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Design of Lateral Acceleration Control Autopilots for Missile Systems

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***Abstract*: The equation defining the three-loop autopilot design and its airframe dynamics of a guided missile using a stable closed loop control system. The airframe dynamics of a guided missile are non-linear, however the non-linearities are in structured form (i.e., quadratic form), and thus novel approach has been considered in expressing the non- linear dynamics in state space form. The derivation of airframe dynamics for a rear controlled autopilot missile has been derived using the aerodynamic transfer function and characteristic equations. The autopilot design is having been directly evaluated according to the gains acquired by the unit step responses. The proposed technique of integrated guidance and control system is implemented with the assistance of linearized state space model which can be further extracted using the linear techniques of modern control theory. Finally, the numerical simulation has been done using MATLAB and demonstrated the results for the effectiveness and feasibility of the proposed technique.**

Keyword: Guided missiles , three-loop autopilots , step response , frequency responses , MATLAB

1. INTRODUCTION TO MISSILE SYSTEMS

Introduction

Any object thrown at a target with the aim of hitting it's a missile. Thus, a stone thrown at a bird may be a missile. The bird, by using its power of reasoning may evade the missile (the stone) by moving either to the Left, right, top or bottom with reference to the flight path (trajectory) of the missile. Thus, the missile, during this case, has been ineffective in its objective of hitting the bird (the target). Now, if the stone is imparted with some intelligence and quick response to maneuver with reference to the bird, to beat aiming errors and therefore the bird's evasive actions and hit it accurately, the stone now becomes a missile .” Thus, “guided missile are self-propelled, unmanned space or air vehicle carrying an explosive warhead whose path are often adjusted during flight, either by automatic self-contained controls or remote human control and are powered either by rocket engines or by reaction propulsion . Missiles could also be aerodynamic, i.e., controlled by aerodynamic surfaces and following a straight-line trajectory to the target, or ballistic, i.e., powered during flight and following a parabolic trajectory.” The trajectory of the missile could be predetermined or continuously varying, based on whether the target is stationary or moving. It is also probable that the missile deviates from its path due to rigging errors, disturbances in the atmosphere, or asymmetrical loading of control surfaces in supersonic flight. In both the situations, a guided missile should be able to maneuver back to its desired path in order to hit the target. One of the tasks of the missile guidance system is to measure the deviations (in both vertical and in horizontal planes) from missile’s desired path. This deviation or error is fed to the control system, which in response to these error signals maneuver the missile quickly and efficiently until these errors are reduced to zero, with the help of various controls available onboard.

1.1 Guidance and Control System

The guidance and control system are a key element that allows the missile to meet its system performance requirements. The objective of the guidance and control system is to force the missile to achieve the steering commands developed by the guidance system. The types of steering commands vary depending on the phase of flight and the type of interceptor. For example, in the boost phase the guidance and control system may be designed to force the missile to track a desired flight-path angle or attitude. In the midcourse and terminal phases, the system may be designed to track acceleration commands to affect an intercept of the target.

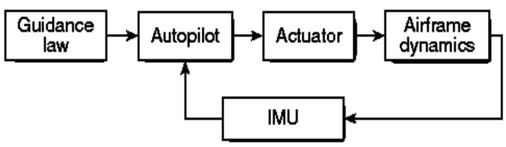

Figure 1 : Guidance and Control System Loop

The four basic elements of the flight control system are shown in the figure 1. The Inertial Measuring Unit senses the inertial motion of the missile. Its outputs and the inputs from the guidance law are combined in the autopilot to form a command input to the control effectors, such as the commanded deflection angle to an aerodynamic control surface. The actuator turns the autopilot command into the physical motion of the control effectors, which in turn influences the airframe dynamics to track the guidance command.

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1.2 Autopilot

An autopilot is a closed loop system and it is a minor loop inside the main guidance loop; not all missile systems require an autopilot. A missile will maneuver up-down or left-right in an apparently satisfactory manner if a control surface is moved or the direction of thrust altered. If the missile carries accelerometers and/or gyros to provide additional feedback into the missile servos to modify the missile motion then the missile control system consisting of servos, control surfaces or thrust vector elements, the airframe, and feedback instruments plus control electronics is usually called an autopilot. Broadly speaking autopilots either control the motion in the pitch and yaw planes, in which case they are called lateral autopilots, or they control the motion about the fore and aft axis in which case they are called roll autopilots. This contrasts with the usual definition of aircraft autopilots; those designed to control the motion in the pitch plane are called lateral autopilots. For instance, an aircraft autopilot designed to keep the heading constant would be called lateral autopilot. For symmetrical cruciform missile however pitch and yaw autopilots are often identical; one injects a 'g' bias in the vertical plane to offset the effect of gravity but this does not affect the design of the autopilot. Roll autopilots serve quite a different purpose and will be considered separately.

