Design of PID Controllers for Decoupled MIMO Systems

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Abstract- **The design of PID controllers for systems with interacting loops is discussed. It is important to deal with the interaction at the lower-level loops, since supervisory control based on for instance MPC seldom has sufficient bandwidth. A new scheme based on modified scalar PID design and static decoupling is developed, where the frequency characteristics of the coupling between the lower-level loops is taken into account. This leads to a design method emphasizing the trade-off between the individual loop performances and the so called interaction indices. The controller is easily implemented, due to its simple configuration based on standard components.**

Keywords: PID controllers, multivariable control systems, decoupling

I INTRODUCTION

Model predictive control (MPC) is becoming the standard technique to solve multivariable control problems in the process industry [12, 5,8]. Practically all MPC systems are however operating in a supervisory mode with PID controllers at the lower level. A substantial portion of the performance improvement credited toMPC is actually due to improvements in the lower-level PID loops. Interaction among the loops causes difficulties when the lower-level loops are closed. There are some difficulties in dealing with the interaction at the MPC level because the bandwidths of the MPC loops are limited: they operate in supervisory mode with sampling intervals that are longer than the PID loops.It is consequently of interest to investigate ways of dealing with interaction at the loop level. The approach we take is to investigate standard PID tuning [2] and see what can be achieved by adding simple interactions between the feedback loops. In many cases the performance of the system can improved, particularly if the coupling in the process is not severe. The proposed scheme is based on a simple decoupling, which implies that it can be easily implemented at the loop level. The advantage by doing this is that it gives performance enhancement in a frequency range that is normally not dealt with by MPC.

II. THE DESIGN PROCEDURE

The PID controller is described by

$$C = K\left(1 + \frac{1}{T_i s} + T_d s\right)$$

where K is the proportional gain, Ti is the integral time and Td is the derivative time.
The design of a decentralized control system with a decoupling matrix can be done combining a diagonal controller Kd (s) with a block compensator D(s), sothat the controller manipulates the variable ui insteadof the ui, as can be appreciated in figure 1, for the 2 x2 case.With this configuration the controller see theprocess as a set of n completely independent processes or with the interaction minimized. The essence of decoupling is the imposition of a calculation net that cancels the existent process interaction, allowing the independent control of the loops. In decentralized design, the question is not to eliminate interaction, but to take it into account. A multivariable system may still experience interactions and responds poorly. The objective in decoupling is to compensate for the effect of interactions brought about by cross coupling of the process variables.
In literature there are different decoupling methods:
Lineal decoupling (Desphande, 1989) is most extended method. In this, the decoupling matrixes try to eliminate interactions from all loops, obtaining following elements for a 2 x 2 system

$$d_{11}=1,\ d_{12}(s) = -\frac{g_{12}(s)}{g_{11}(s)},\ d_{21}(s) = -\frac{g_{21}(s)}{g_{22}(s)},\ d_{22}=1 \quad (1)$$

Implementation of this decoupling matrix has some problems: What happens if numerator has bigger order than denominator?, what if delays exist?, and what if these delays only appear in the denominator? The different works propose some solutions like Pade approximations for delays or steady state decoupling as a first option to prevent this problems. Other solutions consist of applying partial decoupling, setting d12 or d21 null. This fact avoid problems originated in one loop reach the other.

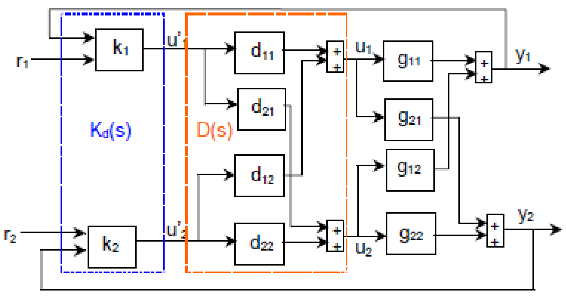

Fig. 1 : General 2 x 2 system with decouplers and single-loop controllers

There are some other methods: one of them looks for diagonal dominance in the system as ALIGN algorithm designed by McFarlane and Kouvaritakis in 1974 and described in (Maciejowski, 1989). Other design by means of singular value decomposition (Desphande, 1989) or by means of inverse decoupling (Wade, 1997) that produces the process input signals by combining one controller output with the other process input signals.Even if decouplers are incorporated, the interaction effects cannot be completely eliminated because of model mismatch. Then, single-loop controllers cannot be tuned independently, and a sequential tuning algorithm that takes interactions into account should be used. Next section describes one of these algorithms.

