Design of Constant Force Compliant Mechanisms

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***Abstract*—Compliant mechanisms realize mechanism functions by utilizing the elastic deformations of flexible components rather than the relative motions of rigid joints. The advantages of compliant mechanisms stem from the removal or replacement of rigid joints, which include the elimination of backlash, friction, wear and lubrication, the reduction of vibration and noise, the decreased manufacturing and assembly cost, and the increased precision. Because of the integrated motion and force behavior and the nonlinearities of large deformations, designing compliant mechanisms is much more challenging than rigid mechanisms. Constant force compliant mechanisms produce an output force that does not change for a large range of input motion and have many different applications. A method is introduced in this paper for designing constant force compliant mechanisms. A designed constant force compliant mechanism is modelled as a network of variable width spline curves which are defined by their interpolation circles. The design of constant force compliant mechanisms is systematized as optimizing the independent parameters of the variable width spline curves. The presented method is demonstrated by the design of a constant force compliant mechanism in the paper.**

Keywords—compliant mechanism; constant force; design; spline interpolation; interpolation circle.

I. INTRODUCTION

Mechanisms are mechanical devices utilized for transferring or transforming motion, force or energy [1]. Conventional mechanisms are rigid mechanisms that consist of rigid links connected by kinematic joints. A desired output motion in the output link of a rigid mechanism is generated by an input motion in the input link through the relative motions of connected rigid links. Conventional rigid mechanisms rely on kinematic joints to generate desired output motions. A kinematic joint (also called kinematic pair) is a connection between two or more links, which allows some relative motion between the connected links [2]. Kinematic joints can be classified in different ways such as contact, degree of freedom, the number of links joined, and physical closure. The type of contact between the connected links can be point, line or surface. Joints with surface contact are called lower pairs. The term higher pair is for joints with point or line contact because of the zero area of point or line and the high contact stress. The number of degrees of freedom allowed by a joint can be one, two, three, four or five. The number of links joined by a joint can be two or more. Joint order is defined as the number of links joined minus one. The type of physical closure of a joint can be either form or force [2].

A form-closed joint keeps all joined links together by its geometry. For example, a pin in a slot is a form-closed joint. The pin can translate along the slot and rotate in the slot, and there are two degrees of freedom allowed by the joint. A slider in a two-sided slot is also a form-closed joint. In this case, the slider can only translate along the slot, but cannot rotate in the slot. Thus, the joint has one degree of freedom. In a form-closed joint, there must be a clearance to allow the relative motion between the joined links. The clearance can only be reasonably small, but cannot be eliminated. The clearance is a potential source for backlash, noise and vibration.

Force-closed joints have no clearance issue, but extra power is necessarily needed to overcome the external force that is used to maintain the closure. In both form-closed and force-closed joints, friction and wear is inevitable and can only be reduced by high surface finish and lubrication. High finish of contact surface in kinematic joints results in high manufacturing cost. Besides inevitable friction and wear, the connected links in kinematic joints have to be assembled together. Assembly time and cost cannot be eliminated.

Inspired by nature, compliant mechanisms take advantage of elastic deformations to realize mechanism functions instead of utilizing kinematic joints. The configuration of a compliant mechanism is usually a piece of elastic material without any kinematic joint. The jointless configurations of compliant mechanisms provide them with merits that include the elimination of backlash, friction, wear and lubrication, the reduction of vibration and noise, the decreased manufacturing and assembly cost, and the increased precision. Compared with traditional rigid mechanisms, compliant mechanisms have advantages of light weight and easy miniaturization that are very helpful in many applications and environments such as aerospace and microelectromechanical systems (MEMS) [3].

Although compliant mechanisms have amazing advantages, they also face tough challenges that have to be surmounted in their designs. Because of the integrated motion and force behaviour in a jointless elastic material and the nonlinearities of large deformations, designing compliant mechanisms is much more difficult than rigid mechanisms. Besides, the motion of compliant mechanisms is often more limited than traditional rigid mechanisms since the deformation of a flexible component is constrained by what it can undergo before failure. In addition, fatigue life needs to be considered for many compliant mechanisms. When the deformation in a compliant mechanism is repeated during its life, fatigue loads are present and the fatigue life must exceed the expected life of the compliant mechanism. All the challenges facing compliant mechanisms have to be considered in the designs of compliant mechanisms [4].

