

Design of Mechanical Drives for a Parabolic Radio Antenna

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Abstract-The paper proposes mechanical drive systems for rotating a radio antenna having dish diameter 5 meters, in the azimuth and elevation axes. The antenna dish is parabolic in shape and is estimated to have a maximum weight of 350 kilograms. Conventional drive systems comprise of a gear pair mechanism for turning the dish through the requisite angle of rotation. These systems are precise and highly efficient, thereby facilitating an extremely high margin of accuracy of the observed data. However, the most significant drawback of a gear-pair drive system is its high initial investment and subsequently, high maintenance cost. To overcome this limitation, a rope and pulley drive system is considered, which reduces the cost exponentially without affecting the accuracy of the data to a large extent.

Keywords - *Rope drives, azimuth drive system, elevation drive system, wind torque.*

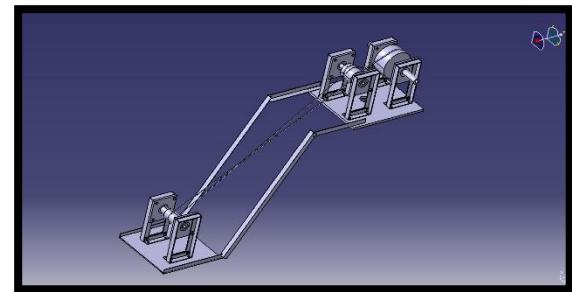
I. INTRODUCTION:

Radio telescope antennas are an important tool in radio astronomy, which relies heavily on observational data. The radio telescope for which the drive system has been designed is used to track celestial bodies and is therefore, required to turn through a maximum rotation of $\pm 270^\circ$ in the azimuth axis and through 0° to 90° in the elevation axis in a continuous, uninterrupted and smooth manner.

The most important consideration that has to be taken into account while designing the drive is the combined torque exerted by the wind on the dish and that due to the mass moment of inertia of the dish itself. The schematic models of the drive assemblies are specified as follows.

A. AZIMUTH DRIVE

Power is supplied by a hand driven winch to the driving pulley, which then transmits the same to the driven pulley. The driven pulley is connected by a shaft to the top flange on which the hub of the dish rests. Rotational motion is thus imparted to the parabolic dish antenna.



B. ELEVATION DRIVE

The general procedure to be followed while designing the drive is as follows:

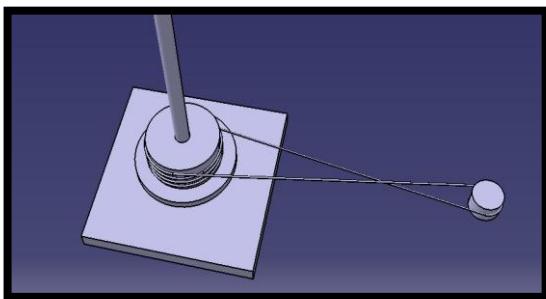
- i. Total torque exerted on the parabolic dish and hence, the azimuth drive, including the wind torque and the torque due to the mass moment of inertia of the dish, is calculated.
- ii. This torque is used to find the actual and design power required to turn the dish through the requisite degree of rotation.
- iii. The power calculated gives the belt tensions exerted on the driving shaft, which in turn is used to determine shaft diameter.
- iv. Furthermore, the dimensions of other supplementary components such as bearings, flanges, bolts, etc. can be found out.

The design process is largely iterative, varying according to the material used and the factor of safety required by individual requirement. The material used for this particular design process is M.S. ($S_{ut} = 400\text{N/mm}^2$) and the factor of safety is assumed to be 1.5.