III. THE METHOD:

Analysis of decentralized methods described in section 1 show several interesting points:

- Obtaining different design specifications for each loop implies solution methods based on some kind of iteration. If designs are carried out loop-by-loop, tuning one of them can detune the others.
- In spite of abundance of methods, there are no simple and general solutions for the multivariable tuning problem. Or methods are used by optimization (Wang et al, 2000) that do not guarantee an appropriate solution and they require several iteration processes, or the solutions are too particular, for concrete models(Ho et al, 1996) or too simple (Shinskey, 1996),(Luyben, 1992) that only get a first approach to the problem.In (Vázquez et al, 1999), a method of tuning PID controllers for systems with decentralized control is presented. Its fundamental characteristics can be summarized in following points:

- It is a generic method for n x n systems with decentralized control.
- It is a method based on successive SISO tuning that does not suppose any additional constraintto obtained controllers (except for their decentralized structure) neither to the transfer functions matrix. This matrix can include the transfer functions of only the process or also including the decoupling net. Present work exploits this characteristic.
- The SISO methodology integrated in the algorithm has certain imposed limitations: it should allow the controller be designed from a frequential description of the process (their frequency response). And it should quantify the specifications achievement by means of a quality index, J. This index controls the iteration evolution of the tuning algorithm.
- The method allows a decoupling matrix to be included between the plant and the controllers.
- The design methodology is divided into two phases that are carried out in a sequential and combined way: the structural decomposition and the controller design. These two phases are described next.

A. *System structural decomposition*

This phase consists of the decomposition of a n x n multivariable system into n SISO systems. To solve this problem the structural decomposition, introduced by Zhu (1996), is used. Let be a n input system controlled by means of a decentralized control strategy. Now, n-1 loops have been closed by means of n-1 controllers. The process 'seen' from the free input to the free output is needed. The scheme corresponds to the figure 2, where loop i has not yet been closed (the pairing problem has been already solved and input-output pairs are in the diagonal).

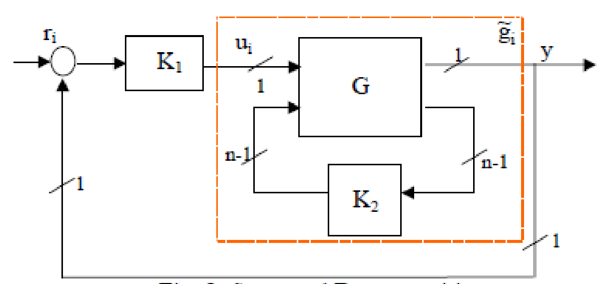

Fig. 2: Structural Decomposition

In the structural decomposition scheme, K1 is the controller that closes the loop between input i and output i. The set of n-1 controllers, whose loops are already been closed, is K2. This way, the controller matrix is:

$$K = \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix} \qquad (2)$$

The process element gii has its feedback loop open. The rest of elements of its same row are called G12. The rest of elements of its same column are called G21. And the rest of elements of G are called G22. Then, the process matrix G can be written as:

$$G(s) = \begin{pmatrix} g_{ii}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{pmatrix} \qquad (3)$$

The process seen from input i to output i when the other loops has been closed is given by

$$\tilde{g}_i = g_{ii} - G_{12}K_2\left(I + G_{22}K_2\right)^{-1}G_{21} \qquad (4)$$

where, using the above notation, g_{ii} is an element of G, that is, a SISO transfer function, G_{12} a transfer function row vector of dimension 1 x (n-1), G_{21} a transfer function column vector of dimension (n-1) x 1, K_2 is a diagonal square matrix of dimension (n-1) and I is the identity matrix of order n-1. In the case of 2 x 2 processes, elements G_{12} and G_{21} are also individual SISO transfer function elements of G, and the calculation of $\tilde{g}_i$ do not imply matrix operations.