A constant force mechanism provides an output force that does not change for a large range of input motion. There are a wide variety of applications for constant force mechanisms that include gripping devices to hold delicate or fragile parts of different sizes, electronic connectors to sustain connection despite disturbances and part tolerances, and coupling devices to apply a constant force between a machine and its end effector [3-4]. Compliant mechanisms have been developed for generating constant force purposes [5-6].

The performance of a constant force mechanism is characterized by the force (*F*) applied to it and the deflection (*D*) which the applied force results in. The slope of the *F-D* curve is its stiffness of the mechanism denoted by *k*. If the relationship between force and deflection is represented by a general function $F = F(D)$, mechanism stiffness can then be defined as $k(D) = dF/dD$. For an ideal constant force mechanism, the force does not change in the entire range of the deflection, i.e., its stiffness is zero from zero deflection (D_0) to maximum deflection (D_m) as shown in Fig. 1. F_d is the desired output force for the mechanism.

The resistance force from an elastic component is zero when there is no deflection in it. Then, the ideal *F-D* curve shown in Fig. 1 is impossible for a constant force mechanism that is based on elastic components since the output force is required to be nonzero for zero deflection. When the starting section of the horizontal line in Fig. 1 is replaced by a steep slope, the *F-D* curve becomes that shown in Fig. 2. The *F-D* curve is now possible for constant force mechanisms that are composed of elastic components to generate. The output force is increased from zero to F_d when deflection changes from naught to D_1. The desired constant force is produced in the deflection range of D_1 to D_m.

There is a sharp corner in the *F-D* curve shown in Fig. 2, which appears at deflection of D_1. It is difficult for an elastic component to have a *F-D* curve with a sharp corner since its force deflection relationship does not change suddenly and is usually a smooth curve. Additionally, the *F-D* curve in Fig. 2 is absolutely horizontal after D_1 and has no fluctuation. It is also difficult that the resistance force from an elastic component does not have any change in the deflection range of D_1 to D_m. The realistic *F-D* curve for a constant force mechanism based on elastic components is that there is no sharp corner in the entire curve and there is only small fluctuation in the deflection range of D_1 to D_m, which is shown in Fig. 3. Although the output force fluctuation from a constant force mechanism cannot be eliminated, it can be reduced or minimized through the optimal design of the mechanism. The desired constant output force is generated by a compliant mechanism in this paper. A design method is presented and discussed in the paper to minimize the output force fluctuation in the required deflection range.

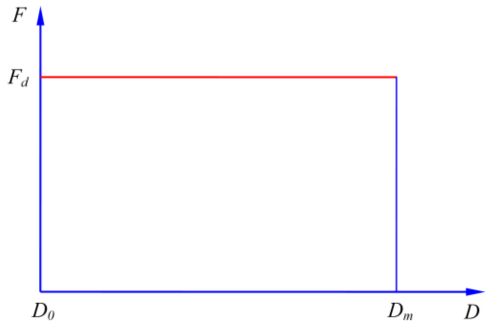

Fig. 1 The ideal *F-D* curve of a constant force mechanism.

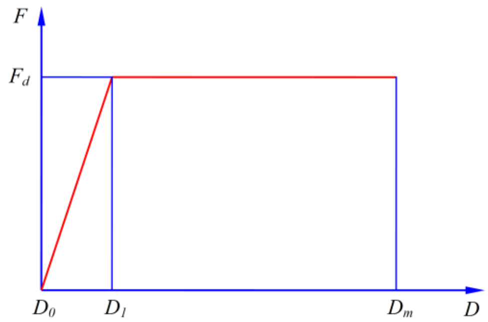

Fig. 2 The desired *F-D* curve of a constant force mechanism.

