

Designing Belleville Spring Washers

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Abstract—Belleville spring washers are elastic washers that have circular shapes. They are flexible axially and deflect like springs when they are compressed. Belleville spring washers provide preload between two fastened surfaces and are used to prevent loose fastening, absorb shocks, eliminate play and uniformize load. Belleville spring washers are often employed when radial space is constrained, and moderate load is applied in the operation. In this paper, a systematic method is presented for designing and analyzing Belleville spring washers. To model a Belleville spring washer, a slanted surface is first utilized to define the axial cross section. The solid model of the Belleville spring washer is then generated through rotating the slanted surface along its central axis. The solid model of a Belleville spring washer is completely decided by its four geometric parameters: the internal and external diameters, the thickness of the washer and its height. The design of a Belleville washer is systemized in this paper as optimizing its four geometric parameters. The analysis of a Belleville washer is conducted based on its solid model. Examples on designing and analyzing Belleville spring washers are presented in the paper to verify the effectiveness and demonstrate the procedure of the introduced method.

Keywords—Belleville spring washer; design; analysis; deformation; reaction force.

I. INTRODUCTION

Washers are usually round-shaped disks that serve to increase the area of contact between the bolt head or nut and the clamped part. A washer is normally used where the bolt compression load on the clamped part needs to be distributed on a larger area than what the bolt head or nut can provide. The use of a washer also prevents damage to the bearing surface adjacent to the fastener by the nut when it is tightened [1]. Washers are vital components in fastening and assembly operations. Besides acting as a seat for bolts, nuts, screws and rivets, washers perform many other functions that include insulating, sealing, locking, spacing, improving appearance, providing spring take-up, aligning, distributing loads. Washers are indispensable to the functions of many machines or devices and of importance to their operations [2].

Spring washers are used in offset situations to provide tension in bolted assemblies, or to furnish the recoil action of springs. Even though their span of deflection is limited because of their size and especially because of their height, a somewhat improved performance may be expected where coupling several washers together. Their stacking ability, along with their compatibility and versatility, makes spring washers quite advantageous where used in confined spaces or where a stabilizing function is needed [3].

Spring washers are classified on the basis of shape, type, and applications. Springs washers have three basic types: curved, waved and Belleville [2-3]. Curved spring washers are

cylindrically curved in one direction and provide a uniform spring rate over a large deflection range. They are also known as crescent or bowed washers. They are best suited for applications that require flexibility, frequent load cycling, and light loads. For a curved spring washer to function properly, the formed portion must be free to slide and bearing surfaces must be hard enough to prevent washer corners from scraping or digging in [4-5].

Waved washers are wavy in the axial direction and have multiple curvilinear lobes. They provide spring pressure when the wavy surface is compressed. Wave spring washers are employed when radial space is constrained, and the moderate load is applied in the operation. The wave uniformity is important because the real load deflection gradient does not start until all waves are evenly loaded. Once all waves are uniformly loaded, a relatively linear spring rate will be obtained until the washer deflects close to its flat position [4-5].

Belleville washers have truncated cone or spherical shapes and are also called as conical or cupped spring washers. Belleville washers provide a small deflection range and have a high load-bearing capacity. A Belleville washer initially has the form of a cone that progressively flattens as the bolt is tightened. In the initial tightening, the load-deflection curve has a constant positive gradient. As the tightening continues, the load-deflection curve will have a negative gradient due to the large geometric change in the shape of the conic washer. A Belleville washer is employed to operate in the region where the gradient is negative. In this way, it will increase the load on the bolt if the bolt is loosened, so that the loosening is counteracted [6-7].

In addition to standard conical shape, Belleville spring washers can have slots. Deflections and stresses of Belleville spring washers are affected by their slots [8-9]. Unlike conventional coil springs that have many books and publications on their designs, Belleville spring washers do not have many publications on their designs. This research is motivated by the current lack of systematic design methods on Belleville spring washers. The research objective of this work is to introduce a systematic method on designing Belleville spring washers.

