Design of Output Feedback Compensator

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Abstract

This paper discusses the problem of stabilization of physical systems of a very high order and the reduced order model of the higher order systems. The methodology uses the concept of parameterization of all compensators that stabilize a given plant. The output feedback compensator is parameterized by one parameter. It stabilizes the higher order system as well as its reduced order model. The compensator design method does not involve higher order system for its computation.

Keywords: Output feedback; Compensator; Reduced order model.

1. Introduction

The mathematical models of many engineering systems are frequently of high dimension. These models are often too large for the purpose of, analysis and controller design. The cost and complexity of the controller increases as the system order increases [1-3]. This problem can be overcome if a 'good' reduced order model is available for the original higher order system and if it is possible to design a controller using the reduced model, which will stabilize the original higher order system when placed in the closed-loop. Hence for the cost and time saving in design, and for simplifying implementation, reduced order models are highly desirable for engineers in analysis, synthesis, and simulation of complicated high order system.

Model reduction concept originated towards the end of the 19th century when Pade approximations were introduced in [4]; interest of researchers in this very important field was spurred after the work of Rosenbrock [5] on distillation columns. Today, there exist a variety of concepts and techniques that have a common goal of reducing the dimension of the mathematical model of a large-scale system in order to simplify the design of control and estimation schemes. Among the important methods are Pade approximation [6], Routh approximations [7], moment matching techniques [8] and balanced realization [9,10]. A good review on the available techniques and comparison can be found in

Mahmoud [11] and Fortuna [12]. Linear matrix inequality based model reduction techniques have been developed in [13] and improved Routh-Pade approximants has been given in [14]. Its limitation is that it works for stable systems only. Reduced order controller has been designed for nonlinear systems also [Astolfi]. A very less effort has been put to use the reduced models for designing controller for the higher order system. This is primarily due to the fact that if a stabilizing controller is designed from the reduced model and applied to the higher order system, it does not guarantee the stability of the closed- loop system [15,16]. In [3], Lamba shows that, if a state feedback control is designed from the Davison [17] and applied to the higher order system, it results in a stable closed-loop system. But the method needs the system states to be fully available for feedback. Such a controller cannot be implemented for a system where some of the state variables are not accessible. Some other methods are reported in [18-20].
This paper describes a technique for designing a stabilizing controller for a higher order system using its reduced order model. The method uses the parameterization of all compensators that stabilize a given plant [21]. It is shown that the output feedback compensator, which is obtained from the reduced model, not only stabilizes the reduced model but also the higher order system.

2. Problem Statement

Let a stable single-input single-output (SISO) linear time-invariant system of order 'n' is represented as

$$G(s) = n_p(s)/d_p(s) \tag{1.1}$$

$G(s)$ represents a higher order system whose reduced order model of order 'm' is represented by

$$M(s) = n_m(s)/d_m(s) \tag{1.2}$$

We assume that there are no pole–zero cancellations. The problem is to find a controller using the reduced model that will stabilize both the higher order system $G(s)$ and its reduced order model $M(s)$ and the order of compensator should be less than the order of the higher order system $G(s)$.
The technique of parameterization of all stabilizing controllers for a system [21] is used to design the controller for higher order system via its reduced model. Assume that a given system $P(s) \in \Re(s)$ and is expressed as a ratio of rational function as:

$$P(s) = n(s)/d(s); n(s), d(s) \in S \tag{1.3}$$

Let $C^{'}(s) \in \Re(s)$ be a stabilizing controller of system $P(s)$ expressed as a ratio of rational functions

$$C^{'}(s) = n_c^{'}(s)/d_c^{'}(s); n_c^{'}(s), d_c^{'}(s) \in S \tag{1.4}$$

where $\Re(s)$ is the set of real rational functions in the variable s and S is the subset of $\Re(s)$ consisting of all rational functions that are bounded at infinity and whose all poles have negative real parts .
The system (1.3) and the stabilizing controller (1.4) are expressed as a ratio of two stable rational functions, which is called factorization approach [21]. Fractional factorization of

rational functions in S is non unique [21,22]. The stable rational functions in (1.3) and (1.4) are selected such that they satisfy Bezout identity.