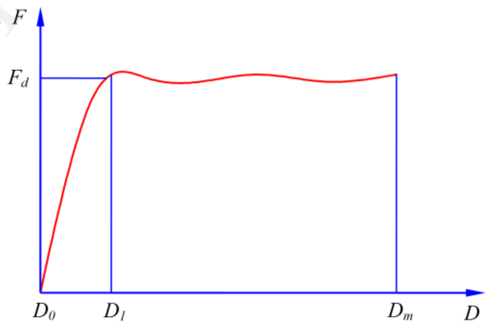

Fig. 3 The realistic *F-D* curve of a constant force mechanism.

The remainder of the paper is organized as follows. The design formulation on constant force compliant mechanisms is provided in section II. The optimization approach of design parameters is presented in section III. Section IV is on the optimal design of a constant force compliant mechanism using the design method introduced in this paper. Conclusions are finally drawn in section V.

II. DESIGN FORMULATION FOR CONSTANT FORCE COMPLIANT MECHANISMS

A constant force compliant mechanism is composed of elastic components. If elastic components are considered as building blocks, a constant force compliant mechanism is then made up of a network of building blocks [7-8]. The configuration of an elastic component is described by its topology, shape and size. The topology of the building blocks used in this paper for constant force compliant mechanisms is curved beams that have variable perpendicular widths, but have no internal holes [9]. The shape of a curved beam is decided by its center curve while its size depends on its perpendicular width. Both its shape and size of a curved beam can be defined by a variable width spline curve [10]. The center curve of a variable width spline curve is a spline curve that interpolates a set of interpolation points.

Besides spline interpolation, Lagrange interpolation is also a popular interpolation approach, but it has a serious intrinsic drawback. The degree of a Lagrange polynomial is the total number of interpolation points minus one, which is undesirably high when the number of interpolation points is not low. High degree polynomials have a strong tendency to oscillate [11]. The Agnesi's cubic (also called versiera or resonance) curve is used here as an example to show the drawback of Lagrange interpolation. The parametric equations of the Agnesi's cubic curve are as follows.

$$x(\theta) = a\tan(\theta) \quad (1)$$

$$y(\theta) = a\cos^{2}(\theta) \quad (2)$$

In (1) and (2), a is the diameter of a circle that is tangent to the x-axis and has its center along the y-axis. The Agnesi's cubic curve is tangent to the peak of the circle and symmetric to the y-axis. θ is in the range of $-\pi/2 < \theta < \pi/2$. The asymptote of the curve is the x-axis. The solid red curve in Fig. 4 shows an versiera curve. Eleven points along the curve are chosen as interpolation points, which are filled small circles in Fig. 4. When Lagrange interpolation is used to interpolate the 11 points, the result is the dotted curve in Fig. 4. Although the dotted curve passes through the 11 interpolation points, it is not close to the solid curve, especially at the left and right ends where the dotted curve vibrates a lot and is far away from the solid curve.

Piecewise polynomials are used in spline interpolation, which leads to the independence between the number of interpolation points and the degree of polynomials. A cubic spline interpolation curve consists of a set of polynomials of degree 3 that are smoothly connected at the interpolation points. Any two adjacent polynomials have continuous slope and curvature at their shared internal interpolation point. The two end points of a spline curve can have different conditions, which include natural end conditions (two end curvatures are set as zero), not-a-knot end conditions (the third derivative is continuous at both the first and last internal points) or clamped end conditions (two end slopes are specified). When spline interpolation is used to interpolate the 11 points in Fig. 4, the result is shown in Fig. 5. The interpolation curve (the dotted curve in Fig. 5) is very close to the target curve (the solid curve). It is difficult to separate them in Fig. 5.

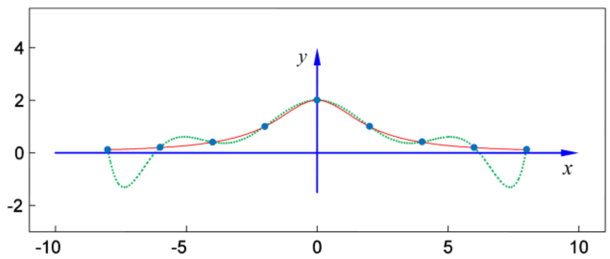

Fig. 4 A versiera curve and its Lagrange interpolation.