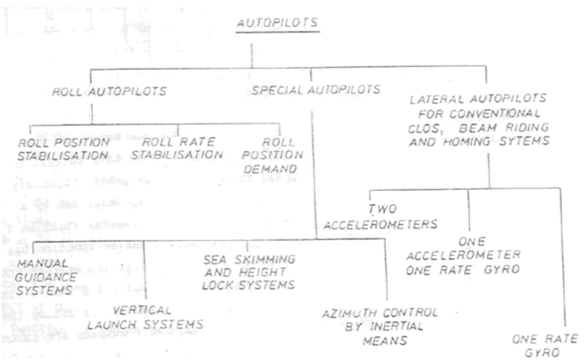

Figure 2 : Types of Autopilots

The autopilots are mainly classified into three types as shown in the Fig: 2 the special autopilot is used in some of special design cases. But the roll autopilot and Lateral autopilot are mainly used. Roll autopilot is meant to maintain particular roll attitude or to keep the roll rate zero to prevent coupling effects between pitch, yaw and roll planes. Lateral Attitude Autopilot is mainly meant for the missile applications where specific attitude is required to be maintained. It is being used in ballistic missiles or strategic missiles. Lateral Acceleration autopilot is mainly meant for maintaining the trajectory. This is mostly used in tactical missiles.

2. METHODOLOGY

According to Newton's laws we will consider the missile and derive the rigid body equations and the following assumptions are considered :

- **Rigid Body** : The body does not go any changes in size and shape.
- Aerodynamic symmetry in Roll
- **Mass** : A constant mass is assumed , dm/dt=0
- Body axes co-ordinate frame is used to write the missile equations.
- A spherical earth rotating at constant angular velocity is assumed .
- Vehicle aerodynamics is nonlinear.
- The undisturbed atmosphere rotates with the Earth.
- The winds are defined with respect to the Earth.
- An inverse-square gravitational law is used for the spherical Earth model.
- The gradients of the low-frequency winds are small enough to be neglected.

Euler's Equation of Motion for a Rigid Body

The rigid body equations are obtained from Newton's second law which states that the summation of all external forces acting on a body is equal to time rate of change of momentum of body, and the summation of external moments acting on the body is equal to time rate of change of moment of momentum (angular momentum)". There are six equations of motion for a body with six degrees of freedom: three force equations and three moment equations. The equations are somewhat simpler if the mass is constant and radii-of-gyration changes slowly. The translation and rotation of a rigid body may be expressed mathematically by the following equations:

Translation: $\sum \mathrm{F} = m\mathbf{a}$ (3.1)

Rotational: $\sum \mathrm{M} = \frac{d}{dt}H$ (3.2)

These motions are shown in Figure, the translations being (u, v, w) and the rotations *(P, Q, R).*

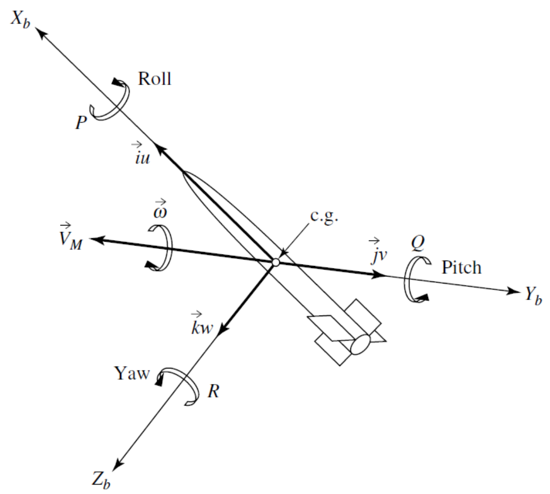

Figure 3 : Representation of the missile's six degrees of freedom

Mathematically, we can write the missile vector velocity, $\mathbf{V}_M$, in terms of the components as

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$\mathbf{V}_M = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$ (3.3)

where ($\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$) are the unit vectors along the respective missile body axes. The magnitude of the missile velocity is given by

$|\mathbf{V}_M| = V_M = (u^2 + v^2 + w^2)^{1/2}$ (3.4)

In a similar manner, the missile's angular velocity vector $\boldsymbol{\omega}$ can be broken up into the components P,Q, and R about the (Xb, Yb , Zb) axes, respectively, as follows:

$\boldsymbol{\omega} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ (3.5)

where P is the roll rate, Q is the pitch rate, and R is the yaw rate.

$\sum \Delta F_x = m(\dot{u} + wQ - vR),$ (3.6)

$\sum \Delta F_y = m(\dot{v} + uR - wP),$ (3.7)

$\sum \Delta F_z = m(\dot{w} + vP - uQ).$ (3.8)

$\sum L = \dot{P}I_x + (I_z - I_y)QR - (\dot{R} + PQ)I_{xz}$ (3.9)

$\sum M = \dot{Q}I_y + (I_x - I_z)PR + (P^2 - R^2)I_{xz}$ (3.10)

$\sum N = \dot{R}I_z + (I_y - I_x)PQ - (\dot{P} - QR)I_{xz}$ (3.11)

Thus the set of Equations (3.6), (3.7), (3.8), (3.9), (3.10), (3.11) represents the complete 6-*Degrees of Freedom* missile equations of motion .

Aerodynamic Transfer functions

To design a missile control system, the linearity of hardware (i.e. the electronics, fin servos, instruments and equations of motion) is generally assumed. Aerodynamic transfer functions are obtained from aerodynamics derivatives defined above. With pitch, roll and yaw dynamics under consideration, aerodynamic derivatives are force derivative if they are used in force equation and moment derivatives if used in moment equation.

The reference axis system standardized in the guided weapons industry is centered on the center of gravity and fixed in the body as follows:

'X' axis, called the roll axis, forwards, along the axis of symmetry if one exists, but in any case, in the plane of symmetry.

