# Design Optimization and Analysis of Fiber Reinforced Composite Cylindrical Shell Structure

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***Abstract*— Composite material usage in underwater applications is increasing in recent years. In this work a composite cylinder is designed which can withstand an external pressure of 5 bar. Stress analysis of the structure is done using Classical Laminate theory and the results obtained are validated using Finite Element Analysis procedure. The health of the composite cylinder is checked by exploring built in failure criteria like Tsai-Wu and Maximum Stress criteria. The work includes determination of optimum design parameters like fiber orientation and ratio of longitudinal to transverse fibers.**

***Keywords—Composite material; Tsai-Wu criterion; Maximum stress criterion; Classical laminte theory.***

## I. INTRODUCTION

Recently the use of cylindrical fiber reinforced laminated composite shells has increased in various fields like piping, remotely operated vehicles, risers etc. [1], [2].Fiber reinforced composites have been successfully used in underwater vehicles and ocean structures replacing conventional metal counterparts due to their light weight , corrosion resistance, high strength to weight ratio. The use of composites which was earlier limited to military and civil aircraft structures has now expanded to underwater applications. This is because of the advancement in composite material manufacturing; that the small sized underwater vehicles can now be manufactured in one piece.

Fiber reinforced composites unlike metals are orthotropic in nature. The elastic properties are different in different directions. The strength of the composite structure is dependent on the fiber directions, stacking sequence, thickness of each ply along with the material properties of the material. Criteria like Maximum Stress and Tsai-Wu are used to determine the failure of such lamina. The work is concentrated on the calculation of lamina stresses in the fiber reinforced composite cylinder using Classical laminate theory and verifying the results by FEA analysis in ANSYS. Laminate configuration for various stacking sequence is analyzed. The ratio of longitudinal to transverse fibers is varied to study effect on the strength of the product.

## II. METHODOLOGY

Stresses in the lamina coordinate system are determined analytically using laminate plate theory. FEA is conducted using ANSYS 15.0. The overall methodology adopted is summarized in the Fig 1

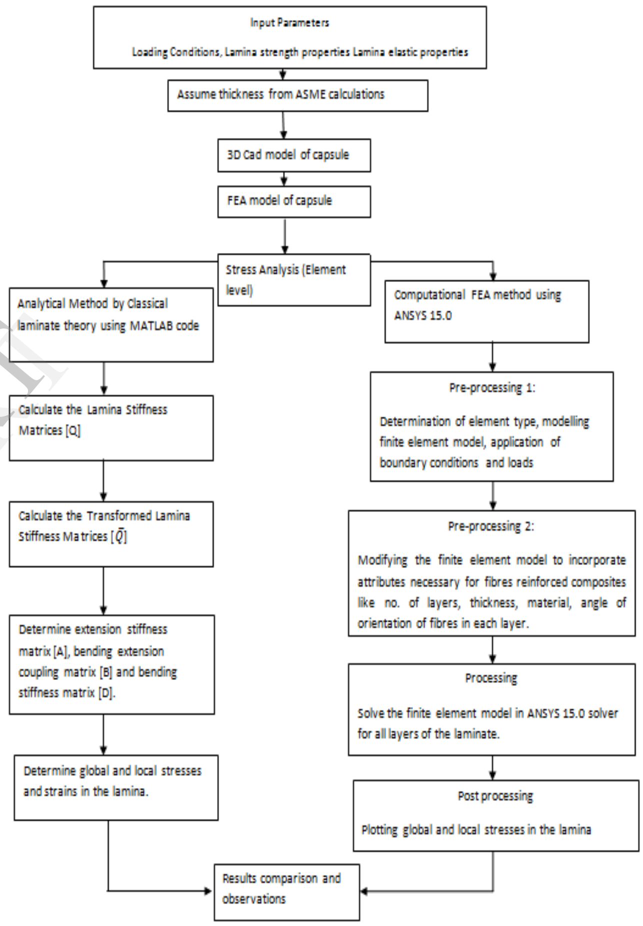


Fig.1 Methodology adopted for failure analysis.

