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Abstract

This paper describes a steady vibration of a passive hydraulic mounts at a relatively low frequency range. A suspended decoupler mechanism is used which affects the start-up & transient response. Also Newton-Raphson method is used to analyze mathematically, treating the mount characteristics as a function of frequency and amplitude. The excitation test is used to simulate an engine shake and an idling vibration which was performed on servo hydraulic machine with a simulated input instead of an actual engine.

During the engine shake, we investigated high damping is needed at an idling frequency ranging between 22-23 Hz. Orifice dimension of the cassette & length of inertia track in hydraulic mounts was tuned to achieve the damping property.

1. Introduction

Unbalanced excitation forces inevitably exist in reciprocating automotive engines since they cannot be completely eliminated due to manufacturing tolerances even in multi-cylinder engines. In particular, in automobiles with a large-displacement four cylinder engine, the unbalanced engine forces are primarily due to the engine secondary forces which are produced by crankshaft and connecting rod motion at twice the crankshaft speed. For instance, the forcing frequency would be 25-200 Hz for an engine with a speed range from 750 to 6000 rpm. Such unbalanced engine forces mainly act in the vertical direction.

The major function of engine mounts, in addition to supporting the engine weight, is to isolate the unbalanced engine forces from the vehicle frame in the frequency range of 25-200 Hz. For this purpose, the engine and gear box unit is usually mounted on three or four engine mounts, and the elastic stiffness of the engine mounts is chosen such that the engine mounting system has a natural frequency of the order of 6-12 Hz in the vertical direction.

First, we will discuss a mathematical model of a hydraulic mount, and identify the parameters for describing the dynamic characteristics & damping of a hydraulic engine mount, considering the dependency on input displacement & frequency.

Second, the equation of motion is converted into a complex-number form, and then solved numerically by the Newton-Raphson method, treating the mount characteristics as a function of the frequency and the input displacement with varying design parameters of orifice & inertia dimensions.

To confirm the validity of this analysis, we carried out two kinds of excitation tests, using a servo hydraulic machine instead of an actual engine. The experimental set-up was excited vertically for an engine shake was provided to the mount axially. Finally, as an approach to achieve & optimize the hydraulic mounting system, we
examined the influence of design parameters such as orifice section area and inertia track length on the damping of 3 cylinder engine shake.

2. Design Features of Hydraulic Mount

Various types of hydraulic mount and their construction details are not identical. However, in practice, their functional characteristics are not much different from each other. The hydraulic mounts employed in this study are the products of Sujan Cooper Standard Anti vibration Systems in Vasai, India. The mount is illustrated in what follows.

![Schematic diagram of the hydraulic mount.](image_url)

(a) Construction details.
(b) Decoupler and decoupler gap.

3. The Dynamic Characteristics of Hydraulic Mount

3.1 Mathematical Model

The dynamic stiffness $k_t$ and the damping coefficient $c_t$ of the hydraulic mount are expressed as follows \[1\][2]. Hydraulic mounts have been reported so far \[1\][2], however, few cases covering both frequency and amplitude dependency have been found \[3\][4][5]. In addition, most research on the engine vibration has been carried out under

\[
k_t = A^2 K_v1 \left[ 1 - \frac{\sigma(1-\lambda^2)}{(1-\lambda^2)^2 + (2\zeta\lambda)^2} \right] + k_r \tag{1}
\]

\[
c_t = \frac{A^2 K_v1}{\omega} \left[ \frac{2\sigma\omega\lambda}{(1-\lambda^2)^2 + (2\zeta\lambda)^2} \right] + c_r \tag{2}
\]

where, $A$: effective piston area, $K_v1, K_v2$: volume stiffness of the primary liquid chamber and secondary liquid chamber respectively, $\sigma=K_v1/(K_v1+K_v2)$, $k_r$: dynamic stiffness of the main rubber, $c_r$: damping coefficient of the main rubber, $\omega$: angular frequency, $\omega_n$: resonant angular frequency of a liquid column, $\lambda=\omega/\omega_n$, $\zeta$: damping ratio. Assuming a laminar flow in the orifice, the damping ratio
ζ can be obtained as follows [4]:

\[
\zeta = \frac{16\nu}{d^2 \omega_n} + \frac{2\kappa \sigma A_0 x_0}{3\pi l s \sqrt{(1 - \lambda^2)^2 + (2\zeta \lambda)^2}}
\]  