'Y' axis, called the pitch axis, outwards and to the right if viewing the missile from behind.

'Z' axis, called the yaw axis, downwards in the plane of symmetry to form a right-handed orthogonal system with the other two .

Table 1 defines the forces and moments acting on the linear and angular velocities, and the moments of inertia; these quantities are shown in fig 4 .The moments of inertia about 'O' are defined as:

$A = \sum \delta m\,(y^2+z^2)$
$B = \sum \delta m\,(z^2+x^2)$
$C = \sum \delta m\,(x^2+y^2)$

The products of inertia are defined as:

$D = \sum \delta m\,yz$
$E = \sum \delta m\,xz$
$F = \sum \delta m\,xy$

The yaw plane is the Oxy plane and the pitch plane is the Oxz plane. The following angles are defined:

α: Incidence in the pitch plane.
β: Incidence in the yaw plane.
λ: Incidence plane angle.
θ: Total incidence, such that:

$$\tan\alpha = \tan\theta\cos\lambda$$
$$\text{and } \tan\beta = \tan\theta\sin\lambda$$

The reason why U, the missile velocity along the x axis is denoted by a capital letter is to emphasize that it is a large positive quantity changing at most only a few per cent per second. The angular rates and components of velocity along the pitch and yaw axes however, tend to be much smaller quantities which can be positive or negative and can have much larger rates of change.

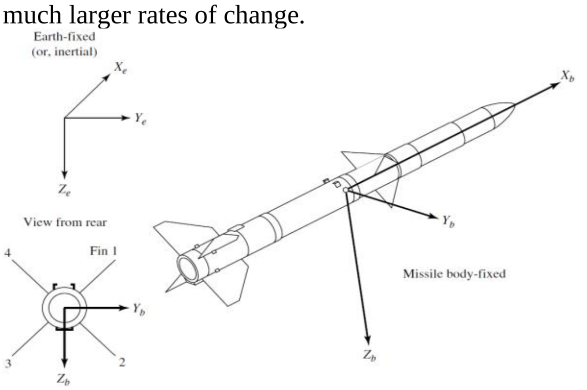

Figure 4 : Earth fixed and Body axes coordinate systems

2.1 Notations and Conventions

The motion of a missile is mostly described using two sets of coordinate systems such as earth-fixed coordinate system and body axes coordinate system. The Earth-fixed coordinate system or inertial coordinate system is fixed to earth and may be used to define missile's position and attitude in a three-dimensional space. While the body axes coordinate system is fixed to the body of missile and is centered at the center-of-gravity (CG) of the missile. Figure 4 illustrates these two axes systems.

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Where **x, y** and **z**-axes are called Roll axis, Pitch axis and Yaw axis respectively. The forces acting on the missile airframe such as aerodynamic, thrust and gravitational forces are resolved along these axes. "The forces and moments acting on the missile, the linear and angular velocities and other motion variables are defined in Table 1. Figure 5 shows these quantities.

Table 1 : Notations

	Roll axis x	Pitch axis y	Yaw axis z
Component of forces acting on missile along each axis	*X*	*Y*	*Z*
Moments acting on missile about each axis	*L*	*M*	*N*
Component of missile velocity along each axis	*U*	*v*	*w*
Component of missile acceleration along each axis	a_x	a_y	a_z
Angular rates	*p*	*q*	*r*
Moments of Inertia about each axis	I_{xx}	I_{yy}	I_{zz}
Products of Inertia	I_{yz}	I_{xz}	I_{xy}

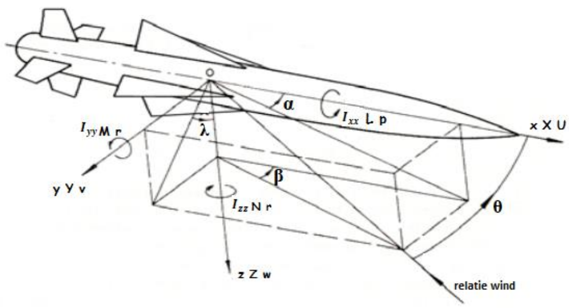

Figure 5 : Convention

These are six equations of motion for a body with six degrees of freedom: three force equations and three moment equations. The equations are somewhat simpler if the mass is constant and the radii of gyration change slowly, in which case the "standard" Euler equations can be used. During boost the mass rate of change can be relatively fast but the radii of gyration usually change slowly. In the great majority of design calculations, the standard equations are used. They are:

$$m(\dot{U} + qw - rv) = X \quad (4.1)$$

$$m(\dot{v} + ru - pw) = Y \quad (4.2)$$

$$m(\dot{w} - qU + pv) = Z \quad (4.3)$$

$$A\dot{p} - (B - C)qr + D(r^2 - q^2) - E(pq + \dot{r}) + F(rp - \dot{q}) = L \quad (4.4)$$

$$B\dot{q} - (C - A)rp + E(p^2 - r^2) - F(qr + \dot{p}) + D(pq - \dot{r}) = M \quad (4.5)$$

$$C\dot{r} - (A - B)pq + F(q^2 - p^2) - D(rp + \dot{q}) + E(qr - \dot{p}) = N \quad (4.6)$$

If an autopilot with acceleration on position due to the higher frequency. If an autopilot with acceleration in position due to the higher frequency. If an autopilot with acceleration feedback is used the effects will be reduced. In any case the guidance loop has a certain stiffness which will be reduced.