## III. GEOMETRY, MATERIAL AND LOADING

Consider a long cylinder shown in Fig 2 having thickness (‘t’)6.4 mm, length (‘l’) of 200 mm and mid surface radius of(‘R’) of 385 mm. These dimensions are obtained from the ASME section VIII procedure using Aluminum 6061in place of composite material under equivalent loading conditions. The uniform 0.5Mpa pressure(‘p’) is applied in the cylinder. The shell considered is constructed of cross ply laminate with even number of layers of equal thickness abbreviated as‘t’.

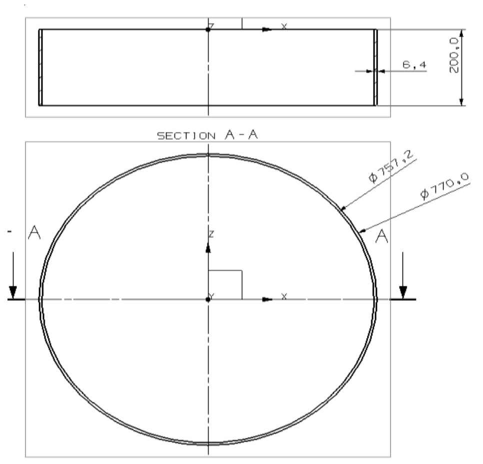


Fig. 2 Cylinder dimensions.

The material selected for analysis is carbon fiber reinforced epoxy. The cross ply laminate construction consists of 16 layers with each layer having thickness of 0.4mm. The fiber direction is defined with respect to the longitudinal axis of the cylinder. The elastic and strength properties are defined in the Table I and Table II respectively having fiber volume fraction of 60% and density 1.5 gm. /cm$^3$. The industrial name of material used isAS4/3501-6.

TABLE I MATERIAL PROPERTIES OFAS4/3501-6[5]

| **Material Elastic Properties in (GPa)** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ex | Ey | Ez | $\upsilon_{xy}$ | $\upsilon_{yz}$ | $\upsilon_{xz}$ | $G_{xy}$ | $G_{yz}$ | $G_{xz}$ |
| 143 | 10 | 10 | 0.3 | 0.3 | 0.52 | 6 | 3 | 5 |

TABLE II STRENGTH PROPERTIES OFAS4/3501-6[5]

| **Strength Properties in (MPa)** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\sigma_{xt}$ | $\sigma_{xc}$ | $\sigma_{yt}$ | $\sigma_{yc}$ | $\sigma_{zt}$ | $\sigma_{zc}$ | $\tau_{xy}$ | $\tau_{yz}$ | $\tau_{xz}$ |
| 2172 | -1558 | 54 | - 186 | 59 | - 186 | 87 | 94 | 124 |

## IV. ANALYSIS USING CLASSICAL LAMINATE THEORY

The cylindrical shell is considered as pressure vessel. Using the formulae for thin shells under pressure the global principle stresses can be calculated by following [3]:

$$\sigma_{hoop}=\frac{p\times R}{t}=\frac{0.5\times385}{6.4}= 30.07 \text{ MPa.} \quad (1)$$

$$\sigma_{axial}=\frac{p\times R}{2t}=\frac{0.5\times385}{2\times6.4}= 15.039 \text{ MPa} \quad (2)$$

For analysis purpose let us consider a small element in the cylindrical shell. The free body diagram of the shell is shown in the Fig.3.The element in the cylinder under pressure is in state of bi axial stress. The radial stress in the thin cylinder is negligible along the thickness as it is thin cylinder and the fiber direction being along the principle direction the shear stress in the plane of lamina is zero[4]. The element under consideration has length of 10mm, thickness of 6.4 mm and unit width. It is like a small element which is bored out of the cylinder and considered for the analysis (as seen in Fig. 3).