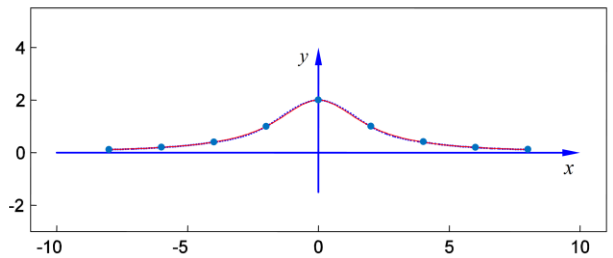

Fig. 5 The versiera curve and its spline interpolation.

A variable width spline curve has a center spline curve and variable perpendicular width along the spline curve. The center spline curve is determined by its interpolation points. If an interpolation width parameter is added to each interpolation point, width can then be interpolated using the same piecewise polynomials like the center spline curve. An interpolation point and its corresponding interpolation width can form an interpolation circle. The center of the interpolation circle is the interpolation point while the diameter of the circle comes from the interpolation width.

Fig. 6 shows five interpolation circles denoted by P_0 to P_4. The variable width spline curve from the five interpolation circles is shown in Fig. 7. There is a chord between the centers of two neighboring interpolation circles in Fig. 6. The cumulative chord length is used as the interpolation parameter (t) in Fig. 7. Both the center spline curve and its variable width are smooth. However, the width can become unsmooth and the variable width spline curve can have an undesirable cusp when half of the width at a point along the center spline curve is greater than the curvature radius of the center spline curve at that point. To avoid any unsmoothness, the following constraint has to be satisfied along the entire center spline curve [10].

$$R(t) > 0.5w(t) \quad (3)$$

$$R(t) = \frac{\left[\dot{x}^{2}(t) + \dot{y}^{2}(t)\right]^{3/2}}{\left|\dot{x}\ddot{y} - \ddot{x}\dot{y}\right|} \quad (4)$$

In (3), $w(t)$ is the perpendicular width at point t along the center spline curve, and $R(t)$ is the curvature radius of the center spline curve at that point.

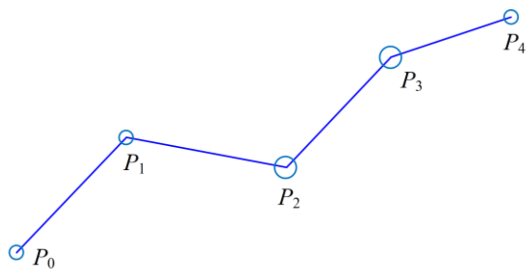

Fig. 6 The five interpolation circles of a variable width spline curve.

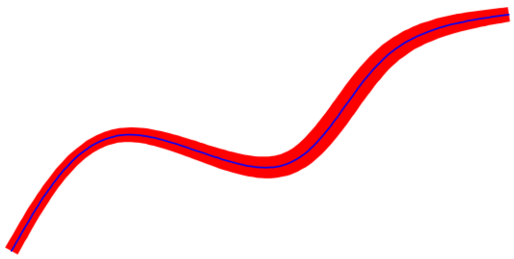

Fig. 7 The variable width spline curve defined from the five interpolation circles of Fig. 6.

III. DESIGN VARIABLE OPTIMIZATION FOR CONSTANT FORCE COMPLIANT MECHANISMS

A constant force compliant mechanism is composed of elastic components. Each elastic component is modeled as a variable width spline curve in this paper. A set of interpolation circles define a variable width spline curve that has a center spline curve and variable perpendicular width along the center curve. Three parameters are needed to decide the location and diameter of an interpolation circle. The design variables for a constant force compliant mechanism are then the parameters used for deciding all interpolation circles. The design of a constant force compliant mechanism is thus to optimize the design variables for the desired constant force generation.

The values of the independent design variables of a constant force compliant mechanism are optimized by using the Global Optimization Toolbox of MATLAB [12-13] in this paper. The Global Optimization Toolbox provides approaches that find the optimal solutions to design problems with multiple local optima. The Global Search Solver in MATLAB's Global Optimization Toolbox is adopted in the paper to search for the optimal design parameters.