The structural decomposition implies some important considerations for the decentralized control: some global system properties, as interaction, stability, integrity, etc, could be deduced from the properties of the obtained n SISO subsystems. Also, no supposition about G has been made, and then, structural decomposition could be applied to every transfer functions matrix. Applying Nyquist theorem to each individual loop obtained with structural decomposition, stability criterion is more interesting from the point of view of applicability to decentralized multivariable systems. Then, the n SISO systems stability implies MIMO stability

Theorem 1: Supposing that individual elements of G(s) and their SISO independent subsystems do not have poles in the right hand plane, the system with decentralized control is stable if and only if the Nyquist contour of the equivalent open loop transfer function i ig ~ k does not encircle point (-1,0), i.e.

$$N\left(-1, k_i\tilde{g}_i\right) = 0 \quad \forall i \qquad (5)$$

The structural direct Nyquist arrays (SDNA) are the representation of the n direct Nyquist diagrams of $N\left(-1, k_i\tilde{g}_i\right) = 0 \quad \forall i$.

B. *Controllers design*

The proposed is a frequency-based method and stability measures as phase and gain margins could be used. But most of existent methods requires a model for obtaining the design parameters. Carrying out the structural decomposition, the resulting transfer function i g ~ could be not a rational function, possibly due to the inclusion of delays in model systems. In order to solve this problem a frequential representation of this transfer function i g ~ it is used, i.e. a frequency response array. But this take implicit another problem: what SISO design method to use? The particular solution has been the mono variable methodology described in (Morilla and Dormido, 2000). This methodology supposes, and in fact requires, a model with the description of the system dynamics. However it leaves open the possibility to be adapted to solve the problems outlined previously it can be modified allowing the controllers to be designed from an array with the frequential representation of the frequency response and also, the obtained results can be compared with the design specifications to present an indicative of quality, indispensable for the proposed solution search method. These two phases cannot be considered isolated, since the design process consists of an iteration of both. This is because, as equation (4) shows, the gi transfer functions, obtained with structural decomposition, incorporate the controller transfer functions in their definitions. Thus, every time a design is done for one of these transfer functions, it will be needed to recalculate the others.From the new i gi, a new controller could be designed, and then, definitely, the algorithm needs an alternation between the two phases

IV. EXAMPLES

One of advantages that present this methodology of multivariable controller design is to be independent of the process model. This has been shown in numerous examples in (Vazquez et al, 1999), which can be repeated with TITO tool. Also, the model does not have reason to be a rational expression in the Laplace operator, but rather it can be a frequency response array. These characteristics allow the immediate extension of this method, using it accompanied by some decoupling strategies (one of the described in section 2 or another), so that in a first step the decoupling matrix is obtained and, in a second step, the controllers for the decoupler+process block designed. Some of the possibilities are shown in following examples.

Example 1: Following shows a system model with a RGA next to 1.7. Some decoupling strategy could be tested. It is a distillation column (Vinante and Luyben, 1972) that describes the existent dynamics between reflux and vapor flow and the temperatures of plates 4 and 17:

$$\begin{pmatrix} R \\ V \end{pmatrix} = \begin{pmatrix} \frac{2.2}{7s+1}e^{-s} & \frac{1.3}{7s+1}e^{-0.3s} \\ \frac{2.8}{9s+1}e^{-1.8s} & \frac{4.3}{9.2s+1}e^{-0.35s} \end{pmatrix} \begin{pmatrix} T_4 \\ T_{17} \end{pmatrix} \quad (6)$$

In order to analyze the effects of designing with decouplers, expressions (1) are used. In this case,dynamic and steady state decoupling coincide:

$$d_{11}=1, \quad d_{12}=-0.59, \quad d_{21}=-0.65 \quad \text{and} \quad d_{22}=1 \qquad (7)$$

Design specifications are phase margin (PM) of 45º and gain margin (GM) of 4 for both loops, and looking for a PID with a limit of the relationship between the derivative and integral constant equal to 0.01. The design using iterative algorithm without decouplers gets following controllers:

$$k_1 = 0.88\left(1+\frac{1}{1.82s}+0.035s\right) \quad \text{and}$$

$$k_2 = 2.70\left(1+\frac{1}{1.79s}+0.073s\right) \qquad (8)$$

Design specifications are met with combined (PM and GM) tuning design in three iterations. Interaction impedes using other methods based on Gershgorin bands, as Ho's method Next, a complete decoupling is chosen. Design specifications are the same as before (PM=45º and GM=4 for both loops). The algorithm converges insame number of iterations as before, obtaining following controllers

$$k_1 = 0.964\left(1+\frac{1}{2.81}+0.057s\right) \quad \text{and}$$

$$k_2 = 0.967\left(1+\frac{1}{0.766s}+0.014s\right) \qquad (9)$$

With this design, specifications are also met, but now, controlled system was decoupler + process block. Time responses are shown in figure 3. Response without decoupler is also superimposed.