In any case the guidance loop has a certain stiffness which will tend to reduce this effect; the missile does not fly open-loop.

Now consider the moment equations. Ideally these should read

$$A\dot{p} = L,\ B\dot{q} = M,\ C\dot{r} = N \quad (4.7)$$

i.e., moments about a given axis produce angular accelerations about that axis. All other terms in these equations are cross-coupling terms and are undesirable from the point of view of system accuracy. We note that three out of four of the cross-coupling terms in each equation disappear if the products of inertia are zero of there are two axes of symmetry, and two will be zero and the missile is reasonably symmetrical about another axis. With two planes of symmetry and a small roll rate therefore these equations reduce to.

$$m(\dot{U} + qw - rv) = X \quad (4.8)$$

$$m(\dot{v} + ru) = Y \quad (4.9)$$

$$m(\dot{w} - qU) = Z \quad (4.10)$$

$$A\dot{p} = L \quad (4.11)$$

$$B\dot{q} = M \quad (4.12)$$

$$C\dot{r} = N \quad (4.13)$$

- The justification for neglecting the terms pq, pr, pv, pw is that the terms q, r, v and w are not large terms and if p is small then their products can be neglected.
- Equation 4.11 Shows that there is zero coupling between the pitch and roll and yaw and roll motions if there are two axes of symmetry (B=C), and unless the missile is very unsymmetrical the cross-coupling should be weak.
- Nevertheless, even with a missile with two axes of symmetry, the principal moments of inertia about the roll axis (A). Moderate roll rates will certainly affect the accuracy of the system.

2.2 *Derivation of Aerodynamic Transfer Functions*

We will now write down the control equations for missile, omitting the force equations along the x-axis as this neither affects the roll, pitch or yaw motion. We will also omit the gravity term and consider forces and moments which are purely of aerodynamics origin.

The forces equation in the direction of Y-axis is given by

$$f_z = \dot{w} - Uq = z_\alpha \alpha + z_\delta \delta + z_q q \quad (4.14)$$

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But for small angles of 'α' tan α is equal to $\frac{w}{U}$,

$$z_{\alpha}\frac{w}{U} = z_w w$$

$$z_w = \frac{z_{\alpha}}{U}$$

Thus, we get,

$$f_z = \dot{w} - Uq = z_w w + z_{\delta}\delta + z_q q$$

(4.15)

Similarly the body rate is equal to

$$\dot{q} = m_{\alpha}\alpha + m_{\delta}\delta + m_q q$$

$$\dot{q} = m_w w + m_{\delta}\delta + m_q q$$

(4.16)

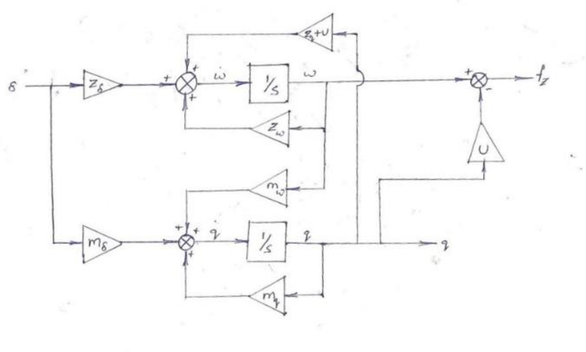

Figure 6 : Signal Flow graph

By using the above signal flow graph and by using Mason's gain Formula:

$$\text{Transfer Function} = \frac{\text{Sum of all the forward loops}}{\text{Characterstic Equation}}$$

Transfer Function $\frac{q(s)}{\delta(s)}$ by using the Mason's gain formula is given by

$$\frac{q(s)}{\delta(s)} = \frac{\frac{m_{\delta}}{s}\left[1 - \frac{Z_w}{s}\right] + \frac{Z_{\delta}m_w}{s^2}}{1 - \left[\frac{m_q}{s} + \frac{Z_w}{s} + \frac{m_w(U+Zq)}{s^2}\right] + \frac{Z_w m_q}{s^2}}$$

$$\frac{q(s)}{\delta(s)} = \frac{m_{\delta}s + (z_{\delta}m_w - z_{\delta}m_w)}{s^2 - (z_w + m_q)s + m_w(U + z_q) + z_w m_q}$$

But $Z_w = Z_{\alpha} / U$, as U is the velocity along X- axis which is very large, Zw tends to zero.

The viscous effect of the air is assumed negligible; Thus, the body rate terms mq, Z_q are negligible.

If we consider a Simple pendulum with initial deflection θ, then for the stable condition we get the frequency as shown below

$$\frac{d^2\theta}{dt^2}\alpha(-\theta)$$

$$\frac{d^2\theta}{dt^2} = -k_{\theta}\theta$$

$$\frac{d^2\theta}{dt^2} + k_{\theta}\theta = k_{\theta}U(t)$$

$$(s^2 + k_{\theta})\theta(s) = k_{\theta}U(s)$$

$$\frac{\theta(s)}{U(s)} = \frac{k_{\theta}}{s^2 + k_{\theta}}$$

$$\frac{\theta(s)}{U(s)} = \sqrt{k_{\theta}}\frac{\sqrt{k_{\theta}}}{s^2 + k_{\theta}}$$

(4.17)

Taking the Inverse Laplace transform we get,

$$\sqrt{k\theta}\sin(\sqrt{k\theta}t + \phi)$$

Where it vibrates with a frequency of $\sqrt{k\theta}$

$$\frac{q(s)}{\delta(s)} = \frac{m_{\delta}s + (z_{\delta}m_w - z_w m_{\delta})}{s^2 + (-Um_w)}$$

If we take the static stability condition then m_w will be a negative term.

$$\frac{q(s)}{\delta(s)} = \frac{m_{\delta}s + (z_{\delta}m_w - z_w m_{\delta})}{s^2 + w_b^{\,2}}$$

Where, $w_b^2 = -Um_w$

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If we take the Center of gravity as the pinned joint and the missile as a simple pendulum as shown above, then for a small disturbance the missile will undergo oscillations with the frequency 'w_b' which is known as the weathercock frequency.