II. DESIGN OF AZIMUTH DRIVE

1. Calculating wind torque on dish:

$$F = \eta_1 \eta_2 PA$$



Where, η_1 = porosity/solidity ratio = 0.31

η_2 = factor taking into account inclination of wind w.r.t. antenna

P = Pressure on dish

A = Area of dish

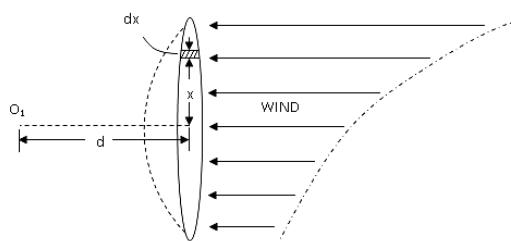
$$P = 1/2 * \rho * v_w^2$$

ρ = air density (1.123 kg/m³)

v_w = wind velocity (80 kmph)

$$P = 277.228 \text{ Pa}$$

For perpendicular attack of wind,



Distribution of Wind Load in Perpendicular Direction

$$\eta_2 = 1$$

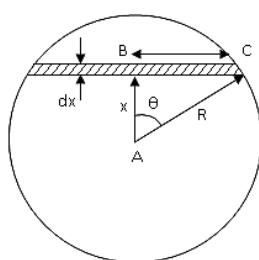
d = depth of antenna + height of hub = 1.055 m

Let, O be the point about which moment is to be calculated.

A be the center of the dish

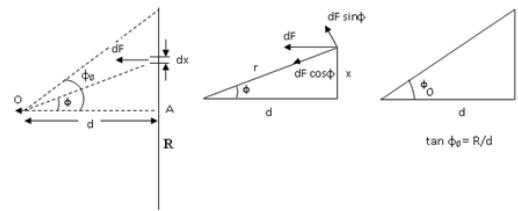
Φ be the angle subtended by differential element at O

Θ be the angle subtended by elemental strip at center of circular cross-section A



$$dA = -2R^2 \sin^2 \Theta d\Theta$$

$$dF = 1/2 * \rho * v_w^2 * \eta_1 * dA$$



$$\tan \phi_0 = R/d = 67.12^\circ$$

$$d\tau_{up} = dF \sin \phi * r \quad (\text{Since } r = d/\cos \phi)$$

$$= dF * \tan \phi * d$$

$$d\tau_{up} = 85.94 * (-2R^2) * \sin^2 \Theta * \tan \phi * d * d\phi$$

$$= -1074.25 * \sin^2 \Theta * d * \tan \phi * d\phi$$

Integrating over limits 0 to τ_{up}

$$\int_0^{\tau_{up}} d\tau_{up} = -1074.25$$

$$\iint_{\phi=0, \theta=0}^{\phi=0, \pi/2} \sin^2 \Theta d\Theta \tan \phi d\phi$$

Solving, $\tau_{up} = 826.225 \text{ Nm}$ - Wind torque for upper half of dish

Assuming same torque on lower half, total torque due to wind = 1652.45 Nm.

2. Torque due to rotation of dish

$$\tau = I * \alpha$$

Mass moment of inertia of dish I = $\int r^2 dm$

$$dm/dA = \text{surface density } (\sigma) = M/A$$

$$\text{Hence, } dm = \sigma dA = \sigma * dx * (H - ax^2)$$

Calculating limits for x, for vertical parabola,

$$y = ax^2$$

$$x = \sqrt{y/a}$$

for y = H i.e. maximum depth of dish and a = 8 from general equation of parabola

$$x = \pm \sqrt{0.655/8} = \pm 2.5$$

Therefore,

$$I = \int_{-2.5}^{+2.5} \sigma x^2 (H - ax^2) dx = 2 \int_0^{+2.5} \sigma x^2 (H - ax^2) dx$$

$$\text{Solving for } I, I = 0.04585 \text{ kg-m}^2$$

Maximum rotation speed = 1 rpm

$$\omega = 0.104719 \text{ rad/s}$$

$$\text{Angular acceleration, } \alpha = \omega^2/2\Theta$$

$$\Theta = 270^\circ$$

$$\alpha = 0.01111 \text{ rad/s}^2$$

Hence calculating torque,

$$\tau = I * \alpha = 5.094 * 10^{-4} \text{ N-m}$$

Total torque exerted on drive = Wind Torque + Torque due to dish

$$= 1652.45 \text{ N-m.}$$

Hence, power required to drive the parabolic dish (P),

$$P = \frac{2\pi NT}{60} = 173.044 \text{ W}$$

3. Design of Crossed Belt Drive

The torque exerted on the driven pulley is the summation of the torque due to wind and that due to the mass moment of inertia of the dish. Given that,