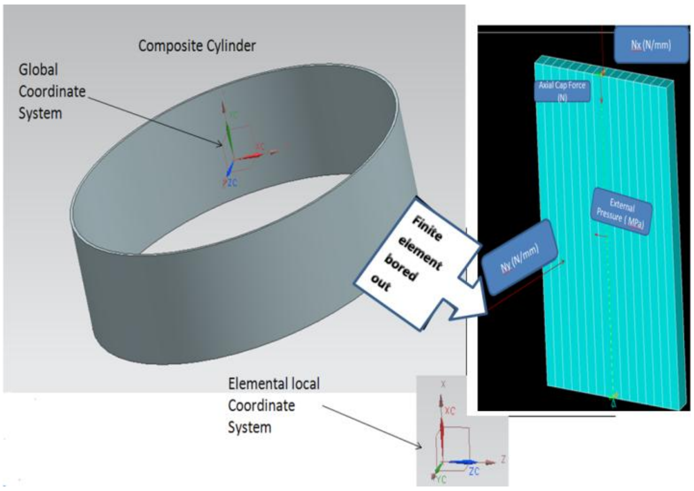


Fig.3 Free body diagram of element in the cylindrical capped vessel under pressure loading in local and global coordinate system

Due to the applied external pressure the force per unit length will be exerted on the laminate in the longitudinal and the hoop direction. The force per unit length on the mid plane is calculated as follows:

$N_y = \frac{\sigma_{hoop}(A_y)}{l} = \frac{30.07\times(6.4\times10)}{10}$ =192.448N/mm($A_y$ is area of face perpendicular to y axis of local coordinate system)

(3)

$N_x = \frac{\sigma_{axial}(A_x)}{l} = \frac{30.07\times(6.4\times1)}{1}$ =96.22 N/mm ($A_x$is area of face perpendicular to x axisof local coordinate system)

(4)

The mid-plane forces and moments are related to the mid-plane strains and curvature by the system of equations (5) given by constitutive equation containing ABD matrix for the considered laminate configuration. Contribution of each lamina to overall properties of the total laminate is incorporated by ABD matrix. A is extensional stiffness matrix, B is coupling stiffness matrix and D is bending stiffness matrix. The extensional stiffness matrix relates resultant force to mid-plane strains and bending stiffness matrix relates resultant moment to plate curvature. The coupling matrix represents the coupling between bending and extension of the laminated plates.

$$\begin{bmatrix} N_1 \\ N_2 \\ N_{12} \\ M_1 \\ M_2 \\ M_{12} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon x \\ \varepsilon y \\ \gamma xy \\ kx \\ ky \\ kxy \end{bmatrix} \quad (5)$$

The B matrix is zero due to symmetric and balanced laminate construction which resulted into no extension and bending coupling. Also due to crossply construction of the laminate of the shell, the$A_{16}$, $A_{26}$, $D_{16}$ and $D_{26}$ in above equation are all zero. The design of balanced and symmetric laminate has resulted in the values of moments to nullify each other and hence zero [2]. Thus equation (5) reduces to

$$\begin{bmatrix} N_1 \\ N_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & 0 & 0 & D_{21} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon x \\ \varepsilon y \\ \gamma xy \\ kx \\ ky \\ kxy \end{bmatrix} \quad (6)$$

The ABD matrix is dependent on the reduced transformed stiffness matrices of each lamina, thickness of each layer and their stacking sequence. Calculations of each of the ij$^{th}$ element in the ABD matrices are given by the following equations:

$$A_{ij} = \sum_{k=1}^{N} \bar{Q}_{ij_k}(z_k - z_{k-1}) \quad (7)$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{N} \bar{Q}_{ij_k}(z_k^2 - z_{k-1}^2) \quad (8)$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} \bar{Q}_{ij_k}(z_k^3 - z_{k-1}^3) \quad (9)$$

Where 'z' is the coordinate from the mid-plane of the 'k$^{th}$' layer and '$\overline{Q}$'is the reduced transformed stiffness matrix.

The stiffness matrix is constructed with help of elastic constants in the plane of the lamina.