In the similar way we can derive the transfer function for the acceleration and the deflection of the control surface.

$$\frac{f_z(s)}{\delta(s)} = \frac{z_\delta s^2 - U(z_\delta m_w - z_w m_\delta)}{s^2 + w_b^2}$$

By doing some manipulations to the two transfer functions we get,

$$\frac{q(s)}{\delta(s)} = \frac{(z_\delta m_w - z_w m_\delta)}{w_b^2} \frac{w_b^2 [T_a s + 1]}{s^2 + w_b^2}$$

$$\frac{q(s)}{\delta(s)} = \frac{k_b w_b^2 [T_a s + 1]}{s^2 + w_b^2} \qquad (4.18)$$

Where,

$$T_a = \frac{m_\delta}{z_\delta m_w - z_w m_\delta}$$

In the similar manner we get,

$$\frac{f_z(s)}{\delta(s)} = \frac{U k_b w_b^2 \left[\sigma^2 s^2 - 1\right]}{s^2 + w_b^2} \qquad (4.19)$$

Where,

$$\sigma^2 = \frac{z_\delta}{U(z_\delta m_w - z_w m_\delta)}$$

Equations (4.18), (4.19) are the required transfer functions. Thus, the transfer functions for the Acceleration, Body rates with respect to deflection of the control surface are derived.

3. AUTOPILOT DESIGN

Missile control system consisting of servos, control surfaces, the airframe and the feedback instruments plus the control electronics, all working together to automatically adjust the orientation of the missile in space can be termed as an autopilot. "In general, an autopilot either control the motion about the fore and aft axis (called Roll Autopilot) or they control the motion in the pitch and yaw planes (called Lateral Autopilot. In contrast to missile autopilot, an aircraft autopilot designed to control the motion in the pitch plane and yaw plane are called longitudinal autopilot and lateral autopilot. Where the lateral autopilot is designed to keep the heading constant. However, in case of a symmetrical cruciform missile, pitch and yaw autopilot are often identical while one injects a ***g*** (acceleration due to gravity) bias in the vertical plane to counter the effect of gravity". But this does not affect the design of the autopilot.

3.1 Design of Lateral Acceleration Autopilot

We use the transfer functions in 4.18 , 4.19 for the design of the three loop Lateral Acceleration Autopilot using pole placement technique. The main objectives for designing the lateral acceleration (LATAX) autopilot:

1. The maintenance of near-constant steady state aerodynamics gain.
2. To increase the weathercock frequency.
3. To increase the weathercock damping.
4. To reduce cross-coupling pitch and yaw motion .
5. To assist in gathering.

3.1.1 A Lateral Autopilot with an actuator:

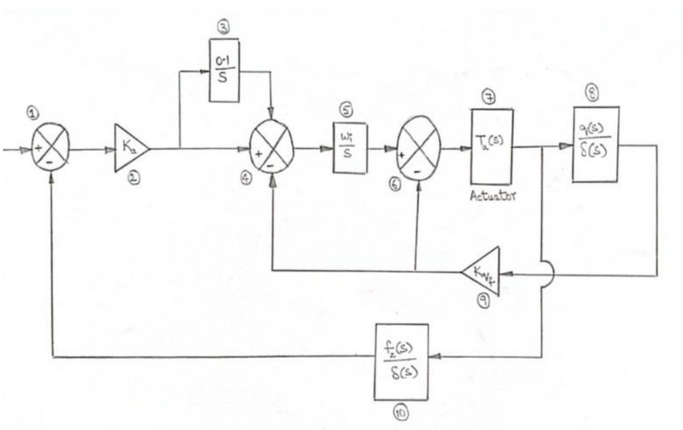

Figure 7 : Signal flow graph for the three-loop autopilot

Let the transfer function of the actuator be $T_a(s)$ which is having a second order characteristic equation.

Characteristic equation for the figure 7 is as follows

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$$=1+k_q T_s(s)\frac{q(s)}{\delta(s)}+k_q\frac{w_i}{s}T_s(s)\frac{q(s)}{\delta(s)}+k_a\frac{w_i}{s}T_s(s)\frac{q(s)}{\delta(s)}\frac{f_z(s)}{\delta(s)}$$

$$=1+\frac{k_q k_b w_b^2(T_a s+1)w_a^2}{(s^2+w_b^2)(s^2+2\zeta_a w_a s+w_a^2)}+\frac{k_q k_b w_b^2(T_a s+1)w_a^2 w_i}{s(s^2+w_b^2)(s^2+2\zeta_a w_a s+w_a^2)}$$