Torque on driven pulley (T_1) = 1652.45 N-m.

Power required to drive the pulley (P_1) = 173.044 W

Speed of driven pulley (N_1) = 1 rpm

Velocity of belt $v = r\omega = 0.0131$ m/s

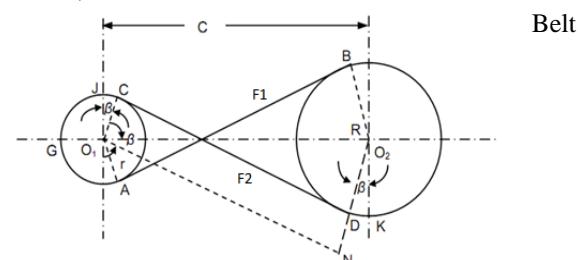
Let, Driven pulley diameter (D_1) = 250 mm.

Driving pulley diameter (D_2) = 50 mm.

Center to center distance (C) = 1000 mm. Since velocity of the driving and driven pulleys is the same,

Design Power = Service Factor (c_s) * Required Power

Hence, $P_{des} = 1.5 * 173.044 = 259.566$ W



Tensions,

$(P_1 - P_2) * v = \text{Power (W)}$

$P_1 - P_2 = 19814.2$

Coefficient of friction (μ) = 0.2

For a crossed belt drive,

$$\beta = \sin^{-1} \frac{(D_1 + D_2)}{2C} = 8.627^\circ$$

Angle of lap,

$$\alpha_1 = \alpha_2 = 180^\circ + 2\beta = 3.4427 \text{ rad}$$

$$\frac{P_1}{P_2} = e^{2.92 * \mu * \alpha}$$

Calculating belt tensions P_1 and P_2

$$P_1 = 22877.95 \text{ N}; P_2 = 3063.75 \text{ N}$$

Calculated Rope Length =

$$L_0 = \frac{\pi * (D_1 + D_2)}{2} + 2 * C + \frac{(D_1 - D_2)}{4 * C} = 2.4713 \text{ m}$$

Shortening the belt length by 1%,

$$L_0 = 2446.58 \text{ mm}$$

4. Shaft Diameter Calculation for Axial load.

Let weight of dish = 350 kg = 3433.5 N

Dead weight of flanges = 12 kg = 117.72 N

Total load on shaft = 3551.22 N

Area of dish = 19.256 m²

Material: M.S. Plain Carbon Steel

$Syt = 380 \text{ N/mm}^2$; $Sut = 680 \text{ N/mm}^2$

$FOS = 2$

Max shear stress = $0.5Sut/FOS = 95 \text{ N/mm}^2$

$\tau = F/A$

$$A = 37.38 \text{ mm}^2 = d^2 * \pi/4$$

$$\text{Hence, } d = 6.8988 \text{ mm} = 10 \text{ mm}$$

5. Shaft Diameter Calculation for Bending load

Total bending load on shaft = $F_1 + F_2 = 25941.7 \text{ N}$

Minimum distance between bearings = 200 mm and shaft length = 1000 mm

Calculating reactions at bearings:

$$R_1 = R_2 = (F_1 + F_2)/2 = 12970.85 \text{ N}$$

$$M_t = (P_1 - P_2) * r = 2476775 \text{ N-mm}$$

$$M_b = R_1 * 100 = 1297085 \text{ N-mm}$$

The material selected for the shaft is M.S.