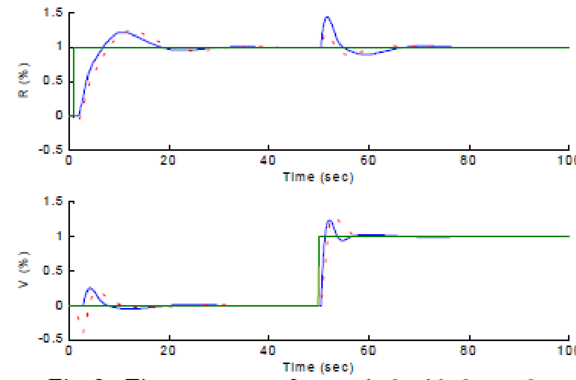

Fig. 3: Time response of example 1 with decoupler and without decouplers (continuous)

A slight modification can be appreciated, mainly in second loop, and an interaction decrease on the first one.Although modifications in time response are minimum, design is good to show methodology effectiveness when decouplers are incorporated.

Example 2: Table 1 shows the results of PID controller design for the system

$$\begin{pmatrix} X_D(s) \\ X_B(s) \end{pmatrix} = \begin{pmatrix} \frac{12.8}{16.7s+1}e^{-s} & \frac{-18.9}{21s+1}e^{-3s} \\ \frac{6.6}{10.9s+1}e^{-7s} & \frac{-19.4}{14.4s+1}e^{-3s} \end{pmatrix} \begin{pmatrix} F_R(s) \\ F_S(s) \end{pmatrix} \quad (10)$$

for different decoupling nets (dynamic, in steady state, partial and total) and for different controllers (PI, PID). All the designs have been carried out with specifications of PM=60º and GM=4 for both loop. It is a water-methanol distillation column (Wood and Berry, 1973) analyzed in numerous later works (Ho et al, 1996), (Toh and Devanathan, 1993), (Vázquez et al, 1999) (it appears in the practical whole of multivariable control references).

Table 1: COMPARATIVE OF DIFFERENT DECOUPLERS AND DESIGN OF EXAMPLE 2

d_{12}	d_{21}	Kp_1	Ti_1	Td_1	Kp_2	Ti_2	Td_2
0	0	0.51	12.62	0	-0.027	2.46	0
18.9/12.8	0	0.48	8.05	0	-0.030	8.94	0
0	6.6/19.4	0.35	9.82	0	-0.031	2.39	0
18.9/12.8	6.6/19.4	0.34	6.11	0	-0.027	9.19	0
$\frac{18.9(16.7s+1)}{12.8(21s+1)}$	0	0.49	8.72	0	-0.038	9.90	0
0	$\frac{6.6(14.4s+1)}{19.4(10.9s+1)}$	0.33	7.60	0	-0.033	2.925	0
$\frac{18.9(16.7s+1)}{12.8(21s+1)}$	$\frac{6.6(14.4s+1)}{19.4(10.9s+1)}$	0.32	5.67	0	-0.031	8.08	0
$\frac{18.9(16.7s+1)}{12.8(21s+1)}$	$\frac{6.6(14.4s+1)}{19.4(10.9s+1)}$	0.34	4.91	0.34	-0.046	9.626	0.759

Figure 4 shows superimposed time responses of the system without decouplers and with total dynamic decouplers, with a PI and a PID (the two last designs of table 1). The PID design has been carried out limiting α to 0.25

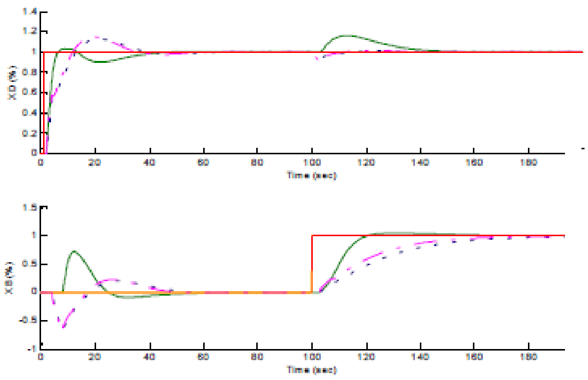

Fig. 4: Time response with decouplers (PI, and PID) and without decouplers (continuous).