The equation (6) gives the value of mid plane strains. These mid plane stains are in global coordinate system which is same throughout the thickness. These strains obtained which are in global X-Y coordinate system are transformed in the local fiber direction (1-2) in particular lamina as shown in the Fig.4. This transformation is done by equation 10.

$$\begin{bmatrix} \varepsilon 1 \\ \varepsilon 2 \\ \varepsilon 3 \end{bmatrix} = T2 \begin{bmatrix} \varepsilon x \\ \varepsilon y \\ \gamma xy \end{bmatrix} \quad (10)$$

$$T_2 = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

(11)

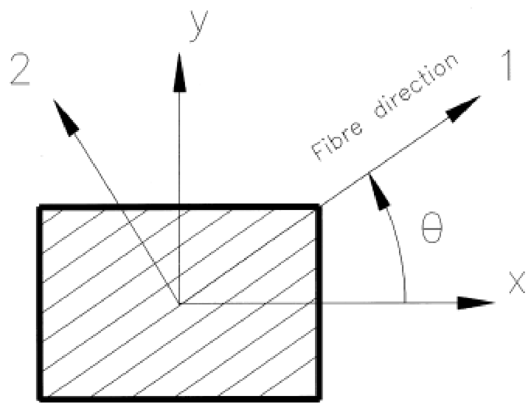


Figure 4- Coordinate transformation from global to local system[4]

Once the strains in the fiber direction are calculated, the constitutive equations relating local stress and strain are used to calculate the stresses in the fiber direction. This relation in the plane lamina is given in equation (14).

$$\begin{bmatrix} \sigma 1 \\ \sigma 2 \\ \tau 12 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{12} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon 1 \\ \varepsilon 2 \\ \varepsilon 3 \end{bmatrix} \quad (14)$$

Using the elastic constant values given in TABLE II the above constitutive equation gets modified as given in equation 15

$$\begin{bmatrix} \sigma 1 \\ \sigma 2 \\ \tau 12 \end{bmatrix} = \begin{bmatrix} 143905.07 & 3019.001 & 0 \\ 3019.001 & 10063.03 & 0 \\ 0 & 0 & 8700 \end{bmatrix} \begin{bmatrix} \varepsilon 1 \\ \varepsilon 2 \\ \varepsilon 3 \end{bmatrix} \quad (15)$$

MATLAB codes were used for all the matrix calculations in the analysis. The resulting ABD matrix and mid-plane external forces are substituted in (6) to get the values of mid-plane strains as

$$\begin{bmatrix} \varepsilon x \\ \varepsilon y \\ \gamma xy \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.000179804 \\ 0.000383531 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

These values of global strains are same for layers 1,3,5,7,9,10,12,14,16 as the orientation of the fibers is zero along the global X axis. Thus stresses in the lamina can be directly calculated by equation (14).The stress values calculated for 0 degree oriented fibers are,

$$\begin{bmatrix} \sigma 1 \\ \sigma 2 \\ \tau 12 \end{bmatrix} = \begin{bmatrix} 143905.07 & 3019.001 & 0 \\ 3019.001 & 10063.03 & 0 \\ 0 & 0 & 8700 \end{bmatrix} \begin{bmatrix} -0.00179 \\ -0.00038 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \sigma 1 \\ \sigma 2 \\ \tau 12 \end{bmatrix} = \begin{bmatrix} -27.091 \\ -4.04 \\ 0 \end{bmatrix}$$

Hence stresses in the fibers oriented at 0° along longitudinal (represented by subscript 1) and lateral (represented by subscript 2) directions in layer no. 1,3,5,7,9,10,12,14,16 are:

$\sigma_{01}$= -27.091 MPa
$\sigma_{02}$ = -4.04 MPa respectively.

But for layers 2,4,6,8,11,13,15 the fibers are oriented in the 90 degrees to the x axis. Hence the global strains need to be transformed in the local material coordinate system. The material coordinate system is such that the 'x' axis is oriented in the fiber axis, the 'y' axis perpendicular to the fiber axis and 'z' axis along the thickness of the laminate.