$$S_{ut} = 400 \text{ N/mm}^2 \text{ FOS} = 1.5$$

$$\tau = 0.5 * S_{ut}/FOS = 133.33 \text{ N/mm}^2$$

$$\tau = \frac{16 * \sqrt{(M_t K_t)^2 + (M_b K_b)^2}}{\pi * d^3}$$

Where, Fatigue factor (K_t) = 1;

Shock factor (K_b) = 1.5

Hence,

$$d = 49.366 \text{ mm} \approx 50 \text{ mm}$$

6. Design of Rigid Flange Coupling

Outer Diameter of hub (d_h) = $2 * d = 100 \text{ mm}$

Length of hub (l_h) = $1.5 * d = 75 \text{ mm}$

PCD of bolts (D_b) = $3 * d = 150 \text{ mm}$

Thickness of flange (t) = $0.5 * d = 25 \text{ mm}$

Protecting rim Thickness (t_1) = $0.25 * d = 12.5 \text{ mm}$

Spigot recess Diameter (d_r) = $1.5 * d = 75 \text{ mm}$

Outer diameter of flange (D_o) = $(4 * d + 2 * t_1) = 225 \text{ mm}$.

7. Design of Bolts

As the shaft diameter is 50 mm,

Number of bolts (N) = 4 (standard value)

σ_c permissible = 200 Mpa

$$\text{Bolt diameter } (d_1)^2 = \frac{8 * M_t}{(\pi * D * N * \tau)}$$

Thus, $d_1 = 8.37 \text{ mm} = 10 \text{ mm}$.

$$\sigma_c = 2 * M_t / (N * t_1 * t * D)$$

$$= 22.0157 \text{ Mpa} < 200 \text{ Mpa}$$

8. Design of Bearings

The vertical shaft is subjected to both radial and axial loads. Hence, taper roller bearings are preferred as compared to ball bearings. The main advantage of using taper roller bearings is that they are able to withstand combined axial and radial loads since the line of action of the resultant reaction on the bearing can be resolved into separate axial and radial components.

Bore of the bearing = 100 mm.

The antenna is to be operated continuously i.e. 24 hrs per day, hence the life of bearing is assumed to be 40,000 hrs.

$$L_{10h} = 40,000 \text{ hrs}$$

$$\text{Consequently, } L_{10} = \frac{60 * n * L_{10h}}{10^6}$$

Where, L_{10} = Rated bearing life (in million revolutions)

n = speed of rotation (rpm)

$$L_{10} = \frac{60 * 1 * 40,000}{10^6} = 2.4 \text{ million revolutions}$$

For taper roller bearings,

$$P = F_r \quad \text{if} \quad \frac{F_a}{F_r} \leq e$$

$$P = (0.4*F_r) + (Y*F_a) \quad \text{if} \quad \frac{F_a}{F_r} > e$$

Where, P = Equivalent dynamic load (N)

F_r = Radial load (N)

F_a = Axial or thrust load (N)

These equations are based on the assumptions that, for taper roller bearings,

- The both bearings are exactly identical and bearings are adjusted against each other to give zero clearance in operation without pre-loading.

$$\text{Now, } F_r = (P_1 + P_2)/2 = 12970.85 \text{ N}$$

$$F_a = \frac{0.5*F_r}{Y} = 4988.788 \text{ N}$$

$$\frac{F_a}{F_r} = 0.3846$$

The bearing selected is Taper roller bearing no. 32020X from the bearing catalogue,

Bearing specifications:

Inner diameter of bearing (d) = 100 mm

Outer diameter of bearing (D) = 150 mm

Axial width of bearing (B) = 32 mm

Dynamic load capacity (C) = 161,000

e = 0.46 Y = 1.3

Winch Specifications:

Weight = 50 Kg; Reduction Ratio = 14:1

$$M_t = 5199833.38 \text{ N-mm.}$$

$$P = \frac{2\pi NT}{60e3} = 544 \text{ W}$$

2. Wind torque for dish at 60°

$$M_t = 3700532.75 \text{ N-mm.}$$

$$P = \frac{2\pi NT}{60e3} = 387.51 \text{ W}$$

3. Wind torque for dish at 90°

$$M_t = 1652450 \text{ N-mm.}$$

$$P = \frac{2\pi NT}{60e3} = 173.044 \text{ W}$$

From the above values, it is clearly seen that the maximum power required is when the dish is inclined at an angle of 30° w.r.t. horizon.

Assumptions for design

The first pulley is considered of diameter of 300mm since it is available easily in market for its size. Besides, there is a space constraint layout to be followed which limits the size of the first pulley mounted on the base yoke.

The wire rope used for this drive system is of following specifications taken from IS 2266:2002 manual

Type: 6x36 (14-7-7-1)

Wire diameter (d_{wire}): 10mm

Rope grade: 1960

Approximate mass: 41.8 kg/100mm

Minimum breaking force: 70 KN

Type of lay: Right Regular Lay.

2. Crossed belt drive design:

a) Stage no.1:

Where D- diameter of first pulley,

N- rpm of first pulley

$$P (W) = (P_1 - P_2) * V$$

- Given:

Diameter of driven pulley (D) = 300 mm

Diameter of driving pulley (d) = 150 mm

Speed of driven pulley (N) = 1 rpm

Center to center distance between pulleys(C) = 475 mm

$$V = \frac{\pi DN}{60e3} = 0.0157 \text{ m/s}$$

$$P (W) = (P_1 - P_2) * V$$

$$P_1 = 34649.68 + P_2$$

- Rope tension ratios:

Where, P_1 and P_2 – Rope tensions

$$P_1/P_2 = e^{fu/\sin(\theta/2)} = e^{(2.92*fu)}$$

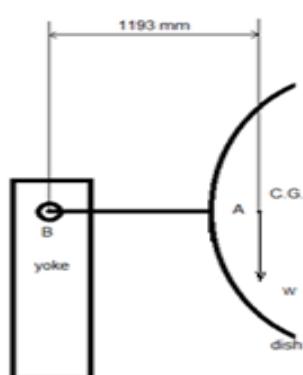
Assume, $\theta = 40^\circ$ $f = 0.2$

$$A = 180 + 2\sin^{-1}\left(\frac{D+d}{2C}\right) + 360 = 10.411 \text{ rads.}$$

$$P_1 = 437.08 P_2$$

$$P_2 = 79.47 \text{ N}; \quad P_1 = 34729 \text{ N}$$

Hence the tensions in the rope wound between 1st and 2nd pulley would be 34729 N on tight side and 79.47 N on slack side.



1. Torque calculations:

Given: Wind torque = 1652.45e3 N-mm.

$$M_t = (\text{Bending moment of dish} * 9.81) + \text{Wind torque}$$

1. Wind torque at 30°

- Design of Shaft 01:

Assumption: The length of shaft = 300mm

$$M_b = \frac{P_1 + P_2}{2} \times \frac{300}{2} = 3111193.82 \text{ N-mm}$$

$$M_t = [P_1 - P_2] [R] = 5197452 \text{ N-mm}$$

$$\sigma = \frac{16}{\pi d^3} \times \sqrt{M_b^2 + M_t^2}$$

$$d = 51.935 \text{ mm (55 mm)} \quad \dots (8)$$

- Bearing selection from the catalogue:

Bearing No: 6211

D: 100mm

B: 21mm

C: 43600

C_o:19600

The antenna is to be operated continuously i.e. 24 hrs per day, hence the life of bearing is assumed to be 50,000 hrs.