The remainder of the paper is organized as follows. The analysis of standard Belleville spring washers is in Section II. Analysing Belleville spring washers with modified shapes is presented in section III. Section IV is on the design of Belleville spring washers. Conclusions are drawn in section V.

II. STANDARD BELLEVILLE SPRING WASHERS

To model a standard Belleville spring washer, a slant rectangle is first created, which is the axial cross section of the Belleville spring washer.

As shown in Figure 1, the slanted rectangle is ABCD that is fully defined four design parameters: the internal diameter (d), external diameter (D), the thickness of the washer (S), and the washer height (h).

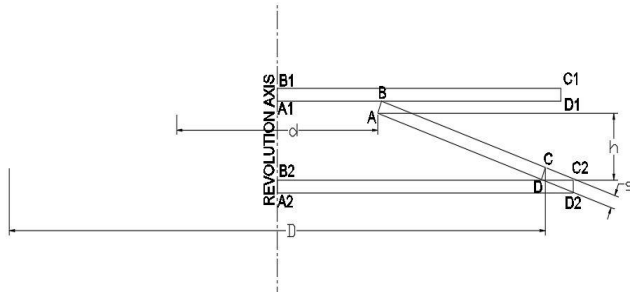


Fig. 1 The four design parameters of a standard Belleville spring washer.

The Belleville washer in Figure 1 contacts two horizontal plates. The axial cross sections of the two horizontal plates are rectangles $A_1B_1C_1D_1$ (top) and $A_2B_2C_2D_2$ (bottom) in Figure 1. The thicknesses of the two plates are the same as that of the Belleville washer, which is S in Figure 1.

The solid model of the Belleville washer is generated by revolving the slanted rectangle (ABCD) along the Belleville washer's vertical axis that is shown in Figure 1. The horizontal rectangular cross sections of the top and bottom plates in Figure 1 have their left lines coincide with the vertical revolution axis. When the three rectangular cross sections in Figure 1 are revolved along the vertical axis in the figure, their solid models are generated.

Figure 2 shows the cross-section sketches of the three rectangles of a standard Belleville spring washer and its contacting top and bottom plates. The following dimensions are used to create the three rectangular sketches in Figure 2: $d = 19$ mm, $D = 38$ mm, $h = 1.4$ mm, $s = 1$ mm.

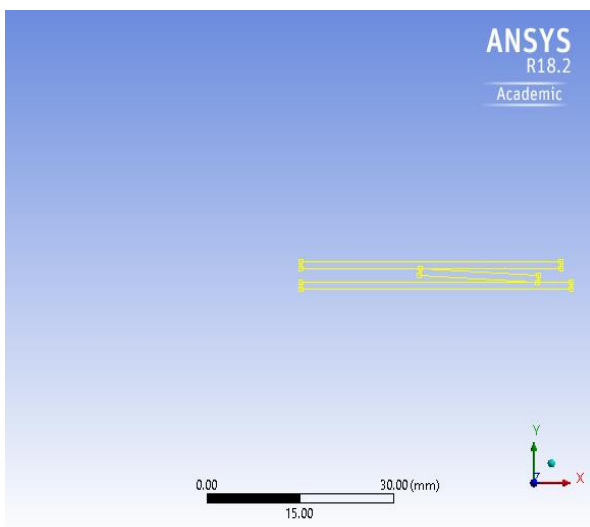


Fig. 2 The cross section sketches of a Belleville spring washer and its top and bottom contacting plates.

After the three rectangular sketches in Figure 2 are revolved about the vertical revolution axis, the solid modes of the standard Belleville spring washer and the two plates are generated. The cross sections of the three solid models (the Belleville spring washer, top and bottom plates) are shown in Figure 3. The solid model of the Belleville spring washer without the two plates is shown in Figure 4 in which the solid models of the two plates are hidden.

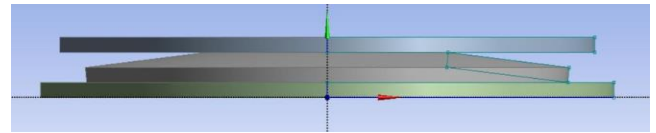


Fig. 3 The cross sections of the Belleville spring washer and its top and bottom contacting plates.

