

Design of A Hybrid Transmission for Electric and Engine Powered Transmission

Transmission Design for A Parallel Hybrid Vehicle

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Abstract— with the increasing amount of pollution and threat of global warming, the Toyota research team did come up with the idea of the Hybrid vehicles in year 1999. The Battery electric vehicle (BEV) is different than Hybrid electric vehicle (HEV) as they use single source of power generation i.e. electric motor. This project was launched as the Toyota Prius 1st Generation. A general hybrid car have two or more engines. Toyota Prius 1st Generation had one internal combustion engine and two electric motor. Both of this electric motors are AC motor-generator (MG) with permanent magnet rotor. One of this motor (MG1) is used to start the IC engine and as the alternator to charge the high voltage battery, while other motor (MG2) is used as the primary drive motor and as the alternator to charge the high voltage battery while car being on the regenerative mode. Honda Hybrid cars started with single motor which later changed to dual motor just like as in Toyota Prius.

There are many advantages for using this kind of vehicles. To start with being more environment friendly, providing higher pick-up and providing better fuel economy. Though with stated advantages, there are also certain disadvantages as such as less power development, poor handling due high weight of high voltage batteries and higher maintenance. This problems are constantly being worked upon and changes are made to hybrid cars to make it more reliable, economical and safe.

Keywords—Transmission design, ANSYS, Solidworks, Gear calculations, Planetary gear-train, Parallel type hybrid vehicle, 30Ni4Cr1, EN30A.

I. INTRODUCTION

According to the Wikipedia, the proper definition for the Hybrid vehicle is as follows,” A Hybrid vehicle is a vehicle that uses two or more distinct type of power source as such as an IC engine and an electric motor. E.g. in Diesel-electric trains using diesel engines and electricity from overhead liner & Submarines that uses diesel when surfaced and batteries when submerged. Other means to store energy include pressurized fluid in Hydraulic Hybrids”^[2]

There are total five types of hybrid vehicles. First is a Mild Hybrid vehicle, which unlike BEV’s uses IC engine, an electric battery and a motor/generator. It is least electrified type of HEV and uses oversized starter motor which can also

be used as the generator called integrated starter-generator (ISG) or belted alternator starter (BAS). Second is Series Hybrid vehicle, where engine is coupled to a generator that charges the battery which provides power to the wheels through transmission or a direct coupling. Third is Parallel Hybrid vehicle, where there are two parallel paths to power the wheels, an engine path and an electrical path. Transmission couples the generator/motor and engine allowing either or both to power the wheels. This is clearly complex mechanism than the series one. Fourth is Series-Parallel Hybrid vehicle, this has both series and parallel energy flow paths. Complexity varies with number of motors used. This one is classified further as complex hybrids, split-parallel hybrids and power-split hybrids. Fifth is Plug-in Hybrid vehicles (PHEV), which can be charged by plugs from electricity walls. They can be easily distinguished by larger size of battery packs than other HEV’s. This type can be made with any Hybrid configuration.^[3]

The topic that is choose by our team is regarding Hybrid vehicle’s transmission design. In this project, the gear box is designed for a Hybrid car with the max speed, acceleration, fuel economy and distance to achieve max speed that will be generated by motor and motor working with the engine. Alongside this calculations, certain essential simulations are also being done with CAD modelling. During this calculations, the assumptions were made considering a passengers car specifications in general. E.g. gross weight was assumed to be 1900kg, final gear reduction as 3:1 and max speed constraint was taken as 55.56m/s.

In this transmission design, we have considered the parallel HEV’s from above discussed types of HEV’s we have used a single motor/generator instead of two motors thus to make design compact and one that can be used in practice. We made use of two planetary gear set whose sun gears are connected to each other through shaft that is connected to the motor/generator at the other end.

II. METHODOLOGY

A. Selection of the gear-drives

A Planetary gear-set has way more advantages than that of the conventional gear-sets. With multiple kinematic combinations, it also provides reduction in volume which proves to be essential criteria for power to weight calculations. Thus we have used a planetary gear-set or to be called epicyclic gear-train instead of the conventional gear-train.