$$+\frac{k_a w_i}{s}\frac{Uk_b w_b^2(\sigma^2 s^2-1)}{(s^2+w_b^2)}\frac{w_a^2}{(s^2+2\zeta_a w_a s+w_a^2)}$$

$$=s(s^2+w_b^2)(s^2+2\zeta_a w_a s+w_a^2)+k_q k_b w_b^2 T_a s^2 w_a^2+k_q k_b w_b^2 s w_a^2$$

$$+k_q k_b w_b^2 T_a s w_a^2 w_i+k_q k_b w_b^2 w_a^2 w_i+k_a w_i U k_b w_b^2 w_a^2\sigma^2 s^2-k_a w_i U k_b w_b^2 w_a^2$$

$$=s^5+(2\zeta_a w_a s)s^4+s^3(w_a^2+w_b^2)+s^2(2\zeta_a w_a w_b^2+k_q k_b w_b^2 w_a^2 T_a+k_a w_i U k_b w_b^2 w_a^2\sigma^2)$$

$$+s(w_a^2 w_b^2+k_q k_b w_b^2 w_a^2+k_q k_b w_b^2 T_a w_a^2 w_i)+k_b w_b^2 w_a^2 w_i(k_q-k_a U)$$

………………(5.1)

As the above characteristic equation is of fifth order, we cannot solve the above equation, so we need a characteristic equation which is having a second order natural frequency and the remaining three variables are placed according to our design conditions.

$$=(s+a)(s+b)(s+c)(s^2+2\zeta_n w_n s+w_n^2)$$

$$=s^5+s^4(a+b+c+2\zeta_n w_n)+s^3[w_n^2+2\zeta_n w_n(a+b+c)+(ab+bc+ca))]$$

$$+s^2[w_n^2(a+b+c)+2\zeta_n w_n(ab+bc+ca)+abc]+s[w_n^2(ab+bc+ca)+2\zeta_n w_n abc]$$

$$+abcw_n^2$$

………(5.2)

We are taking the natural frequency of the actuator value is three times the value of the weathercock frequency. The desired frequency of the desired autopilot is again three times that of the natural frequency of the actuator natural frequency. Thus, we need to place the three variables in between the natural frequency of the autopilot and the frequency of the actuator.

By equating the equations (5.1) and (5.2), we get,
Let a = αw_n.

$$(b+c)=2(\zeta_a w_a-\zeta_n w_n)-\alpha w_n$$

$$a(b+c)+bc=w_a^2+w_b^2-w_n^2-2\zeta_n w_n(a+b+c)$$

$$bc=w_a^2+w_b^2-w_n^2-2\zeta_n w_n(a+b+c)-a(b+c)$$

$$w_i k_a k_b w_a^2 w_b^2\sigma^2+k_q k_b w_a^2 w_b^2 T_a=w_n^2(a+b+c)+2\zeta_n w_n(ab+bc+ca)+abc-2\zeta_a w_a w_b^2+abcw_n^2\sigma^2$$

$$k_q w_i k_b w_a^2 w_b^2 T_a+k_q k_b w_a^2 w_b^2=-w_a^2 w_b^2+w_n^2(ab+bc+ca)+2\zeta_n w_n abc$$

$$k_q w_i P_1+k_q Q_1=R_1$$

$$k_q w_i P_2+k_q Q_2=R_2$$

$$k_q w_i=\frac{R_1-k_q Q_1}{P_1}$$

$$k_q=\frac{\left[R_2-\frac{P_2}{P_1}R_1\right]}{\left[Q_2-\frac{P_2}{P_1}Q_1\right]}$$

\

Thus the above equations are used to solve for the unknown terms (k_q, k_a, w_i, a, b, c) in the signal flow graph.

MATLAB :

A MATLAB code is developed in order to solve the aerodynamic transfer functions we derived and plot the step response of the three loop LATAX control autopilot and frequency response (BODE) plots of the following:

1) Analysis with closed loop
2) Stability Margins with innermost loop open
3) Stability Margins with inner/intermediate loop open
4) Stability Margins with outermost loop open

The program codes to maintain the phase angle between -180° & +180° and the analysis of the design without errors are both inserted in the main code.

The flight parameters for some assumed missiles are taken and are solved using MATLAB.

The MATLAB code and the results are found in the Appendix.

4 . RESULT AND DISCUSSION

Analysis of LATAX control autopilot :

The relations obtained from our three loop LATAX autopilot design with the actuator Ta(s) are solved using MATLAB .

The required control parameters for Lateral Autopilot, with their first estimate values are as listed:

U = 500	**Z_a =0.7**
c = 1	**Z_n= 0.9**
m_ζ =53.0	**Z_w= -2.889**
Z_ζ = 23.06	**m_w= -2.948**

also, with estimated values (on the basis of Unit-step-response) of various gains, natural frequencies and damping

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ratios, which can be optimized are as follows:

K_b = 0.9987 **ω_{ns} = 180**
K_{qf} = 1.2236 **μ_s = 0.5**
K_a = -0.0115

The step response and frequency response plots obtained from MATLAB program code are :