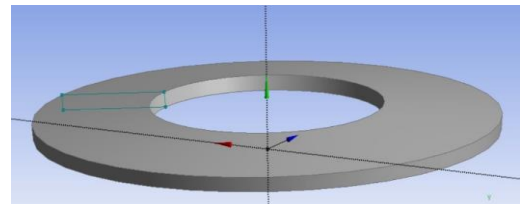


Fig. 4 The standard Belleville spring washer.

The above models are created using ANSYS Workbench [10]. The modeled Belleville spring washer as shown in Figure 4 is analyzed in ANSYS for its stress distribution, reaction force and deformation with a given vertical input displacement from the top contacting plate.

The material used for the analyzed standard Belleville spring and its two contacting plates is carbon steel that has Young's modulus of 210 GPa, Maximum yield strength of 750 MPa, and Poisson's ratio of 0.3. The bottom plate is fixed and has no degree of freedom during the analysis process of the standard Belleville spring washer. The top plate has only one degree of freedom that is vertical input displacement. The three rotation degrees of freedom and two horizontal translation degrees of freedom are restricted.

When 1 mm of vertical input displacement is applied to the top plate, the stress distribution of the analyzed standard Belleville spring washer is shown in Figure 5. The maximum stress (the equivalent von-Mises stress from ANSYS) is 521.55 MPa that occurs near the inner contacting ring between the analyzed Belleville spring washer and the top plate because of its small contacting area. The stress on the outer contacting ring between the analyzed Belleville spring washer and the bottom plate is small because of its large contacting area. The total reaction force between the Belleville spring washer and the top plate is exactly the same as that between the Belleville spring washer and the bottom plate. With the much smaller contacting area between the Belleville spring washer and the top plate than that between the Belleville spring washer and the bottom plate, their stresses have a huge difference. The total reaction between the analyzed Belleville spring washer and any plate (either top or bottom) is 1946 N that is along the vertical direction.

III. BELLEVILLE SPRING WASHERS WITH MODIFIED SHAPES

The standard Belleville spring washer that is analyzed in Section II is used in this chapter as a reference Belleville

spring washer for shape modification. The strategy employed in this chapter for shape modification is to remove some material from the standard Belleville spring washer to make the washer lighter. Compared with the standard Belleville spring washer, the corresponding Belleville spring washer with a modified shape will contain less material and have smaller spring stiffness.

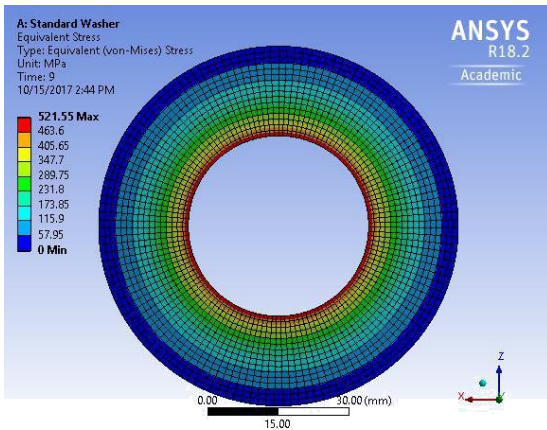


Fig. 5 The stress distribution of the standard Belleville spring washer.

Figure 6 shows a Belleville spring washer with internally slotted shape. The left or right horizontal slot in Figure 6 is used for describing the size of the slot that has the width of 5 mm and the total horizontal depth of 8.5 mm. Eight same slots are evenly distributed along the inner circle of the washer. The material of the eight slots is extruded from the standard Belleville spring washer. The design parameters of the Belleville spring washer with internally slotted shape are the same as those of the standard Belleville spring washer analyzed in Section II. The material used for the Belleville spring washers with modified shapes in this section is the same as the material for the standard Belleville spring washer.