Planetary gear box works on planetary motion principle each stage of the planetary gear box consists of a central Sun Gear meshing with accurately positioned three Planet Gears around it which in turn mesh with the internal teeth of the outer Ring Gear. Normally, the Ring gear is stationary and forms the part of the housing, input is given to the sun gear and output is derived from the three planet gears through a planet carrier. ^[11]

Working of Planetary gear set: Any one of the three members can be used as the driving or input member. At the same time, another member might be kept from rotating and thus becomes the reaction, held, or stationary member. The third member then becomes the driven or output member. Output direction can be reversed through various combinations.

Advantages of Planetary gearbox:

- Compared to conventional gearbox has smaller dimensions.
 - Easier to sort through the constant rounds of shot.
 - Greater durability than conventional bikes in gear.
 - Easy to achieve high transmissions ratio due to the size.
- ^[11]

Disadvantages of Planetary gearbox:

- More expensive than conventional production of gearboxes.
- More complex than conventional transmissions. ^[11]

Special features of Planetary gearbox:

- All shafts are made of special alloy steel and are hardened and tempered.
- Good quality bearings for input and output shafts.
- High efficiency.
- Low noise level.
- No oil leakages.
- Taper roller bearings on output shafts for bigger models.
- Long and trouble free performance. ^[11]

B. Calculations

1) Maximum speed

a) Kinematics of the Mechanism

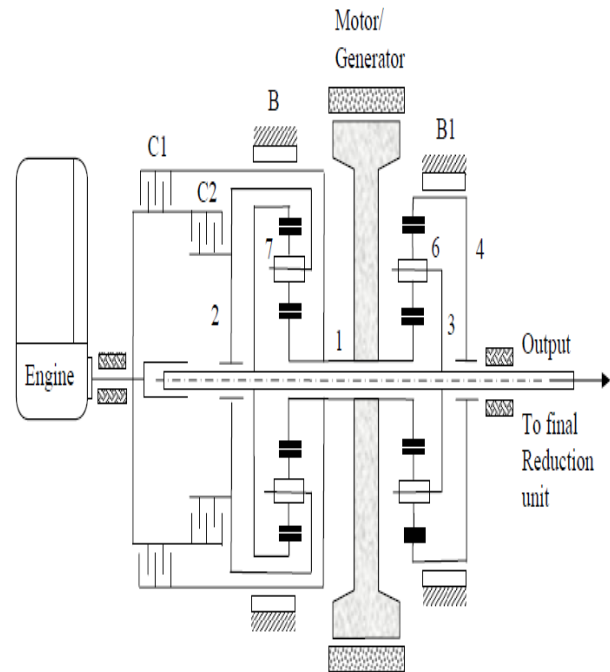


Figure 1: Transmission Gear-box for the Parallel Hybrid Car [5]

The figure above represents the schematic diagram of a motor-integrated transmission mechanism. This transmission is made up of 2 planetary gear-trains and one motor/generator. In the figure, two boldly marked parallel lines indicates a mesh. The gear-train on left is the input planetary gear-train and one on right is output planetary gear-train. The two Sun gears are connected together by link-1 i.e. a common input shaft. This link-1 here is attached to the rotor of the electric motor. The input ring gear is connected to the output carrier shaft i.e. link-3. Link-3 is connected further to the final gear reduction unit of the transmission. The engine crankshaft can be coupled to the link-1 through by a rotating clutch C1 or to the link-2 by another rotating clutch C2. The output ring gear can be grounded to the casing by a band clutch B1 and the electric motor can be held stationary by a band clutch B2.

On basis of which clutches are engaged, 5 modes of operation are made possible, namely motor mode (M), power mode (P), engine mode (E), engine/charge mode (EC) and regenerative mode (R). This modes are further classified as shown in the table below into 16 sub-modes. Note that here one of the charge modes provide CVT capability and 'X' signifies the engaging of that clutch in that mode.

Now applying the fundamental circuit equation for the kinematic analysis of the hybrid transmission, let i and j be the gear pair and k be the carrier and thus forming a fundamental circuit. The equation for the same can be written as,

$$\omega_i - \omega_k = \pm N_{ji} (\omega_j - \omega_k) \quad (1)$$

Where ω_i , ω_j and ω_k denote angular velocities of link i , j and k respectively and N_{ij} represents the gear ratio between this gears i.e. $N_{ij} = T_j / T_i$ where T_j and T_i represents the number of teeth on respective gears. The sign of this equation is positive or negative depending on whether the gear mesh is internal or external.