L_{10h} = 50,000 hrs

$$\text{Consequently, } L_{10} = \frac{60 * n * L_{10h}}{10e6}$$

$$L_{10} = \frac{60 * 1 * 50,000}{10e6} = 3 \text{ million revolutions}$$

- Design of key:

For a flat key:

$$\text{width (b)} = \frac{d}{4} = 16.25 \text{ mm}$$

$$\text{height (h)} = \frac{d}{6} = 10.833 \text{ mm}$$

$$\text{length (l)} = 1.5d \text{ for } 2.1d = 136.5 \text{ mm}$$

$$l = \frac{2M_t}{\tau_{db}} \dots \text{For shear forces}$$

$$\tau = 72.07 \text{ N/mm}^2 < 110$$

$$\tau < \tau_{\text{permissible}}$$

$$l = \frac{4M_t}{\sigma_c dh} \dots \text{For torsional forces}$$

$$\sigma_c = 216.298 \text{ N/mm}^2 < 220$$

$$\sigma_c < \sigma_{c\text{permissible}}$$

Hence the key is safe.

Dimensions of the key are:

$$b = 18 \text{ mm} \quad h = 12 \text{ mm} \quad l = 138 \text{ mm}$$

b) Stage no 2:

Diameter of driven pulley D = 300mm

Diameter of driving pulley (d) = 150mm

Speed of driven pulley (N) = 2 rpm

Center to center distance between pulleys(C) = 1212mm

$$V = \frac{\pi \times 150 \times 2}{60e3} = 0.0157 \text{ m/s}$$

$$\alpha = 180 + 2\sin^{-1}\left(\frac{D+d}{2C}\right) = 3.3067 \text{ radians}$$

$$P = (P_1 - P_2) * V$$

$$P_1 = 34649.68 + P_2$$

$$P_1/P_2 = e^{(2.92 * f\alpha)}$$

$$P_1 = 6.6298 P_2$$

Solving the equations,

$$P_1 = 40524.5 \text{ N} ; P_2 = 5875.24 \text{ N}$$

- Design of Shaft 02:

Assumption: The length of shaft = 300mm.

$$M_b = \frac{P_1 + P_2}{2} \times \frac{300}{2} = 3479980.79 \text{ N-mm}$$

$$M_t = [P_1 - P_2][R] = 2598694.5 \text{ N-mm}$$

$$\sigma = \frac{16}{\pi d^3} \times \sqrt{M_b^2 + M_t^2}$$

$$\text{So, } d = 46.49 \text{ mm (50 mm)}$$

- Bearing selection from the catalogue:

Bearing No: 6210; L_{10h} = 50,000 hrs.

D: 90mm B: 20mm

C: 35100 C_o: 19600

$$L_{10} = \frac{60 * n * L_{10h}}{10e6} = 6 \text{ million revolutions}$$

- Design of key:

For a flat key:

$$b = \frac{d}{4} = 16.25 \text{ mm} \approx 18 \text{ mm}$$

$$h = \frac{d}{6} = 10.833 \text{ mm} \approx 12 \text{ mm}$$

$$l = 1.5d \text{ for } 2.1d = 136.5 \text{ mm} \approx 138 \text{ mm}$$

$$l = \frac{2M_t}{\tau_{db}} \dots \text{For shear forces}$$

$$\tau = 72.07 \text{ N/mm}^2 < 110$$

$$\tau < \tau_{\text{permissible}}$$

$$l = \frac{4M_t}{\sigma_c dh} \dots \text{For torsional forces}$$

$$\sigma_c = 216.298 \text{ N/mm}^2 < 220$$

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Hence the key is safe.

Dimensions of the key are:

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IV. CONCLUSION

The rope-pulley drive thus designed, is found to be accurate, easy to operate and economically and ergonomically viable. The design is also found to be safe based on analytical calculations performed.

V. REFERENCES

- [1] Analytical Model of Wind Disturbance Torque on Servo Tracking Antenna-Tushar Golani and Suresh Sabhapathy
- [2] Design of Machine Elements - V.B.Bhandari