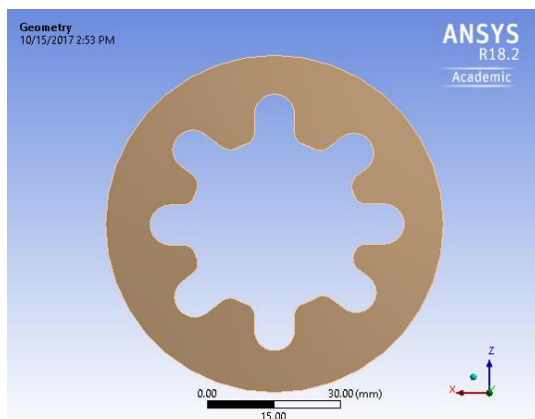


Fig. 6 The internally slotted Belleville spring washer.

Same as the standard Belleville spring washer, the internally slotted Belleville spring washer has top and bottom contacting plates. The bottom one is fixed to support the washer while the top one can only have vertical displacement for input movement. When a vertical input displacement of 1 mm is applied to the top plate, the stress distribution within the internally slotted Belleville spring is shown in Figure 7.

The maximum stress occurs near the inner contacting ring between the internally slotted Belleville spring washer and the top plate and has value of 411.2 MPa that is lower than that (521.55 MPa) of the standard Belleville spring washer. The total reaction force between the internally slotted Belleville spring washer and bottom supporting plate is 1570 N that is lower than that (1946 N) of the standard Belleville spring washer. The lower reaction force from the internally slotted Belleville spring washer is what we expect because some material has been removed from the standard Belleville spring washer. The total material volume of the internally slotted Belleville spring washer is 2764.2 mm³ that is lower than that (3419.8 mm³) of the standard Belleville spring washer. The internally slotted Belleville spring washer is lighter and softer than the standard Belleville spring washer.

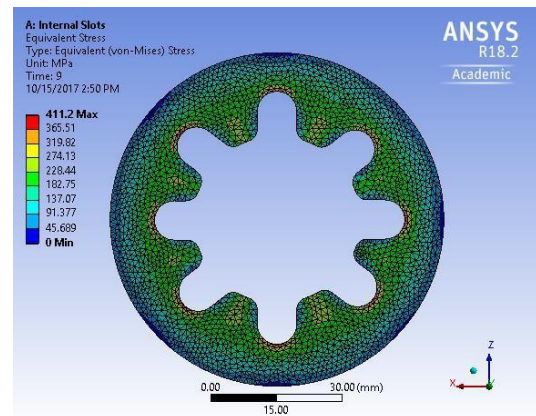


Fig. 7 The stress distribution of the internally slotted Belleville spring washer.

Figure 8 shows a Belleville spring washer with externally slotted shape. The four design parameters of the externally slotted Belleville spring washer are the same as those of the internally slotted Belleville spring washer analyzed before. Different from the internally slotted Belleville spring washer, the eight evenly distributed slots are along the outer circle of the washer.

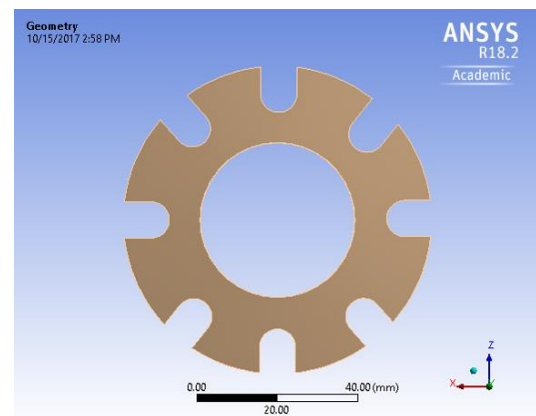


Fig. 8 The externally slotted Belleville spring washer.