Note that interaction effects decrease not very significantly. This same reduction could be obtained tuning the second loop in a less aggressive way (for example PM=80º), which would reduce its effect on first loop.

Example 3: This third example studies a process proposed by Niederlinski (1971) and analyzed with a set of decouplers proposed by Shiu and Hwang (1998). The process function transfer matrix is:

$$\begin{pmatrix} y_1(s) \\ y_2(s) \end{pmatrix} = \begin{pmatrix} \dfrac{0.5}{(0.1s+1)^2(0.2s+1)^2} & \dfrac{-1}{(0.1s+1)(0.2s+1)^2} \\ \dfrac{1}{(0.1s+1)(0.2s+1)^2} & \dfrac{2.4}{(0.1s+1)(0.2s+1)^2(0.5s+1)} \end{pmatrix} \begin{pmatrix} u_1(s) \\ u_2(s) \end{pmatrix} \quad (11)$$

Shiu proposes the following decoupler, obtained after identifying the i g ~ functions applying a relay method:

$$d_{12} = \frac{2(0.587s+1)(0.0676s+1)}{(0.379s+1)(0.2x0.0676s+1)} \quad \text{and}$$

$$d_{21} = -\frac{0.417(1.06s+1)(0.197s+1)}{(0.48s+1)(0.2x0.197s+1)} \quad (13)$$

In order to compare time responses with and without decoupler, the tuning algorithm has been used with the same design specifications (PM = 60º for both loops). Without decoupler, controllers are

$$k_1 = 1.11\left(1+\frac{1}{0.67s}\right) \quad \text{and}$$

$$k_2 = 0.27\left(1+\frac{1}{0.88s}\right) \quad (13)$$

obtained in six iterations of the algorithm. With decouplers, the controller equations are

$$k_1 = 0.91\left(1+\frac{1}{0.23s}\right) \quad \text{and}$$

$$k_2 = 0.26\left(1+\frac{1}{0.37s}\right) \quad (14)$$

obtained in only three iterations, because, in general,the algorithm converges quicker the more dominant the system is. Figure 5 shows time response, with and without decouplers. Note that interaction decreases. If this fact is quantified by means of some measure as the IAE test, it can be proven that in the response without decouplers, IAE is 3.42, while when decouplers are applied it is 0.98.

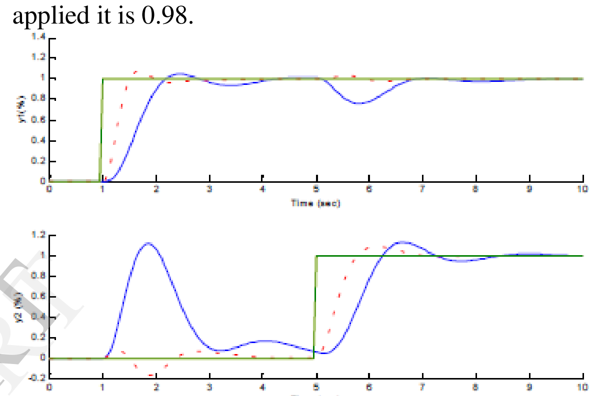

Fig. 5: Time response of the system of example 3, with and without decouplers (continuous).

V. Conclusions

The suggested control scheme is a quite simple multivariable controller. However, it handles a large number of practical control problems, although it can be implemented with regular PID controllers together with a proportional controller D. It is desirable to have efficient design methods for decoupled PID control. Such a method has been developed in the paper, under the assumption that the interaction is not too severe. It takes advantage of methodology proposed in (Vázquez et al, 1999), and it does not need a transfer function matrix with the system model but a representation with frequency response. Independently of how decouplers had been obtained, they can be included between process and controllers. Then, a decentralized PID controller is designed, using some of the different possibilities of proposed algorithm: design with only phase margin specifications for both loops, only gain margin, and a combination of PM and GM specifications.

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