The layer wise stresses are indication of whether the lamina will withstand the stresses. Tsai-Wu and Maximum Stress failure criteria are used to determine the health of the layers in the applied loading condition. According to maximum stress criteria if any of the tensile, compressive or shear stresses induced exceed the allowable values then the layer is considered to be failed. In the Tsai-Wu criteria a single polynomial strength criteria is used for failure determination. It considers effect of all the stresses induced simultaneously. If this criteria exceeds the value 1 then the lamina is considered failed. Values of both the criteria are obtained from ANSYS.

## V. FINITE ELEMENT ANALYSIS

Finite element Analysis is well accepted for many industrial applications. Complex geometry or highly computationally demanding problems or indeterminate structures are solved by FEA. The FEA model is developed considering an element in the cylinder with appropriate material, loading and constrains. Consider following finite element model shown in the Fig 5.

One end of the element is fixed in the axial direction of the cylinder while the other end is free. The axial force due to pressure is applied at the free end and is calculated as

$F= (\pi \times r^2 \times p) = \pi \times 385^2 \times 0.5=232713.25$ N

i.e. projected area multiplied by exerted pressure. The rotation about global Z direction is also fixed. The displacement in X direction which is the circumferential direction is fixed at both the element nodes. Nodes are also coupled in the radial direction as the element is bounded by other elements and thus displacement in Z direction will be coupled.

One dimensional axi-symmetric shell 208 is used in this element model. This element is suitable for modeling thin to moderately thick axi-symmetric shell structures. The x direction in the elemental coordinate system represents the direction along the axis; the z represents the thickness or the radial direction while the elemental y axis represents the tangential or the hoop stress direction.

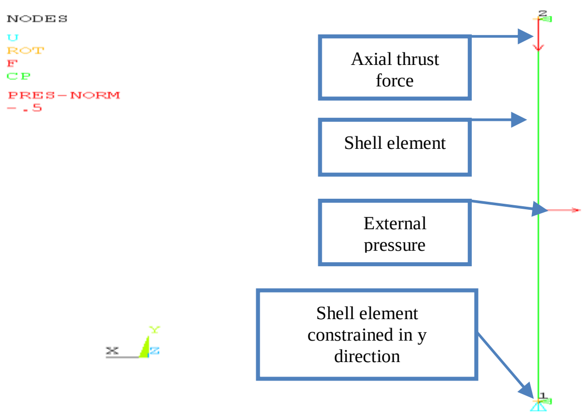


Fig. 5 Finite Element Model for cylinder under external pressure

Section command is used to define each layer for giving fiber directions, no. of layers, stacking sequence and integration points. The fibers are oriented with respect to x axis of the element coordinate system and stacking of the thickness takes place in the positive z direction. Stacking sequence is shown in the Fig.6.

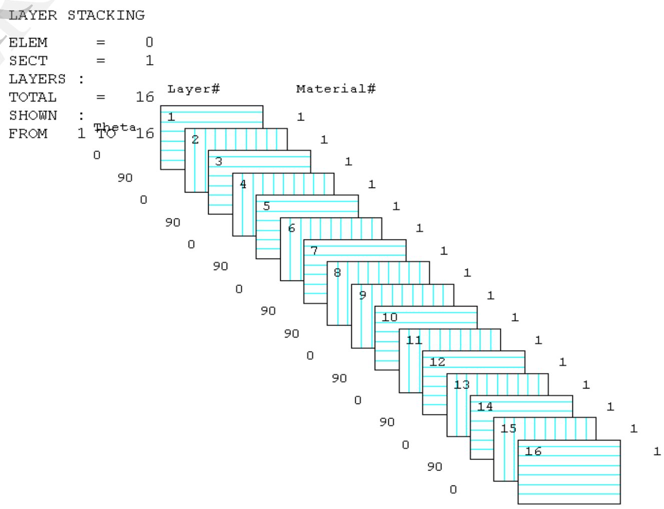


Fig. 6 Detailed materials and orientation of the laminate

The results obtained from FEA are mentioned in the TABLE III

The above FEA analysis is verified using conventional shell elements SHELL41, SHELL181 modifying the nodal coordinate system to cylindrical coordinate system along with appropriate constraints and results obtained were in good conformance with theoretically calculated results by classical laminate theory. These values obtained by the ANSYS simulation is compared and validated with the theoretical calculated stress and strains values with help of MATLAB 11.0 (as shown in the Table III).