When a vertical input displacement of 1 mm is applied to the top plate, the stress distribution within the internally slotted Belleville spring is shown in Figure 9. The maximum stress occurs near the inner contacting ring between the

externally slotted Belleville spring washer and the top plate and has value of 456.47 MPa that is lower than that (521.55 MPa) of the standard Belleville spring washer, but higher than that (411.2 MPa) of the internally slotted Belleville spring washer. The total reaction force between the externally slotted Belleville spring washer and bottom supporting plate is 1406 N that is lower than that (1570 N) of the internally slotted Belleville spring washer. This is because some material has been removed from the outer circle of the Belleville spring washer where the fixed bottom plate contacts and supports the Belleville spring washer. The total material volume of the externally slotted Belleville spring washer is 2754.7 mm³ that is a little lower than that (2764.2 mm³) of the internally slotted Belleville spring washer. The externally slotted Belleville spring washer is a little lighter and softer than the internally slotted Belleville spring washer. But its maximum stress is higher than that of the internally slotted.

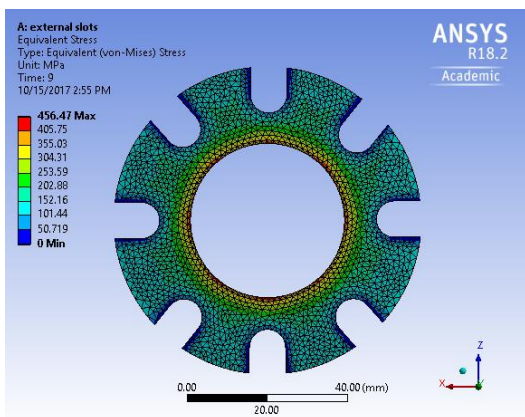


Fig. 9 The stress distribution of the externally slotted Belleville spring washer.

Figure 10 shows a Belleville spring washer with middle holes. The four design parameters of the Belleville spring washer with middle holes are the same as those of the externally slotted Belleville spring washer analyzed last section. The eight holes have diameter of 5 mm and are evenly distributed along a circle that is in the middle between the inner and outer rings of the Belleville spring washer.

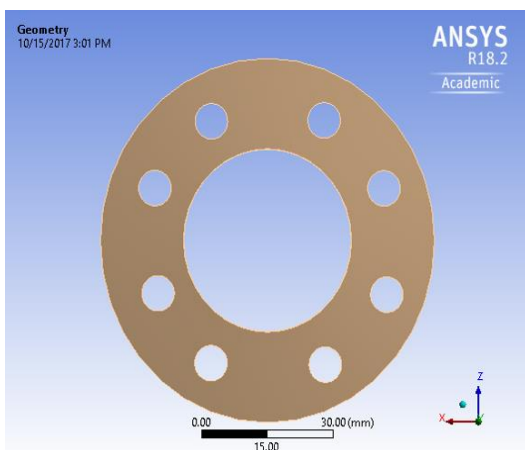


Fig. 10 The Belleville spring washer with middle holes.

When a vertical input displacement of 1 mm is applied to the top plate, the stress distribution within the Belleville spring is shown in Figure 11. The maximum stress occurs near the inner contacting ring between the Belleville spring washer and the top plate and has value of 521.02 MPa that is almost the same as that (521.55 MPa) of the standard Belleville spring washer, but higher than that (456.47 MPa) of the externally slotted Belleville spring washer. The total reaction force between the Belleville spring washer with middle holes and bottom supporting plate is 1650 N that is lower than that (1946 N) of the standard Belleville spring washer, but higher than that (1570) of the internally slotted Belleville spring washer. This is because the removed material is away from the inner or outer ring of the spring washer. The total material volume of the Belleville spring washer with middle holes is 3071.5 mm³ that is a little higher than that (2764.2 mm³) of the internally slotted Belleville spring washer. The Belleville spring washer with middle holes is harder than the internally slotted Belleville spring washer, but softer than the standard Belleville spring washer. Its maximum stress is higher than that of the internally slotted Belleville spring washer.

