Design Optimization of the Spur Gear Set

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***Abstract*—Gears are one of important modes for transmission of mechanical power in different mechanical systems. Optimization plays an important role in gear design as reducing the weight or volume of a gear set will increase its service life and improve the bearing capacity. In this paper the basic design parameters for gear like module and number of teeth on the pinion are considered as the variables and centre distance is considered as the objective function. The bending stress limit and the surface stress limit are considered as the constraints. The Genetic Algorithm technique of optimization is used to obtain the optimized values of module and number of teeth on the pinion and the results are compared as that obtained using the traditional method.**

Keywords—Spur Gear, optimization, Genetic Algorithm

I. INTRODUCTION

When the number of design variables and constraints are large, solving engineering problems can be complex and a time consuming process. In an optimization problem the design variables are determined to obtain the best measurable performance of the objective function under the given constraints. Optimization algorithms are becoming popular in engineering design activities, primarily because of availability and affordability of high speed computers. The improvement of faster computer has given chance for more robust and efficient optimization methods. Genetic algorithm is one of these methods. The genetic algorithm is a search technique based on the idea of natural selection and genetics [1].

Gears are the toothed machine elements which transmit power and motion from one shaft to other by successive engagement of teeth.In spur gears teeth are parallel to the axis of the shaft so they are used only when the shafts are parallel. In the past many researchers considered the optimized design of different gear drives choosing the best parameters to improve the bearing capacity and to reduce the weight and cost ,hence prolonging the service life.[2][3][4]. The design of gear trains was considered for minimum overall size in refrence [5] .Savage, Coy and Townsend [5] presented a method for the optimal design of compact standard spur gear sets by considering the aspects of scoring, pitting, bending and involute interference.

The work presented here establishes the mathematical model for the optimization of design of spur gear pair for a given set of input parameters. A design space is defined in terms of the number of teeth and the module. The space is then combined with the objective of minimum centre distance to obtain an optimal design region using optimization technique of genetic algorithm. The results are compared with the Conventional design method.

II. MATHEMATICAL OPTIMIZATION MODEL FOR SPUR GEAR

A. Design-Parameters

Following parameters may be taken while designing a spur gear pair:

Number of teeth on the pinion(N_1), Module(m)

Generally a normal pressure angle of 20º is used. Once the specifications about materials and their heat treatment are defined, the material properties are assumed to be constant. Also,as long as the tooth load remains uniform ,gear mesh face width is directly proportional to the resulting strength of gear set.For tooth load to be uniform ,a common criterion is to limit the length to diameter ratio,λ,of the line of contact.This is the ratio of the gear face width to the pitch diameter of smallest gear in meshand is related to face width by the number of teeth on the pinion and the module as:$b=\lambda N_1 m$

In view of the above consideration, the design space consist of two design variables: Number of teeth on the pinion (N_1),

The standard gear design problem for obtaining compact spur gear pair made up of a chosen material can be reduced to a two dimensional design problem where two parameters are:

The number of teeth on pinion ($N_{1}) = x_{1}$ and module (m) $=x_2$

B. Objective Function

The purpose of the present work is to develop a procedure to design a compact spur gear set. Designing minimum size gear set provides benefits such as minimal weight, lower material costs, small housing and smaller inertial loads.

Choosing the gear mesh center distance, C, as the measure of gear size .The objective function is to minimize the center distance C, given by

$C=R_1+R_2$

$C=R_1 (1+R_2/R_1$

$C=N_1 m (1+m_g)/2$

In terms of variables

$C=x_1x_2(1+m_g)/2$

C. Design constraints

Constraints are conditions that must be met be met in the optimum design and include restrictions on the design variables. These constraints define the boundaries of the feasible and infeasible design space domain. The constraints considered are following:

1) Bending Stress Constraint:

To avoid the tooth failure in bending, the bending stress should be less than the allowable bending stress.

Or, $\sigma_{b} \leq \sigma_{b,all}$ where, $\sigma_{b,all}$=allowable bending stress

In terms of variables

$$\frac{2T_p}{\lambda x_1^2 x_2^3 (0.484 - \frac{2.865}{x_1})} \le \sigma_{b,all}$$

2) Surface Stress Constraint:

The surface failure of gear teeth is an important concern in gear design. Surface failure modes include pitting, scoring and wear. The contact stress should be less than surface strength of softer material for safe design.

i.e $\sigma_c \le \sigma_{c,all}$

In terms of variables

$$\left\{ \frac{4T_P * (1+1/m_g) * (E_1E_2/E_1+E_2)}{\pi\lambda x_1^3 x_2^3 (1-\mu^2)\cos\alpha\sin\alpha} \right\}^{1/2} \le \sigma_{,all}$$

III. METHODOLOGY OF GENETIC ALGORITHM

GA begins its search with a population of randomly selected set of initial solutions usually coded in binary string structures. Instead of working with a single solution in each iteration ,GA begins its search with a population of randomly selected set of initial solutions usually coded in binary string structures representing the problem's decision variables .This initial population is subjected to genetic evolution to create the next generation of the candidate solution. The progenies are then then evaluated and tested for termination. The procedure is continued till the termination criteria is met.