## VI RESULTS AND DISCUSSIONS:

The layer no.1 as shown in the Fig.7 has zero degree orientation of the fiber while the layer no. 2 has the 90 degree orientation. The result for layer 1 is reported for local stress values as shown in Fig.7.It can be seen from the figure that the global transverse and longitudinal stresses are same throughout the laminate.

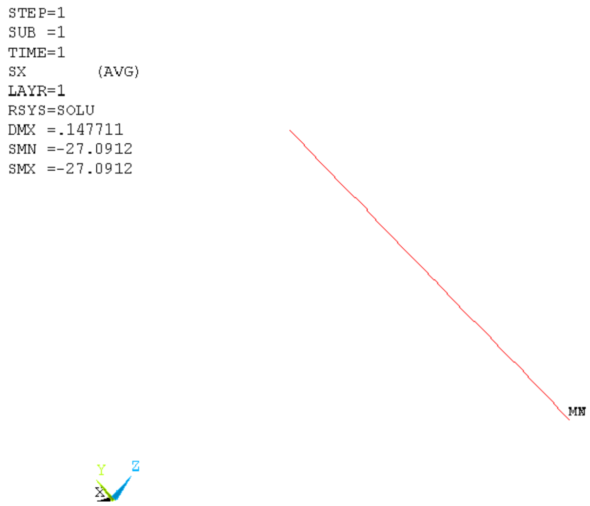


Fig.7 Layer 1(0 degree fiber), stress in x direction:

Stress along fiber direction for each layer is shown in Fig. 8 while stress in the direction perpendicular to the fiber axis is shown in Fig 9.The local strain being directly proportional to the local stress as seen in equation 14, the variation of strain along thickness would be similar. These stress variation can be compared with allowable stresses and strains in each lamina and thus load at which first ply failure initiates in one of the laminas may be calculated based on Maximum Stress Theory.