The gear-train in fig. 1 contains 4 fundamental circuits: (7, 3, 2), (7, 1, 2), (6, 4, 3) and (6, 1, 3). The circuit equations can be written as:

$$\begin{aligned} \omega_7 - \omega_2 &= N_{37} (\omega_3 - \omega_2) & (2) \\ \omega_7 - \omega_2 &= -N_{17} (\omega_1 - \omega_2) & (3) \\ \omega_6 - \omega_3 &= N_{46} (\omega_4 - \omega_3) & (4) \end{aligned}$$

Table 1: Clutching Condition for Different Modes and Condition of Motor Operation at that Time [5]

Operating Mode		Working clutches			
		C1	C2	B1	B2
Motor				X	
Power	P1	X		X	
	P2		X	X	
	P3	X	X		
Engine	E1	X		X	
	E2		X	X	
	E3	X	X		
Engine/Charge	EC1	X		X	
	EC2		X	X	
	EC3	X	X		
	CVT		X		
Regenerative	R0			X	
	R1	X		X	
	R2		X	X	
	R3	X	X		

$$\omega_6 - \omega_3 = -N_{16}(\omega_1 - \omega_3) \quad (5)$$

From equation (4) and (5),

$$\omega_1 + N_{41}\omega_4 - (1 + N_{41})\omega_3 = 0 \quad (6)$$

Where $N_{31} = N_{37}/N_{17}$. Similarly, equation (2) and (3) gives,

$$\omega_1 + N_{31}\omega_3 - (1 + N_{31})\omega_2 = 0 \quad (7)$$

The equation (6) and (7) relate the angular velocities of 4 co-axial links 1, 2, 3 and 4. With the angular velocities of any 2 links, we can solve equation (6) and equation (7) for the other two.

The motor will be running in the Motor and the Power mode whereas it will be free-wheeling in the Engine mode. When the car will be propelled in Engine/Charge mode or Regenerative mode, the motor will act as the generator.

b) Static Torque and Power Flow Equations

Consider the coupled planetary gear-set as a system then under the steady-state conditions, torque will be exerted on link 1, 2, 3 and 4 about the central axis of the gear set that must be summed up to zero. i.e.

$$\tau_1 + \tau_2 + \tau_3 + \tau_4 = 0 \quad (8)$$

Where τ_i denotes an external torque that has been exerted on link i. Similarly, the net power flows into the system via the four co-axial links that must sum up to be zero. i.e.

$$\tau_1\omega_1 + \tau_2\omega_2 + \tau_3\omega_3 + \tau_4\omega_4 = 0 \quad (9)$$

Eliminating the τ_3 from equation (8) and (9) and using equation (6) and (7),

$$N_{41}\tau_1 + (1 + (N_{41} / 1 + N_{31})) \tau_2 + \tau_3 = 0 \quad (10)$$

Eliminating τ_4 from equation (8) and (9) and using equations (6) and (7),

$$(1 + N_{41})\tau_1 + (1 + (N_{41} / 1 + N_{31})) \tau_2 + \tau_3 = 0 \quad (11)$$

This equations (10) and (11) relate torques exerted on the links 1, 2, 3 and 4 in terms of the gear ratios N_{31} and N_{41} . Given torques exerted on any of the two links, we can find those exerted on the other two.

c) Operating Modes Details

The data for the process are given as follows:

Input Planetary gear-train: $T_1=20, T_7=16$ and $T_3=52$.
 Output Planetary gear-train: $T_1'=20, T_6=16$ and $T_4=52$.
 Final reduction ratio=3:0
 Tire radius= 0.235m

• *Motor mode*

To start a vehicle from stationary conditions, the band clutch B1 is engaged and all the other clutches are disengaged. Using the output ring member, the electric motor drives the vehicle all by itself through the output planetary gear-train. Meanwhile, the input planetary gear-train spins freely. Thus, the output ring gear $\omega_4 = 0$. Substituting this value in equation (6) and (7) gives,

$$\omega_1 = (1 + N_{41})\omega_3 = 3.6\omega_3 \quad (12)$$

$$\omega_2 = (1 + (N_{41} / 1 + N_{31}))\omega_3 = 1.72\omega_3 \quad (13)$$

By the equation (12), it's clear that the speed ratio between the electric motor and the output link is 3.6. As the input planetary gear-train moves freely, we take $\tau_2 = 0$ and substituting its value in the equation (10) and (11) gives,

$$\tau_4 = N_{41}\tau_1 = 2.6\tau_{motor} \quad (14)$$

$$\tau_3 = -(1 + N_{41})\tau_1 = -3.6\tau_{motor} \quad (15)$$

By the equation (14) and (15), it's observed that reaction torque exerted on the link 4 is 2.6 times of the motor torque and the output torque is equal to 3.6 times that of the motor torque. The negative sign in equation (15) is the indication that the torque is being applied on the final reduction unit by the output shaft.