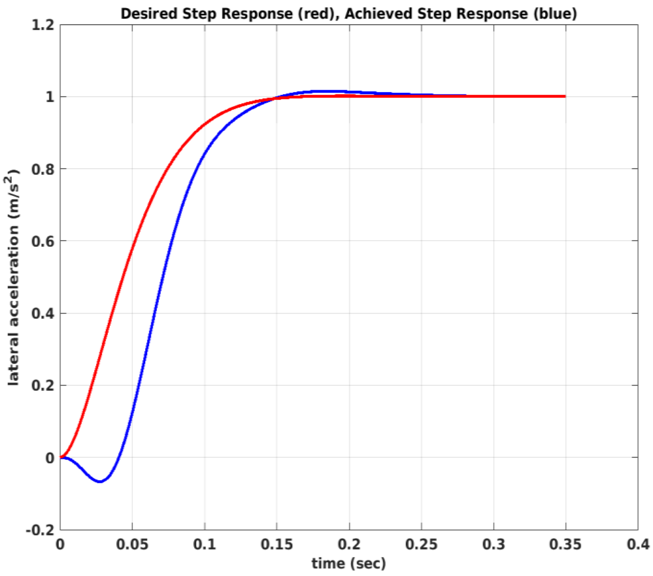

Figure 8 : Step Response of LATAX Control Autopilot

This gives a unit step response with the following parameters:

Rise Time = 0.1803 sec
Settling Time = 0.0848 sec (within 2%)

Frequency Response Plots :

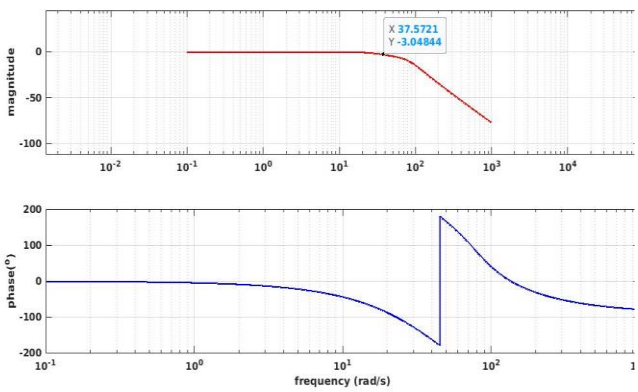

Figure 9 : Frequency Response Plot of Closed Loop Autopilot

In the frequency response of the closed loop, the change in magnitude is observed at frequency value of 37.57 rad/s

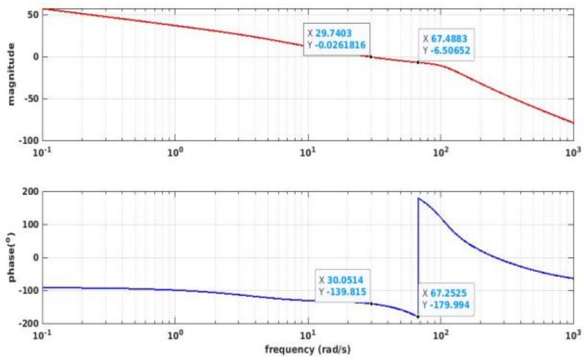

Figure 10 : Frequency Response Plot with Inner Loop Open

For the bode plot with the inner loop open , in bandwidth of -0.026rad/s and -0.65 rad/s the gain margins are observed . The phase margin frequencies are 30.05 rad/s and 67.25 rad/s with the phase angles at -139.88° and -179.899° respectively.

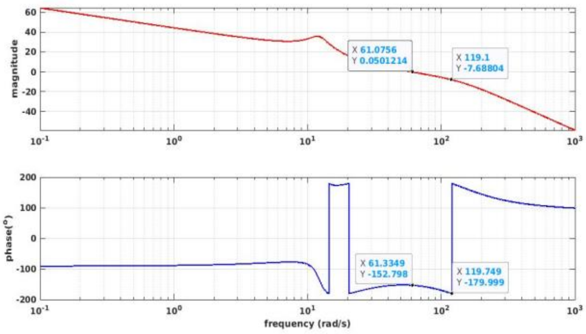

Figure 11 : Frequency Response Plot with Inner most Loop Open

With the inner most loop open , the magnitude is 0.05012 and -7.688 at gain crossover frequencies at 61.07 rad/s and 119.1 rad/s. The phase angles are -152.7° and -179.99° with the frequencies of 61.33 rad/s and 119.749 rad/s respectively.

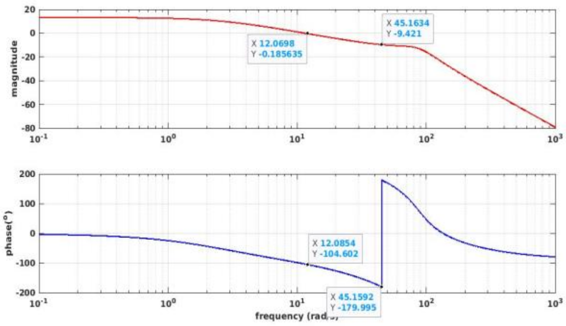

Figure 12 : Frequency Response Plot with Outer Loop Open

In the frequency response with outer loop open , the gain margin frequencies are 12.06 rad/s and 45.163rad/s along with the phase angles at -104.602° and -179.995° respectively.