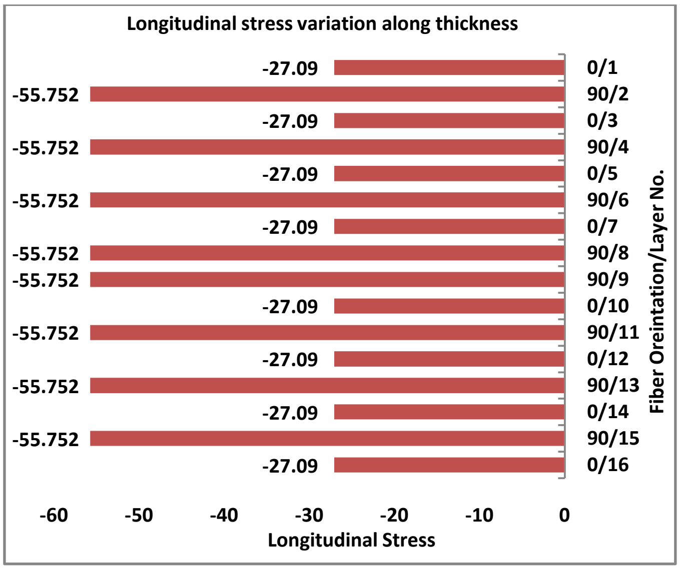


Fig. 8 Longitudinal stress variation along thickness of the laminate

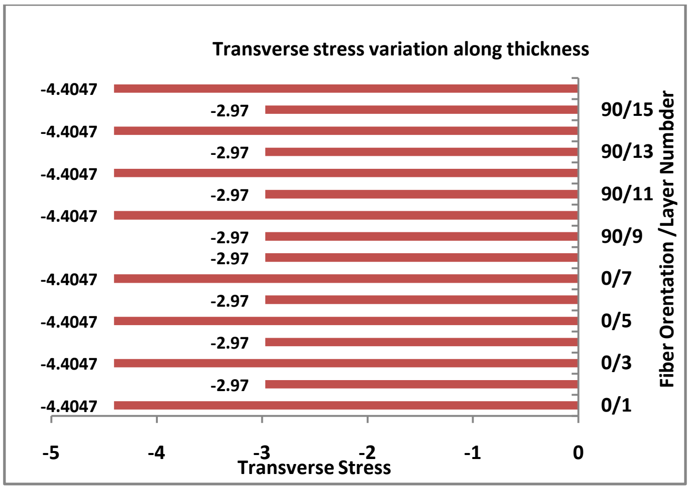


Fig.9. Transverse stress variation along the thickness of the laminate.

TABLE III
RESULT FOR STRESSES AND FAILURE CRITERIA

| Orientation | Layer No. | Longitudinal stress (MPa) | | Transverse stress (MPa) | | Max stress | Tsai Wu |
|---|---|---|---|---|---|---|---|
| | | CLT | FEA | CLT | FEA | | |
| 0 | 1,3,5,7, 10,12,1 4,16 | -27.09 | -27.091 | -4.4 | -4.405 | 0.023 | 0.051 |
| 90 | 2,4,6,8, 9,11,13, 15 | -55.68 | -55.75 | -2.97 | -2.971 | 0.035 | 0.028 |

The effect of various configurations like [(± 0)s,(± 15)s,(± 25)s,(± 45)s,(± 55)s, (± 65)s, (± 75)s, (± 85)s, (± 90)s],quasi-isotropic and cross ply on the strength of the shell is studied. Table IV lists the various test cases and the resulting maximum stress criteria values.

TABLE IV EFFECT OF STACKING SEQUENCE TO FAILURE STRENGTH OF LAMINATED CFRP CYLINDER SUBJECTED TO EXTERNAL PRESSURE.

| Laminate Arrangement | Maximum stress criteria |
|---|---|
| $(\pm 0)_s$ | 0.16 |
| $(\pm 15)_s$ | 0.141 |
| $(\pm 25)_s$ | 0.14 |
| $(\pm 45)_s$ | 0.086 |
| $(\pm 55)_s$ | 0.027 |
| $(\pm 65)_s$ | 0.036 |
| $(\pm 75)_s$ | 0.064 |
| $(\pm 85)_s$ | 0.078 |
| $(\pm 90)_s$ | 0.08 |
| CROSSPLY | 0.035 |
| QUASI ISOTROPIC | 0.042 |

As can be seen in the graph shown in Fig.13 the orientation increases from 0 degree, gradually the max stress criteria value decreases or it moves to safer values till it reaches 55 degree. If the orientation angle is further increased the maximum stress criteria value increases further. Thus the optimum orientation of fibers is around 55 degrees. It is also observed from the graph that the maximum stress criterion value obtained for cross ply laminate is relatively very close to the optimum value as compared to other configurations.