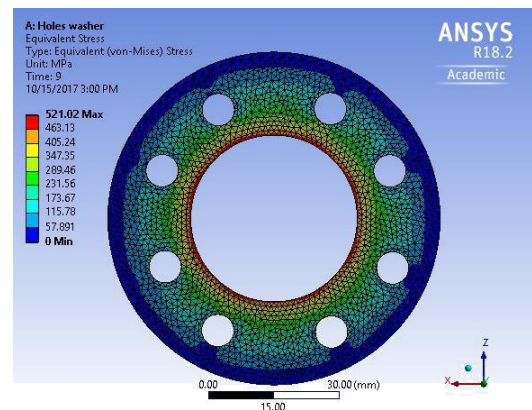


Fig. 11 The stress distribution of the Belleville spring washer with middle holes.

Figure 12 shows a Belleville spring washer with mixed holes and internal slots. The four design parameters of the Belleville spring washer are the same as those of the Belleville spring washer with middle holes that is analyzed before. The sizes and distributions of the eight holes and the eight internal slots are the same as those discussed before in this section.

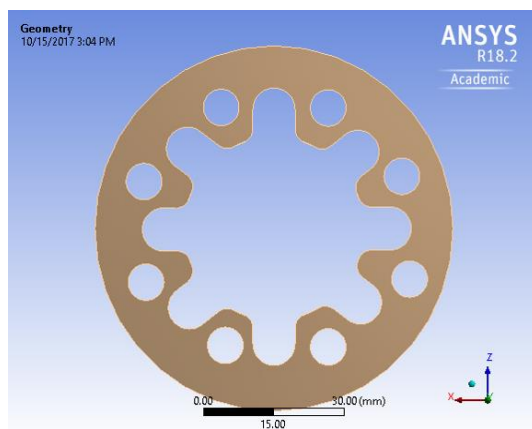


Fig. 12 The Belleville spring washer with mixed holes and internal slots.

When a vertical input displacement of 1 mm is applied to the top plate, the stress distribution within the Belleville spring is shown in Figure 13. The maximum stress occurs near the inner contacting ring between the Belleville spring washer and the top plate and has value of 380.9 MPa that is lower than that (411.2 MPa) of the internally slotted Belleville spring washer. The total reaction force between the Belleville spring washer with mixed holes and internal slots and the bottom supporting plate is 1352 N that is lower than that (1406 N) of the externally slotted Belleville spring washer. This is because the removed material includes both holes and slots. The total material volume of the Belleville spring washer with mixed holes and internal slots is 2399.1 mm³ that is a lower than that (2764.2 mm³) of the internally slotted Belleville spring washer. The Belleville spring washer with mixed holes and internal slots is softer than the internally or externally slotted Belleville spring washer. Its maximum stress is also lower than that of the internally or externally slotted Belleville spring washer.

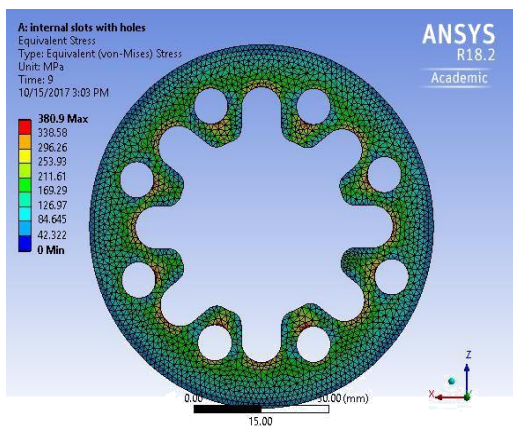


Fig. 13 The stress distribution of the Belleville spring washer with mixed holes and internal slots.

Figure 14 shows a Belleville spring washer with mixed holes and external slots. The four design parameters of the Belleville spring washer are the same as those of the Belleville spring washer with middle holes that is analyzed before. The sizes and distributions of the eight holes and the eight external slots are the same as those discussed before in this section.