• *Power Mode*

When the vehicle in climbing a hill or it has the requirement of the maximum acceleration to be achieved, the vehicle goes into the power mode. For this mode to be active, the engine must be in the running condition. There are 3 sub-modes available in this mode, P1, P2, P3 & P4.

P1 mode has the rotational clutch C1 and the band clutch B1 engaged while the rest clutches are disengaged. It uses the output ring member, motor and the engine to drive the vehicle through the output planetary gear-train. Meanwhile, the input planetary gear-train moves freely. As both of the motor and engine are connected to the same link which is grounded and have the same speed, the equation (12) and (13) remains valid whereas equation (14) and (15) becomes,

$$\tau_4 = N_{41}\tau_1 = 2.6(\tau_{motor} + \tau_{engine}) \quad (16)$$

$$\tau_3 = -(1 + N_{41})\tau_1 = -3.6(\tau_{motor} + \tau_{engine}) \quad (17)$$

Hence, from the equation (16) and (17), we can observe that the reaction torque is 2.6 time the combined motor and engine torques and the output torque is equal to the 3.6 times that of the combined motor and engine torques. Moving forward and neglecting the frictional losses, the output power can be obtained as follows:

$$P_{out} = -3.6(\tau_{motor} + \tau_{engine})\omega_3 \quad (18)$$

Here, the negative sign signifies that the power is flowing out of the transmission.

P2 mode has the rotational clutch C2 and a band clutch B1 engaged while the rest of the clutches are disengaged. It uses the output ring member, motor and the engine to drive the vehicle simultaneously at the second reduction. However, the planetary gear-train is no longer free-wheeling. The engine transmits the power through entire gear-train whereas the motor transmits the power through the output planetary gear-train. The speed ratios for this are given by equation (12) and

(13) and substituting $N_{31} = N_{41} = 2.6$ in equations (10) and (11), we get,

$$\begin{aligned} \tau_4 &= N_{41}\tau_1 + (1 + (N_{41}/1 + N_{31})) \tau_2 \\ &= 2.6\tau_{motor} + 0.72\tau_{engine} \end{aligned} \quad (19)$$

$$\begin{aligned} \tau_3 &= -(1+N_{41})\tau_1 - (1 + (N_{41}/1 + N_{31})) \tau_2 \\ &= -(3.6\tau_{motor} + 1.72\tau_{engine}) \end{aligned} \quad (20)$$

Hence, from the equation (19) and (20), we can observe that the motor torque is amplified by 3.6 times and the engine torque is amplified by 1.72 times at the output shaft. Thus, the overall power output can be obtained as,

$$P_{out} = -3.6(\tau_{motor} + \tau_{engine})\omega_3 \quad (21)$$

Here, the maximum power occurs at the different point though the formula is same as the P1 mode as the speeds of the electric motor and the engine are related to the output shaft.

P3 mode has both rotating clutches C1 and C2 in engaged condition whereas the rest of the clutches are in rest. In this condition, the gear set locks up as the rigid body moving the vehicle with 1:1 ratio. The output torque and output power are given as follows:

$$\tau_3 = -(\tau_{motor} + \tau_{engine}) \quad (22)$$

$$P_{out} = -(\tau_{motor} + \tau_{engine})\omega_3 \quad (23)$$

- **Engine Mode**

One of the following modes are used when the demand for power is low and battery state is sufficiently high to handle accessory loads.