(3)

where, \(d\): hydraulic diameter of the orifice, \(l\): orifice length, \(s\): orifice section area, \(\nu\): dynamic viscosity of liquid, \(\kappa\): pressure loss factor at the orifice inlet and outlet, \(x_0\): amplitude of engine vibration. The equation (3) can be derived from the concept of equivalent damping, in that the energy consumption of damping during a cycle should be equal to the work of liquid column performed by the pressure difference [3]. Equation (3) includes \(\zeta\) in the second term of the right-hand side, so the equation is nonlinear with respect to \(\zeta\), and the value of \(\zeta\) can be obtained by numerical calculation. Substituting the value of \(\zeta\) into Equation (1) and (2), dynamic stiffness \(k_t\) and the damping coefficient \(c_t\) of a hydraulic mount can be expressed numerically as a function of the frequency and the amplitude, because Equation (3) includes the vibration amplitude \(x_0\) as well as the frequency ratio \(\lambda\) in its right-hand side. A compressive preload \(F\) coinciding with a given mass is applied to the mount through the hydraulic actuator. In this study, \(F=-1200\) N unless specified otherwise. The actuator is now excited with a sinusoidal stroke \(x\) under closed-loop control \([x(t)=X \sin \omega t]\).

4. Identification of Mount Characteristics

Concerning parameters related to the dynamic characteristics, \(d\), \(l\), \(s\) are the orifice dimensions known at the design stage, while \(\nu\) is also known. However, we can neither predict nor directly measure the parameters \(A\), \(Kv1\), \(Kv2\), \(\kappa\). Therefore, in this paper, those parameters were determined by the identification method as follows, using the measured dynamic characteristics data. First of all, \(\omega_n\) can be found from the intersecting point on a plot of \(k_t-k_r\) curves versus the frequency over various amplitudes. Setting \(\lambda=1\) in Equation (1) and (2), we obtain

\[
k_{t1} = A^2Kv1(1 - \sigma) + k_{r1}
\]  

(4)

\[
c_{t1} = A^2Kv1\sigma/(2\zeta_1\omega_n) + c_{r1}
\]  

(5)

Providing \(\lambda = \infty\) in equation (1), then

\[
k_{\infty} = A^2Kv1 + k_{\infty}
\]  

(6)

From Equation (4) and (6),

\[
\sigma = 1 - \frac{k_{r1} - k_{r1}}{k_{\infty} - k_{\infty}}
\]  

(7)

Then, \(Kv1\) can be written as follows:

\[
Kv1 = \frac{\sigma \omega_n^2 \rho \rho / l}{s}
\]  

(8)

where \(\rho\): liquid density.
For the definition of $\sigma$, $K_v2$ is written as:

$$K_v2 = \frac{1-\sigma}{\sigma} K_v1$$

(9)

From the equation (4),

$$A = \frac{k_{r1} - k_{r2}}{(1-\sigma)K_v1}$$

(10)

From equation (5) & (6); $\zeta$ at $\lambda=1$ can be expressed as follows:

$$\zeta_1 = \frac{\sigma}{2\omega_n} \cdot \frac{k_{r2} - k_{r1}}{c_{r1} - c_{r2}}$$

(11)

Finally submitting $\zeta_1$ into equation (3) & letting $\lambda=1$; $\kappa$ can be obtained as follows:

$$\kappa = \frac{3\pi \zeta_1^6}{2 \lambda_0^2 \omega_n} \left( \frac{16}{d^2 \omega_n} \right)$$

(12)

The actual characteristics were measured at room temperature by a servo-hydraulic type dynamic loading tester (maximum force: ± 200 N, maximum dynamic displacement: ±10 mm, frequency range: 5-1000 Hz). The parameters of the hydraulic mount, determined by the afore-mentioned method, are shown in Table 1. Using the determined parameters, the calculated results of the dynamic stiffness and the damping coefficient are shown in Figure 3 (a & b), and the calculated results agree closely with the measured data. Therefore, we can say that the numerical model of a hydraulic mount would be available with reasonable accuracy for reproducing the dynamic characteristics of the mount, including the frequency and the amplitude dependency. We treated the characteristics of the major principal elastic axis of a hydraulic mount in the manner mentioned above. However, for the other principal directions of a hydraulic mount or in case of a rubber mount, a multiple regression analysis was applied to represent the characteristics in this paper. The following equations show that the stiffness $k$ is expressed as a function of the frequency $f_r$ and amplitude $x$, and the damping coefficient $c$ is treated as a function only of the frequency $f_r$.

$$k(f_r, x) = p_0 + p_1 f_r + p_2 x + p_3 f_r^2 + p_4 f_r x$$

(13)

$$c(f_r) = q_1 f_r + q_2 f_r^2$$

(14)