The performance of a design candidate has to be evaluated in the optimization process. In this paper, finite element analysis software ANSYS is used for the evaluation of a designed constant force compliant mechanism [14-15]. With the provided parameters for the interpolation circles, the force, deflection and stress of the related constant force compliant mechanism are analyzed by ANSYS. An ANSYS batch file is first created in MATLAB on nodes, elements, material properties, boundary conditions and input information. The batch file is then called from MATLAB and executed in ANSYS. An output file on the performance of the designed constant force compliant mechanism is generated in ANSYS after executing the batch file. MATLAB inputs the ANSYS output file and calculates the objective and constraint functions for optimization. The data exchange between MATLAB optimization and ANSYS finite element analysis is based on ANSYS Parametric Design Language in the paper.

IV. DESIGN OF A CONSTANT FORCE COMPLIANT MECHANISM

A constant force compliant mechanism is to produce an output force that does not change as the mechanism deflection progresses. The desired constant force (*F*) is 20 N in this design example. The range of deflection (*D*) of the designed constant force compliant mechanism is from 0 (D_0) to 20 mm (D_m). The output force is required to be almost constant from the deflection of 5 mm (D_1) to 20 mm (D_m).

Fig. 8 shows the design domain, which is 100 mm x 100 mm. The constant force compliant mechanism is composed of two elastic components that are symmetric to the middle horizontal line of the design domain. Each elastic component is modelled by a variable width spline curve and defined by five interpolation circles (P_0 to P_4) as shown in Fig. 8. The right ends of the elastic components are fixed at the right edge of the design domain (the solid vertical line in Fig. 8) and marked by filled circles in Fig. 8, which are either at the middle of the solid line (the two filled circles coincide in this case) or at the ends of the solid line (the two filled circles are separated in this case). There is a horizontal T-shaped bar at the left end of the design domain. The bar is rigid and has a height of 50 mm, and can translate along the middle horizontal line of the design domain. The force and deflection are transferred from the rigid bar to the two elastic components. The left ends of the elastic components are fixed at the rigid bar, which are either at the middle of the rigid bar (the two interpolation circles coincide in this case) or at the ends of the rigid bar (the two interpolation circles are separated in this case).

The material for the constant force compliant mechanism is engineering plastic with yield strength of 71 MPa and modulus of elasticity of 2200 MPa. The out-of-plane thickness is set at 4 mm. The in-place width is varied from 1.0 mm to 3.0 mm. The two variable width spline curves are symmetric, so only the top variable width spline curve is lettered in Fig. 8. Each of the internal interpolation circles (P_1, P_2 and P_3) has three independent design variables (two location variables and one diameter variable). Any of the two end interpolation circles (P_0 and P_4) has only one independent design variable (diameter variable) since its location has been set. Thus, there are totally 11 parameters to be optimized, which are represented as a design variable vector *X*.

$$X = \lfloor w_0 \quad p_{1x} \quad p_{1y} \quad w_1 \quad p_{2x} \quad P_{2x} \quad w_2 \quad P_{3x} \quad P_{3y} \quad w_3 \quad w_4 \rfloor \tag{5}$$

In (5), *w*'s are the diameters of the corresponding interpolation circles. The compliant mechanism is designed to generated a constant output force. As shown in Fig. 8, force *F* is desired to be 20 N when deflection *D* is from D_1 (5 mm) to D_m (20 mm). The design objective is to minimize the error between the actual force from the compliant mechanism and the desired force when deflection *D* is from D_1 to D_m. This error is measured by the average deviation at four deflections (D_1, D_2, D_3, D_4, which are 5, 10, 15 and 20 mm, respectively) as follows.

$$FE = \frac{1}{4}\sum_{j=1}^{4}\left|F_{a,\,j} - F_{d,\,j}\right| \tag{6}$$

FE is the average force error. $F_{a,\,j}$ is the actual force generated by the compliant mechanism when deflection D_j is input to the compliant mechanism while $F_{d,\,j}$ is the desired constant force (20 N). The maximum stress in the compliant mechanism is constrained to be below its allowable value.