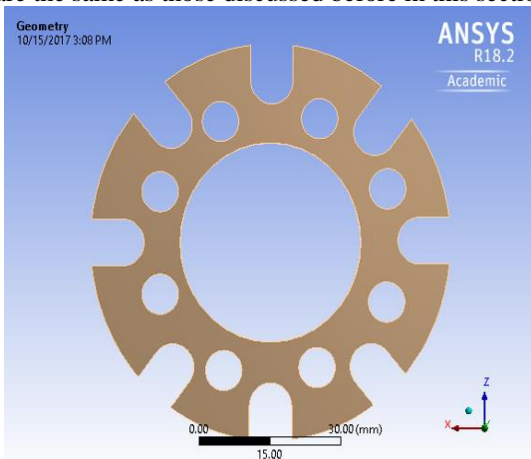


Fig. 14 The Belleville spring washer with mixed holes and external slots.

When a vertical input displacement of 1 mm is applied to the top plate, the stress distribution within the Belleville spring is shown in Figure 15. The maximum stress occurs near the inner contacting ring between the Belleville spring washer and the top plate and has value of 416.91 MPa that is lower than that (456.47 MPa) of the externally slotted Belleville spring washer. The total reaction force between the Belleville spring washer with mixed holes and external slots and the bottom supporting plate is 1211 N that is lower than that (1406 N) of the externally slotted Belleville spring washer. This is because the removed material includes both holes and slots. The total material volume of the Belleville spring washer with mixed holes and external slots is 2387.4 mm³ that is a little lower than that (2399.1 mm³) of the Belleville spring washer with mixed holes and internal slots. The Belleville spring washer with mixed holes and external slots is softer than the Belleville spring washer with mixed holes and internal slots. Its maximum stress is higher than that of the Belleville spring washer with mixed holes and internal slots.

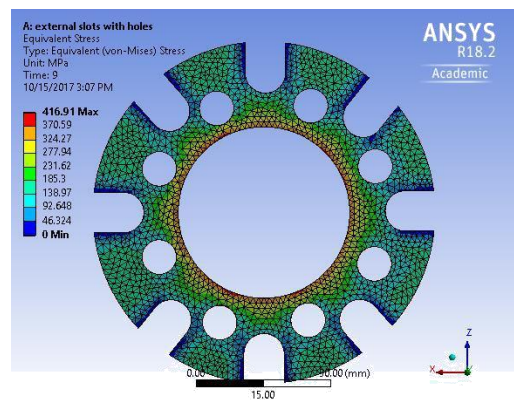


Fig. 15 The stress distribution of the Belleville spring washer with mixed holes and external slots.

Among the six Belleville spring washers analyzed above, the standard Belleville spring washer has the highest reaction force under the same input displacement, and also contains the largest amount of material. The Belleville spring washer with the mixed holes and internal slots has the lowest maximum stress. The Belleville spring washer that has the least amount of material is the one with the mixed holes and external slots. That Belleville spring washer also has the lowest reaction force among all analyzed washer in this chapter.

IV. DESIGN OF BELLEVILLE SPRING WASHERS

The material used for the designed Belleville spring washer is carbon steel with its Young's modulus, Poisson's ratio and yield strength of 210 GPa, 0.3, and 750 MPa, respectively.

The designed Belleville spring washer is desired to generate the reaction force of 1350 N under the input displacement of 1 mm. The thickness of the designed Belleville spring washer is specified as 1 mm together with its height of 1.4 mm. The design parameters to be selected are the internal diameter (d) and the external diameter (D) of the Belleville spring washer. The maximum stress within the designed Belleville spring washer is not allowed to exceed the yield strength of the material. Based on the application, the

ranges of the internal diameter and the external diameter are: $17 \text{ mm} \leq d \leq 20 \text{ mm}$ and $36 \text{ mm} \leq D \leq 40 \text{ mm}$, respectively.

To design a Belleville spring washer, the solid models of the designed washer and its corresponding top and bottom plates are first modeled in ANSYS Design Modeler using their design parameters. The created solid models are then analyzed in ANSYS Mechanical. Based on the analysis results, the objective and constraint functions are computed for design parameter optimization. The objective function in this research is to make the generated reaction force from the Belleville spring washer have its desired value (1350 N) under the specified input displacement (1 mm). The design parameters of the Belleville spring washer are optimized in ANSYS Design Exploration through the Direct Optimization Toolbox [10].