$$n_c^{'}(s)n(s)+d_c'(s)d(s)=1 \tag{1.5}$$

Assume that $P(s)\in\Re(s)\dfrac{n!}{r!(n-r)!}$ and let $C'(s)\in\Re(s)$ be a stabilizing controller of $P(s)$. Let $n(s)/d(s)$ and $n_c^{'}(s)/d_c^{'}(s)$ be fractional factorizations of $P(s)$ and $C'(s)$ in S. The set of all stabilizing controllers of $P(s)$ is given by

$$Stab(P(s))=\left\{\frac{n_c'(s)+rd(s)}{d_c'(s)-rn(s)}: r\in S \ and \ d_c'(s)-rn(s)\neq 0\right\} \tag{1.6}$$

To each $r\in S$ corresponds a stabilizing controller and to each stabilizing controller corresponds a stable rational function $r\in S$.

3. Compensator Design

In this section, the above concept of controller parameterization is used to derive a controller for higher order system using its reduced order model.

As the higher order system is stable we can choose rational functions as:

$n_1(s)=G(s), d_1(s)=1, x_1(s)=0$ and $y_1(s)=1$,

then they satisfy Bezout identity $x_1(s)n_1(s)+d_1(s)y_1(s)=1$.

Similarly for reduced order plant, if we choose

$n_2(s)=M(s), d_2(s)=1, x_2(s)=0$ and $y_2(s)=1$

then they satisfy Bezout identity $n_2(s)x_2(s)+d_2(s)y_2(s)=1$.

We can take $G/1$ and $M/1$ as the fractional factorization of the system and its reduced model respectively. As G and M are stable, there exists constants k_1 and k_2 which stabilize the system and the model respectively as shown in Fig. 1.1 and Fig. 1.2.

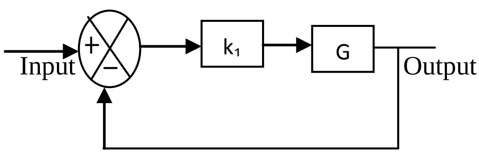

Fig 1.1: Higher order system with constant gain k_1

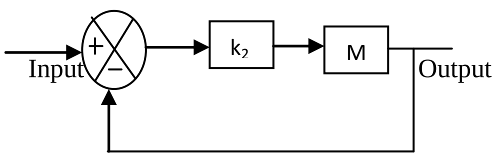

Fig 1.2: Reduced order model with constant gain k_2

Let the fractional factorization of k_1 and k_2 be $k_1/1$ and $k_2/1$ are trivial factorization respectively. From (1.6), the set of stabilizing controller for the system G(s) is

$$Stab(G(s)) = \left\{ \frac{k_1 + r_1}{1 - r_1 G} : r_1 \in S \right\} \tag{1.7}$$

Similarly the set of stabilizing controller for the reduced model is

$$Stab(M(s)) = \left\{ \frac{k_2 + r_2}{1 - r_2 M} : r_2 \in S \right\} \tag{1.8}$$

Given the plant and reduced model G(s) and M(s), which are to be stabilized,the parameterization of two compensators, which individually stabilize those plants, can be equated to find the common stabilizing compensator. We get, $\frac{k_1 + r_1}{1 - r_1 G} = \frac{k_2 + r_2}{1 - r_2 M}$

Solving for r_1, $$r_1 = \frac{k_2 + r_2 + k_1 r_2 M - k_1}{1 - r_2 M + G k_2 + r_2 G} \tag{1.9}$$

Theorem

If $d_m d_p - n_m d_p + n_p d_m k_2 + n_p d_m$ is Hurwitz then there exists an mth order compensator $C(s) = \frac{(k_2 + 1) d_m}{d_m - n_m}$, which stabilizes the higher order system and its reduced order model.