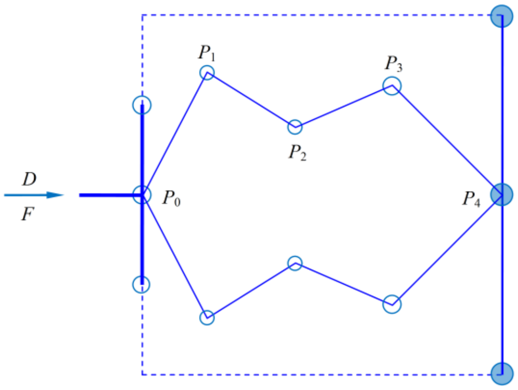

Fig. 8 The design domain of the constant force compliant mechanism.

When both the left and the right ends of the variable width spline curve are located at the middle, the synthesis result is shown in Fig. 9. The design variable vector X for this solution is

$$X = [2.78 \quad 32.98 \quad 14.71 \quad 1.06 \quad 45.74 \quad 10.05 \quad 1.26 \quad 74.09 \quad 13.83 \quad 1.14 \quad 2.66] \tag{7}$$

In this solution, the desired and actual forces at D_0, D_1, D_2, D_3 and D_4 are: (0, 0), (20, 17.30), (20, 19.02), (20, 19.46) and (20, 20.00). The spline curve that interpolates the actual forces is shown in Fig. 10 as the red curve. The maximum stress in the compliant mechanism is 68.96 MPa, which happens when the compliant mechanism has deflection of 20 mm. Fig. 11 shows the undeformed and deformed beam elements of the compliant mechanism, which is from ANSYS with deflection of 20 mm.

When the force, deflection and stress of the compliant mechanism are analyzed in ANSYS, the input deflection of 20 mm is divided into 4 even load steps and geometric nonlinearity command “NLGEOM” is turned on. The compliant mechanism is discretized into beam elements and modeled by BEAM188 that allows tapered beam cross-sections.

When the left end of the variable width spline curve is located at the end of the rigid bar and the right end is at the middle, the synthesis result is shown in Fig. 12. The design variable vector X for this solution is

$$X = [2.52 \quad 24.38 \quad 36.92 \quad 1.40 \quad 51.37 \quad 26.00 \quad 1.34 \quad 84.74 \quad 23.73 \quad 1.58 \quad 2.96] \tag{8}$$

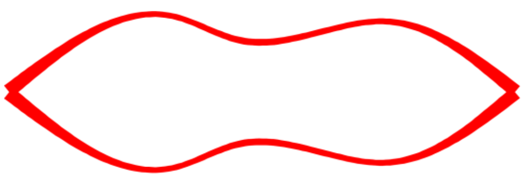

Fig. 9 Design 1 of the constant force compliant mechanism.

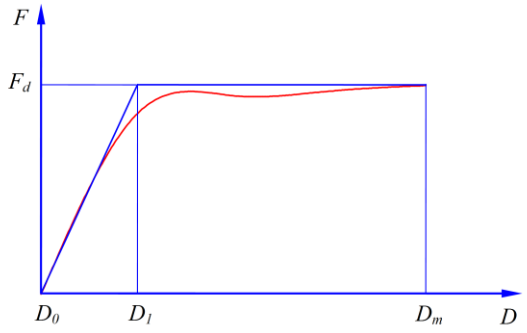

Fig. 10 The desired and actual *F-D* curves of Design 1.

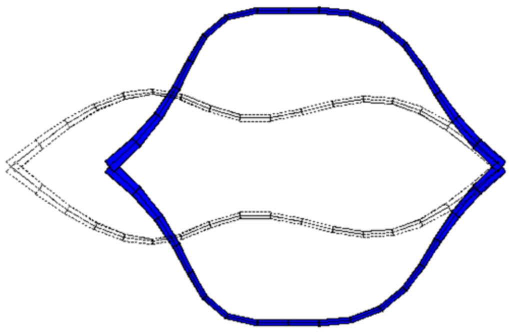

Fig. 11 The deformed compliant mechanism of design 1.