E1 Mode is very much similar to the P1 mode but the motor here is on off mode and is free-wheeling. Using the link-4 as reaction member, equation (12) and (13) counts just right for the speed ratios of the four co-axial links. As the motor is running freely, τ_{motor} is zero and thus equation (16) and (17) comes out to be:

$$\tau_4 = N_{41}\tau_1 = 2.6\tau_{engine} \quad (24)$$

$$\tau_3 = -(1 + N_{41})\tau_1 = -3.6\tau_{engine} \quad (25)$$

Here, you can see that the engine torque is amplified by 3.6 times at the output shaft. This corresponds to the first gear of the conventional automatic transmission. The output power is given as:

$$P_{out} = -3.6\tau_{engine}\omega_3 \quad (26)$$

E2 Mode is very much similar to the P2 mode but the motor here is on off mode and is free-wheeling. Using output ring gear as the reaction member while the engine is the only driving member for the vehicle through the entire gear train at the second reduction. The speed ratios for four co-axial links are given by equation (12) and (13) and substituting $\tau_1 = 0$ into the equation (19) and (20) gives,

$$\tau_4 = (N_{41}/1 + N_{31})\tau_2 = 0.72\tau_{engine} \quad (27)$$

$$\tau_3 = -(1 + (N_{41}/1 + N_{31}))\tau_2 = -1.72\tau_{engine} \quad (28)$$

And the output power is given by,

$$P_{out} = -1.72\tau_{engine}\omega_3 \quad (29)$$

E3 Mode locks up the planetary gear-train as the single rigid body and transmits the power to the output shaft at 1:1 speed ratio i.e. the direct drive. In this mode, both C1 and C2 clutches are engaged. The output torque and power are given by,

$$\tau_3 = -\tau_{engine} \quad (30)$$

$$P_{out} = -\tau_{engine}\omega_3 \quad (31)$$

- **Engine/Charge Mode**

EC1 Mode has a feature where when the motor is working as the generator, the engine also supports by giving power to the vehicle simultaneously. The engine also powers the electrical accessories and also charge the batteries. The kinematic, torque and power relationships is same as that of the P1 mode, only difference is that the applied torque is negative.

EC2 Mode has a feature where when the motor is working as the generator, the engine also supports by giving power to the vehicle simultaneously. The engine also powers the electrical accessories and also charge the batteries. The kinematic, torque and power relationships is same as that of the P2 mode, only difference is that the applied torque is negative.

EC3 Mode has a feature where when the motor is working as the generator, the engine also supports by giving power to the vehicle simultaneously. The engine also powers the electrical accessories and also charge the batteries. The kinematic, torque and power relationships is same as that of the P3 mode, only difference is that the applied torque is negative.

- **Continuously variable transmission (CVT) Mode**

It is used at moderate to high speed or as the E4 Mode. Here, the rotating clutch C2 is engaged and all rest of the clutches are disengaged. The motor is switched into a generator for charging the batteries. The clutches position makes the input planetary gear-set to work as a power splitting one input two output device. The part of the power that the engine has produced is directed to the input ring gear to drive the vehicle and the rest of the part is used to drive the input sun gear to drive the generator. In this mode, engine can run at optimum efficiency point while regulating the speed of the vehicle by controlling the speed and load on the generator. Thus, the vehicle runs as a CVT. The speed ratios for the four co-axial links can be given by the equation (6) and (7) and substituting the value 2.6 for N_{31} in equation (7) gives,

$$\begin{aligned} \omega_3 &= (-\omega_1 + (1 + N_{31})\omega_2) / N_{31} \\ &= -0.3846\omega_{generator} + 1.3846\omega_{engine} \end{aligned} \quad (32)$$

So, it is possible to run the engine at an optimal operating speed by proper control of the generator speed for and given output shaft speed ω_3 . Say that a car is moving at the speed of 100 Km/hr and output shaft speed is 2775 rpm. Now, if we want engine to move at 2400 rpm, the generator must be running at 1742 rpm. Here, link-4 spins freely and the output planetary gear-train carries no load. Substituting value of τ_4 as 0 in the equations (10) and (11), we get,

$$\tau_1 = (-1 / 1 + N_{31})\tau_2 = -0.27\tau_{engine} \quad (33)$$

$$\tau_3 = (-N_{31} / 1 + N_{31})\tau_2 = -0.72\tau_{engine} \quad (34)$$

$$P_{out} = -0.72\tau_{engine}\omega_3 \quad (35)$$

$$P_{generator} = -0.27\tau_{engine}\omega_1 \quad (36)$$

- **Regenerative Mode**

When the brakes are applied, this motor works as the generator and meanwhile 4 regenerative modes are possible.

