i \);
\( C \) is Output matrix of the system;
\( B \) is Input matrix of the system;
\( l_i \) is Right eigenvector corresponding to eigenvalue \( i \);
\( r_i^H \) complex conjugate of the left eigenvector corresponding to eigenvalue \( i \).

The maximum value obtained for the residue corresponding to \( u_k \) and \( y_i \) associated with mode ‘i’ are the most efficient signal for damping the low frequency oscillations.

### IV. APPLICATION

The power system used in this paper is shown in Fig.1. This is the two fully symmetrical area linked by 230 kV lines. Each area is outfitted with two indistinguishable round rotor generators rated 20 kV/900 MVA. Each generator is equipped with the identical speed regulators in addition with fast static exciter(K=200). The parameter of the machines are provided in table I. For more details refer to the book Prabha Kundoor’s ‘Power System stability and control’(example 12.6). Under the given conditions, the state space model of the two-area system is acquired. The dimension of the state vector \( x \), the output vector \( y \) and the input vector \( u \) are (64×1), (8×1) and (4×1) respectively. The inputs are the voltage reference, \( v_{ref} \), and the outputs are the rotor angle deviations \( \Delta \delta \) and rotor speed deviation \( \Delta \omega \) of the four generators (G1,G2,G3 and G4). The investigation of the system exhibits two local area modes and one inter-area modes. The frequency and damping ratio of these eigenvalues are given in table II.

<table>
<thead>
<tr>
<th>Machines and Bus data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gen.</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>G1</td>
</tr>
<tr>
<td>G2</td>
</tr>
<tr>
<td>G3</td>
</tr>
<tr>
<td>G4</td>
</tr>
<tr>
<td>B7</td>
</tr>
<tr>
<td>B9</td>
</tr>
</tbody>
</table>

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The values for geometric approach and for residue approach are shown in the table III and table IV respectively taking speed deviation as the control feedback signal. Both the measure are giving same loop selection index. However, the value for loop selection index in residue approach is very high in amplitude.

Table III
Geometric approach

<table>
<thead>
<tr>
<th>No.</th>
<th>Input/output</th>
<th>∆ω1</th>
<th>∆ω2</th>
<th>∆ω3</th>
<th>∆ω4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Generator 1</td>
<td>4.34e-009</td>
<td>2.9e-009</td>
<td>6.68e-009</td>
<td>5.44e-009</td>
</tr>
<tr>
<td>2</td>
<td>Generator 2</td>
<td>2.8e-004</td>
<td>1.9e-004</td>
<td>4.4e-004</td>
<td>3.8e-004</td>
</tr>
<tr>
<td>3</td>
<td>Generator 3</td>
<td>2.96e-009</td>
<td>1.99 e-009</td>
<td>4.5 e-009</td>
<td>3.99 e-009</td>
</tr>
<tr>
<td>4</td>
<td>Generator 4</td>
<td>2.32e-007</td>
<td>1.56 e-007</td>
<td>3.57 e-007</td>
<td>3.13 e-007</td>
</tr>
</tbody>
</table>

Table IV
Residue approach

<table>
<thead>
<tr>
<th>Gen</th>
<th>Input / Output</th>
<th>∆ω1</th>
<th>∆ω2</th>
<th>∆ω3</th>
<th>∆ω4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gen 1</td>
<td>0.3e-04</td>
<td>0.23e-04</td>
<td>0.53e-04</td>
<td>0.471e-04</td>
</tr>
<tr>
<td>2</td>
<td>Gen 2</td>
<td>2.4</td>
<td>1.6</td>
<td>3.8</td>
<td>3.3</td>
</tr>
<tr>
<td>3</td>
<td>Gen 3</td>
<td>0.23e-04</td>
<td>0.16e-04</td>
<td>0.36e-04</td>
<td>0.3e-04</td>
</tr>
<tr>
<td>4</td>
<td>Gen 4</td>
<td>0.002</td>
<td>0.001</td>
<td>0.003</td>
<td>0.002</td>
</tr>
</tbody>
</table>

V. RESULTS
The magnitudes of the loop selection index from both the approaches is on y-axis and x-axis represents the generator from whom the output is taken for the synthesis of the proper input-output pair. The result show that the calculated loop selection index is different for the two approaches. From the figure we can see that the magnitude of the geometric approach is negligible as compared to the residue approach. However, for the control loop gen2-∆ω3 residue approach and geometric gives the same result.

Fig.2 Geometric versus residue

Table III shows the joint controllability/observability measure corresponding to mnth input and nth output combination. For global control the signal from the different area is desired as the global signal is more effective in controlling the inter-area modes as studied in [5]. From the table we can say that for generator 1 and generator 2, the global signals are ∆ω3 and ∆ω4. Similarly, for generator 3 and generator 4, the global signals will be ∆ω1 and ∆ω2. We can see from the table, the loop select index corresponding to input generator 2 and output ∆ω3 gives the most elevated LSI.

Table IV shows the magnitude obtained with the residue approach. It is observed that the global signal ∆ω3 corresponding to generator 2 has the highest magnitude which is same as in the case of result obtained from geometric approach. Therefore, according to the residue measure, the maximum value of magnitude gives the most efficient damping to mode ‘i’.

V. CONCLUSION
The geometric approach and the residue approach were used to select the most effective control loop for the controller to damp the 0.62 Hz inter-area mode of the simple two area system. The analysis shows that, contrary to the geometric measure, the residue approach provides high amplitude signals. The result shows that both the approaches gives the same result for the effective control signal for the designing of the wide area speed PSS where speed deviation will be taken as the control signal for designing the robust controller.

VI. REFERENCES