In this solution, the desired and actual forces at D_0, D_1, D_2, D_3 and D_4 are: (0, 0), (20, 17.08), (20, 19.80), (20, 20.04) and (20, 20.35). The actual *F-D* curve of this solution is shown in Fig. 13. The maximum stress is 67.97 MPa when the compliant mechanism has the maximum deflection of 20 mm. The deformed compliant mechanism of this solution is shown in Fig. 14.

The two designs of the constant force compliant mechanism shown in this paper produce close *F-D* functions, but have different locations of the left end of the elastic component in the design domain. One design can be chosen based on the application.

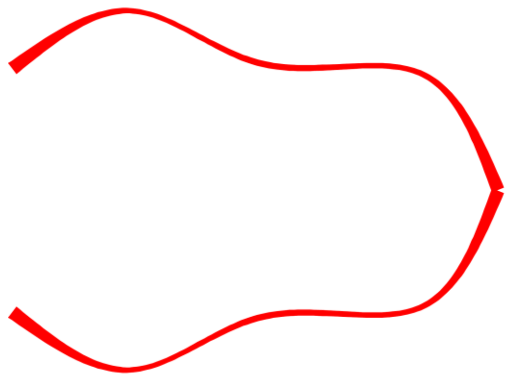

Fig. 12 Design 2 of the constant force compliant mechanism.

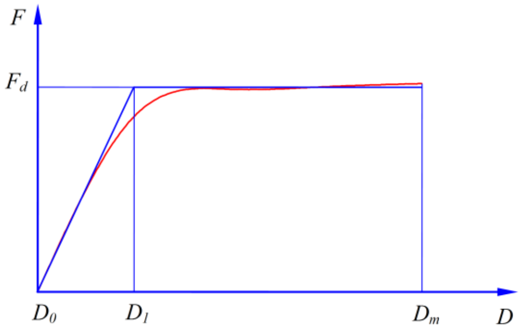

Fig. 13 The desired and actual *F-D* curves of Design 2.

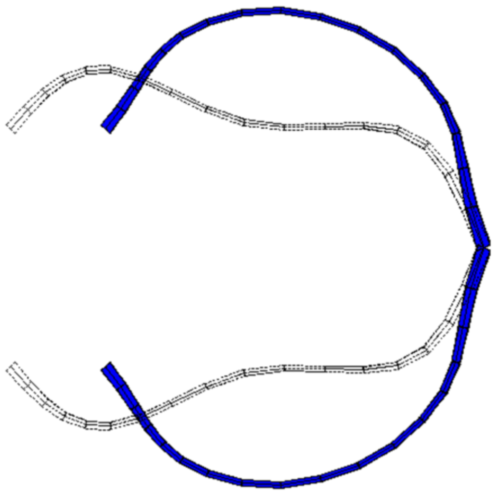

Fig. 14 The deformed compliant mechanism of design 2.

V. CONCLUSIONS

A design method of constant force compliant mechanisms is presented in the paper. A compliant mechanism is composed of elastic components. Each elastic component is modeled as a variable width spline curve. A designed constant force compliant mechanism is represented by a network of variable width spline curves in this paper. A variable width spline curve includes a center spline curve and variable perpendicular width, and is defined by its corresponding interpolation circles. Three parameters are needed to fully decide an interpolation circle (its location and diameter). The design variables for a constant force compliant mechanism is the independent parameters for all related interpolation circles. The design objective is to minimize the deviation between the desired constant force and the actual output from the designed compliant mechanism.

The Global Optimization Toolbox of MATLAB is employed by the authors of the paper for the optimization of the design variables. The design objective or force deviation is measured by the average difference between the desired constant force and the actual output force from the compliant mechanism under certain deflections. The maximum stress in the compliant mechanism is constrained below the yield strength of the material. The force, deflection and stress of a designed compliant mechanism are analyzed using ANSYS. The data exchange between MATLAB and ANSYS is based on ANSYS Parametric Design Language. A constant force compliant mechanism is designed in the paper to verify the effectiveness and demonstrate the procedure of the presented method.

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