A. Genetic operators

The three genetic operators are Reproduction, Cross over and mutation for procreation of the next generation.

1) Reproduction:

It is usually the first operator applied on a population and is also called the selection operator as it selects the individuals based on their fitness values relative to that of the population. It picks up above average strings from the current population and their multiple copies are inserted in mating pool in a probabilistic manner. Each individual string is selected with a probability proportionate to its fitness .The probability of selecting the i^{th} string is

$P_i = F_i / \sum F_i$

2) Crossover:

In crossover, new strings are created by exchanging information among strings of mating pool. First two individual strings are selected at random from the mating pool generated by the reproduction operator. Next a crossover site is selected at random along string length.The binary digits are swapped between the two strings following the crossover site,e.g.if two design vectors (parents),are given by

(Parent1)00|000
(Parent2)11|111

Strings after crossover
(Offspring1)00111
(Offspring2)11000

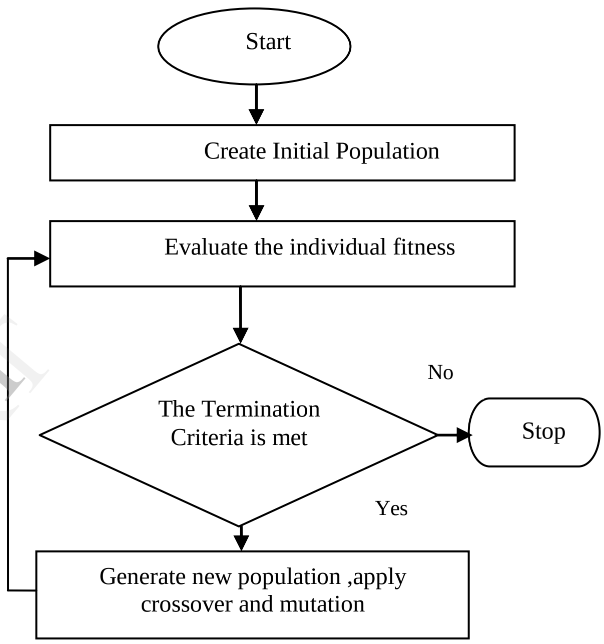
All the strings in the mating pool are not used in crossover and a crossover probability pc is used.

3) Mutation:

Mutation is the occasional random alteration of binary digit from 0 to 1 or vice versa.It is rationally aimed to maintain diversity in the population and fulfills the need for local search around current solution. Mutation is performed with a small mutation probability p_m.

0110 1011 00010110
01111100 00111101

B. Flow chart of GA



IV. METHODOLOGY OF GENETIC ALGORITHM

Table 1 shows the basic gear parameters for a sample spur gear set to be studied and Table 2 shows the GA input data.The gear parameters taken are same as taken by Savage, Coy and Townsend [5] so that the results can be validated.The problem is also solved with the conventionaldesign approach using the Lewis equation and determining the values of relevant coefficients like Lewis form factor and velocity coefficient from the design data book.

TABLE I. GEAR PARAMETERS

S.NO	Gear Parameter	Value
1	Pressure angle, α	20^o
2	Gear ratio, m_g	5.0
3	Length to diameter ratio, λ	0.25
4	Pinion torque, T_P	113N-m
5	Modulus of Elasticity for gear and pinion,E	205Gpa
6	Poisson's ratio, μ	0.25
7	Surface stress limit ($\sigma_{C,all}$	1.38GPa
8	Bending Stress limit ($\sigma_{b,all}$)	414MPa

TABLE II. GA INPUT DATA.

S.No	GA Parameter	Value
1	Population Size	20
2	Total Number of generations	35
3	Total string length	40
4	Cross over probability	0.8
5	Mutation probability	0.01
6	Number of binary coded variables	2
7	Upper and lower bounds for variables	$15 \le x_1 \le 200$ $1mm \le x_2 \le 20mm$

V. RESULTS AND DISCUSSION

(i)To apply GA a software code is developed in C for obtaining the optimal design of spur gear .Once the gear design parameters from Table 1 and GA input data from the Table 2 are applied the following output is obtained:

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Mutations(binary)=275;Crossovers=292
Best ever fitness:107.82(from generation :33)
Variable vector: Binary Real>33.805- 1.063
Best_ever String=01111011101101011000-
01111111101100010000

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(ii)As the number of the teeth cannot be in fraction so taking N_1=34 and choosing the nearest standard module from the recommended series of module ie m=1.06 mm, the value of centre distance becomes C=108.4 mm which is less than 152mm as obtained by Savage, Coy, Townsend[5],hence showing the improvement in results.

(iii)GA results are compared with the results of classical method of optimization design ad conventional design in the

TABLE III.

Design Method	Module (mm)	No. Of teeth on pinion	Centre Distance (mm)
Conventional Design	2	42	252
Classical Optimization Method	1.49	34	152
Genetic Algorithm	1.063	34	108.4

VI. CONCLUSION

By studying the above case of an optimization of a spur gear set we conclude that Genetic Algorithm is an important tool to optimize the problems of the mechanical system design .As compared to the conventional methods which use a single design point, GA use a population of points and gives the global results when compared to the classical method of optimization GA is quite fast and sophisticated.

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