5. Vibration Analysis

5.1 Equation of Motion

The equation of motion of an engine can be written by Equation (15), regarding that a rigid body vibrates within a

$$[M]\{\ddot{u}_G\} + [C]\{\dot{u}_G\} + [K]\{u_G\} = \{f\}$$

(15)

Where $[M]$, $[C]$, $[K]$ are matrices of mass, damping and stiffness respectively, and $\{f\}$, $\{u_G\}$ are vectors of force and displacement at the centre of gravity.
<table>
<thead>
<tr>
<th>ITEM</th>
<th>SYMBOL</th>
<th>VALUE (no. of inlet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIQUID DENSITY</td>
<td>ρ</td>
<td>1.03 g/cm³</td>
</tr>
<tr>
<td>DOUBLE</td>
<td>1.03 g/cm³</td>
<td></td>
</tr>
<tr>
<td>DYNAMIC VISCOSITY</td>
<td>ν</td>
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</tr>
<tr>
<td>DOUBLE</td>
<td>7 mm²/s</td>
<td></td>
</tr>
<tr>
<td>ORIFICE SECTION AREA</td>
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</tr>
<tr>
<td>DOUBLE</td>
<td>100.8 mm²</td>
<td></td>
</tr>
<tr>
<td>LENGTH</td>
<td>l</td>
<td>209 mm</td>
</tr>
<tr>
<td>DOUBLE</td>
<td>189 mm</td>
<td></td>
</tr>
<tr>
<td>HYDRAULIC DIA</td>
<td>d</td>
<td>55 mm</td>
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<tr>
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<tr>
<td>RESONANT FREQUENCY</td>
<td>f_n</td>
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<td>EFFECTIVE PISTON AREA</td>
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<tr>
<td>VOLUME STIFFNESS</td>
<td>K_v1</td>
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<tr>
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<td>VOLUME STIFFNESS</td>
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<td>PRESSURE LOSS FACTOR</td>
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</tr>
</tbody>
</table>

TABLE - 1 : PARAMETERS OF HYDRAULIC MOUNT

6. Experimental Method

In this experiment, we used the servo hydraulic machine for the mass and moment of inertia were known, instead of an actual engine. To simulate an engine shake, the experimental set-up was directly laid on the floor of a large-scale electric hydraulic exciter, and the machine load cell was vertically excited to provide vibration to the experimental device.

Figure 2 : Experimental set up for testing
7. Results:

7.1 Correlation between experimental results and simulation

The simulated results coincided fairly closely with the experimental frequency responses, all of which behaved differently according to the amplitude. Therefore, the analysis with respect to the frequency and the amplitude dependency is effective in predicting behaviour in engine shake.

![Graph (a)](image1)

**Figure 3:** Correlation between Calculation & Actual measurement in dynamic characterization of Hydraulic Engine Mount (courtesy: Sujan Cooper Standard AVS)

(a): Loss Factor of Hydraulic Mount
(b): Dynamic Stiffness of Hydraulic Mount

7.2 Influence of amplitude on dynamic characterization

The excitation test for dynamic characterization showed that the amplitude markedly affected the loss factor ($\phi_k$) & dynamic stiffness at around 13 Hz, 0.05 mm amplitude is reduced further than that of the 1 mm amplitude. We conclude that as the amplitude decreases, the loss factor & dynamic stiffness, as shown in Figure 4.

![Graph (a)](image2)

**Figure 4:** Influence of amplitude in Hydraulic Engine Mount (courtesy: Sujan Cooper Standard AVS)

(a) Loss Factor
(b) Dynamic Stiffness
7.3 Effect of two Orifice / two inlet to the inertia track:

Figure 5 shows the frequency responses in engine shake for number of orifice section areas of the hydraulic mount. As the number of orifice increased to two, the loss factor displays a shift of 7-9 Hz. Therefore, the current two inlet is considered to be most preferable in this system.

![a) Single Orifice](image1)

![b) Double Orifice](image2)

Figure 4: Cassette with single & double orifice (courtesy: Sujan Cooper Standard AVS)

![Figure 5: Effect of no. of orifice at low frequency dynamic stiffness spectra](image3)

7. Conclusion

1. Some parameters of a hydraulic mount were identified by its measured dynamic characteristics, and the characteristics regenerated by the mathematical model based on the parameters agreed closely with the original data.
2. It is imperative to consider the frequency and amplitude dependency of the mount characteristics to predict the shaking behaviour, especially in case where hydraulic mounts are included in the engine suspension system.
3. As for an idling vibration, both the excitation test and the simulation showed approximately the similar frequency response.
4. We performed analysis of loss factor directly with the no. of orifice of the hydraulic mount as an approach to mounting system design.
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