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The unknown parameters of the derived characteristic equations have been acquired by using MATLAB and are as follows :

P1 =791.8308	P2 = 1.0460e+06
Q1 =1.0460e+06	Q2 = 2.9053e+06
R1 = 1.3035e+06	R2 = 3.4645e+07

Table 2 : Characteristic Equation Solution Values

5. CONCLUSION

The accelerometer location relative to the CG greatly affects the measured acceleration and may be wont to shape the zeros within the acceleration transfer function. The missile autopilot design methodology should consider the unfavorable aspect of on-board hardware feature in high frequency region and employs open-loop crossover frequency to constrain high frequency dynamics. The pole placement technique used has produced with the proper values of the control parameters and the desired step response was achieved within short period of time. The actuator portion plays an important role in the autopilot by sending continuous signals to the control system loop and causing the necessary deviations of the missiles in its aerial path . It allows the missile to guide towards the target with no room for errors in trajectory , path or in any sort that may cause failures. These responses obtained in our design of LATAX says that these three loop autopilots are very much reliable in the missile systems in both ways of attacking and counter-attack modes of the military.

FUTURE SCOPE

Further improvements of these three loop autopilots, in which the application of RSS technology on missile design can introduce much benefit to missile maneuverability and full flight performance. However, the missile with lower static stability requires a complicated control strategy. Both the two-loop and three-loop topologies can stabilize a static unstable missile and achieve desired performance requirement. The exhibited performance of the classical three loop topology is analogous to an inertial loop, and can be modified for the wants of next generation air-to-air agile missile. The Iron Dome is an example of successful missile defense system with 90 percent accuracy , but it is for short-range. It requires a lot of efforts to develop a long-range autopilot missile system which is accurate. Much more research and analysis will help in building a missile control system design of such accuracy and efficacy.

6. APPENDIX

MATLAB CODE :

```
clear all;
close all;

%%% plant parameters
zq=0.0; mq=0.0;

mdelta =53.0;

zdelta = 20.06;

mw =  -0.2948;

zw =  -2.889;

U =500;%%%% Missile speed

Ta=mdelta/(zdelta*mw-zw*mdelta)

wbsq = -U*mw

kb=(zdelta*mw-zw*mdelta)/wbsq

sigmasq=zdelta/(U*(zdelta*mw-zw*mdelta))

ls=0.0;%xcg-xsense

%c=1;

%for ki=0.0:0.1:30

%wa=2*pi*wa;

%%%%% desired system % specified system

zn=0.9
wn=sqrt(abs(wbsq))*3.0%1.2*sqrt(abs(wbsq));
wnsq=wn*wn
znwn=zn*wn

sys_des = tf([wnsq],[1 2.0*znwn wnsq]);%%% Transfer
function of desired system

%%%% actuator part

%wa=2*pi*10;
%wa=3.5*wn;
zawa=3*znwn
za=0.7
wa=zawa/0.7
wasq=wa*wa

%%%%%%%%%%%%%%

alp=(1.0-znwn/zawa);

a=alp*(2*zawa)*0.5
```

```
%%%%%%%%%%%%%%

apbpc=2*(zawa-znwn)

bpc=2*(zawa-znwn)-a

bc=wasq+wbsq-wnsq-2*znwn*apbpc-a*bpc

abpbcpca=a*bpc+bc

abc=a*bc

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

P1=kb*wasq*wbsq*(U*sigmasq-ls*Ta)/U

Q1=kb*wasq*wbsq*Ta

R1=wnsq*apbpc+2*znwn*abpbcpca+abc*(1+wnsq*(U*sig
masq-ls*Ta)/U)-2*zawa*wbsq

P2=kb*wasq*wbsq*(Ta-ls/U)

Q2=kb*wasq*wbsq

R2=-wasq*wbsq+wnsq*abpbcpca+2*znwn*abc-
(abc*wnsq*ls)/U


%%%%%%%%%%%%%%%%%%%%%%%

kqf=(R2-(P2/P1)*R1)/(Q2-(P2/P1)*Q1)

kqfwi=(R1-kqf*Q1)/P1

wi=kqfwi/kqf

kawi=(kqfwi/U)-(abc*wnsq)/(U*kb*wasq*wbsq)

ka=kawi/wi



%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%

CE=[1,...
   2*zawa,...
   wbsq+wasq,...

2*zawa*wbsq+kawi*U*kb*wasq*wbsq*sigmasq+kqf*kb*
wasq*wbsq*Ta,...

wasq*wbsq+kqf*kb*wasq*wbsq+kqfwi*kb*wasq*wbsq*T
a,...
   kb*wasq*wbsq*(kqfwi-U*kawi)]

BCE=[1,...
   apbpc+2*znwn,...
   wnsq+2*znwn*apbpc+abpbcpca,...
   wnsq*apbpc+2*znwn*abpbcpca+abc,...
   wnsq*abpbcpca+2*znwn*abc,...
   abc*wnsq]

%%%%%%%%%%%%%%%%%%%%%

rootsCE=roots(CE)

rootsBCE=roots(BCE)

analysis;%%%% to carry out time and frequency domain
analysis

%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%

%lroots(c,:)=rootsBCE;

%c=c+1;

%end
%%
%
%   for x = 1:10
%
%   for x = 1:10
%
%   for x = 1:10
%
%   for x = 1:10
%       disp(x)
%   end
%
%       disp(x)
%   end
%
%       disp(x)
%   end
%
%       disp(x)
%   end
%

% %ki=-0.025:-0.2:-30;
% %zn=0.5:0.01:1.5
% % wa=0.5:0.1:30;
% ki=0.0:0.1:30;
% % % %
% % % %
% figure;
%
% plot(ki,lroots);gzh;
```

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