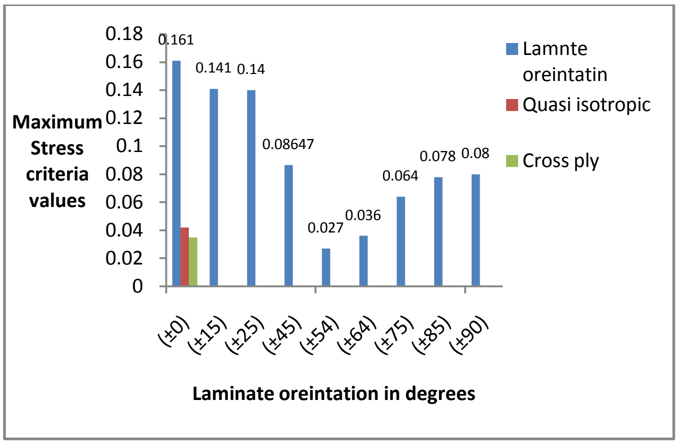


Fig. 10 Stacking orientation V/s Maximum stress criteria

The cross ply so selected is further investigated for the effect of ratio of longitudinal fibers to transverse fibers on the strength of the structure as seen in the Fig.10. To study the effect of change in ratio of longitudinal fibers to transverse fibers the numbers of transversely oriented fibers are increased symmetrically. The following cases were investigated as shown in the Table V in this process.

TABLE V EFFECT OF RATIO OF LONGITUDINAL FIBERS TO TRANSVERSE FIBERS TO FAILURE STRENGTH OF LAMINATED CFRP CYLINDER SUBJECTED TO EXTERNAL PRESSURE

| No. of layers | 16 | 16 | 16 | 16 | 16 |
|---|---|---|---|---|---|
| No. of Longitudinal to transverse layers | 8:8 | 6:10 | 4:12 | 2:14 | 0:16 |
| Ratio of longitudinal to transverse fibers | 1 | 0.6 | 0.3 | 0.14 | 0 |
| Maximum stress Criteria | 0.161 | 0.141 | 0.0865 | 0.027 | 0.036 |

Fig.11 shows that the Maximum stress criteria value decreases as we increase the number of fibers oriented in transverse direction. This value is lowest when the ratio is between 0.3 and 0.6. The failure criteria increases further as the ratio rises above 0.6. Thus the optimum ratio of longitudinal to transverse fibers is between 0.3 and 0.6.

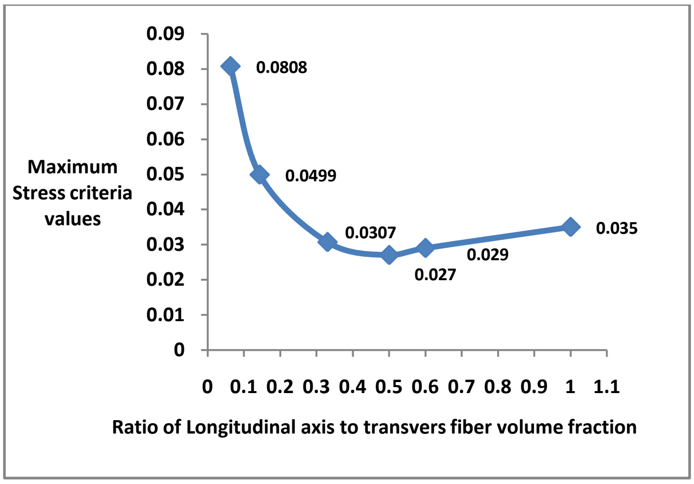


Fig. 11 Ratio of Transverse to longitudinal fiber volume fraction V/s Maximum stress criteria.

## VII . CONCLUSION AND FUTURE SCOPE:

Stress analysis was carried out using Classical Laminate Theory and validated using FEA methodology. These results were in very much conformance with each other which can be seen in Table III. Results obtained for failure criteria are very much safe for desired operating conditions of 0.5 bar.

The stress plot along the thickness indicates that the majority of the stress is taken along the fiber axis, resulting in very less stress in the direction perpendicular the fiber. This advantage is obtained because of selecting the configuration having orientation of the fibers in the direction of the principal stresses. This shows the advantage of fiber composites as they can be tailored according to the external conditions or load acting on the structure.

The above analysis carried out shows that fiber orientation near about 55 degrees gives most optimum results considering the strength criteria. In addition it was also observed that cross ply laminate configuration also gave results which were relatively very much close to the optimum value.

Further investigation was carried out for optimizing the ratio of longitudinal fibers to transverse fibers in cross ply laminate structure. It was observed that the optimum value of the ratio was between 0.3 and 0.6.

This work can be further extended for practical validation by fabrication of prototype and its hydrostatic testing.

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