Each of the six Belleville spring washer configurations (one standard plus five modified) that were analysed in Sections II & III is designed to meet the requirements for the Belleville spring washer. The design results are shown in Table 1. Each of the six design solutions can generate exactly the desired reaction force (1350 N) under the specified input displacement (1 mm). However, the maximum stress within the deformed washer and the washer material volume are different among the six design solutions. Table 2 shows the maximum stress and material volume values of the six different washer configurations.

Table 1 Design results for the Belleville spring washer

Belleville Washer Configuration	Internal Diameter	External Diameter
Standard Washer	17.42 mm	39.16 mm
Washer with Holes	17.42 mm	38.54 mm
Internally Slotted	19.18 mm	36.32 mm
Externally Slotted	17.84 mm	36.82 mm
Internally Slotted with Holes	19.04 mm	38.10 mm
Externally Slotted with Holes	19.97 mm	37.09 mm

Table 2 Maximum stress and material volume of the Design results for the Belleville spring washer

Belleville Washer Configuration	Maximum Stress	Material Volume
Standard Washer	496.39 MPa	3671.1 mm ³
Washer with Holes	488.67 MPa	3215.2 mm ³
Internally Slotted	447.52 MPa	2281.4 mm ³
Externally Slotted	497.04 MPa	2623.6 mm ³
Internally Slotted with Holes	378.3 MPa	2380.9 mm ³
Externally Slotted with Holes	453.37 MPa	2000.2 mm ³

Although the standard Belleville spring washer design is the most convenient washer to manufacture, it contains more material than any of the five other designs that have modified washer shapes. Among all six feasible designs, the Belleville spring washer with mixed holes and external slots contains the least amount of washer material. The maximum stress from

the design of the Belleville spring washer with mixed holes and internal slots has the smallest value.

There are six solutions for the specified design conditions among which one design can be chosen based on practical requirements or considerations.

V. CONCLUSIONS

Belleville spring washers are elastic washers and have circular shapes. Their axial cross section has rectangular shape. To design a Belleville spring washer, its solid model needs to be established. To model a Belleville spring washer, a slanted rectangle is first created that is defined by four design parameters of the Belleville spring washer: internal and external diameters, washer height and thickness. The internal and external diameters are measured perpendicular to the Belleville spring washer's axis while the washer height is along the washer axis. The thickness of the washer has the value of the short length of the slanted rectangle. With the slanted rectangle created, the solid model of the Belleville spring washer can be established by revolving the slanted rectangle with respect to the washer axis. Designing a Belleville spring washer is systemized in this research as the optimization of the four design parameters for the designed Belleville spring washer to meet its desired performance requirements.

Standard Belleville spring washers have been widely used for different applications because of their convenience to manufacture. However, standard Belleville spring washers have a disadvantage that is their material volume or weight. To reduce material volume or weight for Belleville spring washers, five different shape modifications (internal slots, external slots, middle holes, mixed holes and internal slots, mixed holes and external slots) are presented in this research. Belleville spring washers with modified shapes are analyzed and compared with their corresponding standard Belleville spring washer. A Belleville spring washer with mixed holes and external slots is found to contain much less material than its corresponding standard Belleville spring washer. The maximum stress within a standard Belleville spring washer is much higher than that of its corresponding Belleville spring washer with mixed holes and internal slots when they are under the same input displacement.

For the specified design conditions of an application, there are usually multiple feasible design solutions that come from either a standard or a modified Belleville spring washer. One design can be chosen from the available candidates based on practical requirements or other considerations to better meet the needs. A standard Belleville spring washer can be chosen if manufacturing cost has priority and washer weight does not have priority. A Belleville spring washer with mixed internal slots and holes or only internal slots can be chosen if the maximum stress needs to be low and manufacturing cost does not have priority. When holes are mixed with internal slots, the maximum stress is further reduced but the manufacturing cost increases. A Belleville spring washer with mixed external slots and holes or only external slots can be chosen if the washer needs to have light weight and manufacturing cost does not have priority. When holes are mixed with external slots, the washer weight is further reduced but the manufacturing cost increases.

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