Proof: From (1.9) we have $r_1 = \frac{k_2 + r_2 + k_1 r_2 M - k_1}{1 - r_2 M + G k_2 + r_2 G}$

for simplicity we take $r_2 = 1 \in S$ then $r_1 = \frac{k_2 + 1 + k_1 M - k_1}{1 - M + G k_2 + G}$

as $M = n_m / d_m$ and $G = n_p / d_p$, we get

$$r_1 = \frac{(k_2 d_m + d_m + k_1 n_m - k_1 d_m) d_p}{d_m d_p - n_m d_p + n_p d_m k_2 + n_p d_m} \tag{1.10}$$

Hence if $d_m d_p - n_m d_p + n_p d_m k_2 + n_p d_m$ is Hurnitz then $r_1 \in S$. From (1.8).

$Stab(M(s)) = \left\{ \frac{k_2 + r_2}{1 - r_2 M} : r_2 \in S \right\}$ as $r_2 = 1$ and $M = \frac{n_m}{d_m}$

so mth order compensator

$$C(s) = \frac{(k_2 + 1) d_m}{1 - n_m} \tag{1.11}$$

stabilizes both the system and the reduced order model of the system. □

Remark 1: From (1.11) we see that, for designing a controller for higher order system, only a reduced order model and checking Hurwitzness of a certain polynomial is involved in computation. Hence, the complexity in obtaining a controller for higher order system is greatly reduced and further we get a controller of lower order.

Remark 2: The compensator in (1.11) is parameterized by one parameter k_2. The range of k_2 for which the feedback system shown in Fig.1.2 is stable can be obtained. Similarly the range of k_2 for which the polynomial $d_m d_p - n_m d_p + n_p d_m k_2 + n_p d_m$ is stable can be obtained. From these the range of k_2 can be obtained for which $C(s) = \frac{(k_2 + 1) d_m}{1 - n_m}$

stabilizes the system and the reduced order model. In this approach k_1 is not required to be computed.

4. Example

Consider the continuous time system having three poles of relatively different order of magnitude described by the following transfer function [23]

$$G(s)=\left[\frac{1}{(s+5)(s+1)(s+0.1)}\right]$$

The transfer function of reduced order model is

$$M(s)=\frac{-0.1835s+0.1938}{s+0.0969}$$

The range of k_2 for which reduced model is stable is $k_2>-0.5$.

From the polynomial $d_m d_p - n_m d_p + k_2 n_p d_m + n_p d_m$,the constraints on k_2 for the stability are: $-0.1662k_2+5.8622>0$,

$$\frac{-0.1662k_2^2+4.9977k_2+5.8046}{-0.1662k_2+5.8622}>0 \quad \text{and} \quad 0.0485+0.0969k_2>0$$

To satisfy all the constraints $k_2>-0.5$

Thus a compensator $C(s)=\dfrac{(k_2+1)(s+0.0969)}{(1.1835s-0.0969)}, k_2>-0.5$

is a stabilizing controller for the given higher order system and its reduced order model.

If $k_2+1=k$ then $C(s)=\dfrac{k(s+0.0969)}{(1.1835s-0.0969)}, k>0.5$

The closed-loop responses of the higher order system and its reduced order model with this compensator for $k=1.6$ is shown in Fig. 1.3.

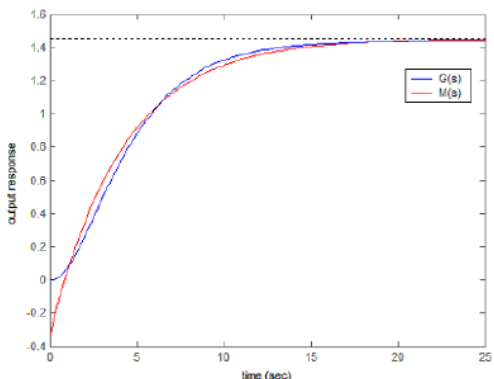

Figure 1.3: Closed loop responses of the system G(s) and reduced model M(s)

5. Conclusions

The effectiveness of a reduced order model is established, as a controller designed from the reduced model works satisfactory when applied to the higher order system. The controller design method does not involve higher order system for its computation and it ensures the stability of the closed-loop system. Another feature of this method is that the order of the compensator is equal to the order of the reduced model. The efforts required to design the compensator through the reduced model are very less as compared to the one, which will involve higher order system for its computation.

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