R0 Mode where only clutch B1 is engaged and rest are disengaged the way the kinematic energy of the whole vehicle is directed through the output planetary gear-set for charging of those batteries. During this, the power flows in opposite direction of the motor mode. Maximum recovery of the kinematic energy is achieved when no brakes are applied on the vehicle.

R1 Mode has clutch C1 and B1 engaged and rest of the clutches disengaged. Part of the kinetic energy of whole vehicle is directed to output planetary gear-set for charging of the batteries while the rest of the kinematic energy is directed to the engine through the entire gear-set. During this, the power flows in opposite direction of the P1 mode. Thus, both engine and generator are responsible for generating the braking efforts on the vehicle.

R2 Mode has clutch C2 and B1 engaged and rest of the clutches disengaged. Part of the kinetic energy of whole vehicle is directed to output planetary gear-set for charging of the batteries while the rest of the kinematic energy is directed to the engine through the entire gear-set. During this, the power flows in opposite direction of the P2 mode. Thus, again, both engine and generator are responsible for generating the braking efforts on the vehicle.

R3 Mode has clutch C1 and C2 engaged and the rest of the clutches are disengaged. In this setup, the gear-set locks up together and forms a rigid body. The power flows in the opposite direction to that of P3 mode. Both engine and the generator will provide the braking efforts to the vehicle at 1:1 ratio.

d) *Shifting between the modes*

We know by knowledge of above mentioned details that shifting from M to R0 mode can be achieved very easily by merely changing the motor into a generator and vice-versa also hold true. Similarly, shifting amongst P1, E1, EC1 and R1 modes; P2, E2, EC2 and R2 modes; and P3, E3, EC3 and R3 requires only changing of motor to free-wheeling state or into a generator. Hence, we will consider M and R0 modes as the motor mode, P1, E1, EC1 and R1 modes as the first gear, P2, E2, EC2 and R2 modes as the second gear, P3, E3, EC3 and R3 as the direct drive. So re-arranging the table X on basis of above mentioned classification, we get table X.

It is also possible for a parallel hybrid to have two different driving schedules i.e. Performance schedule and economy schedule. Where first option provides good performance characteristics on expense of less fuel efficiency, the other option provides better mileage feature. In both cases, transmission starts with motor mode and then shifts to first, second, direct and then over-drive or CVT mode.

CVT mode can be applied to moderate to high speeds for charging of the batteries. When the batteries are charged enough, the transmission is further shifted to overdrive to increase the fuel economy. In each gear, the transmission can be shifted from the power mode to the engine mode or engine/charge mode when power demand is low and to regenerative mode when the brakes are applied. Below is the example for a typical performance schedule, economy schedule and engine charge schedule respectively: [5]

- M → P1 → P2 → P3 or E3 → CVT
- M → E1 → E2 → E3 → CVT
- EC1 → EC2 → EC3 → CVT

Table 2: Results for Max. Speed and Max. Acceleration for different Schedules and their respective Modes

Operating schedule	Power Modes	Max. Speed (m/s)	Max. Acceleration (m/s ²)
Performance schedule	M	14.804	7.548
	P1	11.386	13.129
	P2	16.121	10.083
	P3	45.914	2.904
Economy schedule	CVT	47.476	0.243
	M	14.804	7.5485
	E1	11.386	5.1467
	E2	23.803	2.0431
	E3	40.994	0.7677
	CVT	47.288	0.24285

Table 3: Clutching Condition for Different Modes re-arranged and Condition of Motor Operation at that Time [5]

Operating Mode		Working clutches				Motor mode
		C1	C2	B1	B2	
Motor	M			X		Motor
	R0			X		Generator
First Gear	P1	X		X		Motor
	E1	X		X		Free-wheeling
	EC1	X		X		Generator
	R1	X		X		Generator
Second Gear	P2		X	X		Motor
	E2		X	X		Free-wheeling
	EC2		X	X		Generator
	R2		X	X		Generator
Direct Drive	P3	X	X			Motor
	E3	X	X			Free-wheeling
	EC3	X	X			Generator
	R3	X	X			Generator
CVT	CVT		X			generator

2) *Number of teeth*

The criteria while choosing the number of teeth was to make overall reduction from Sun gear to Carrier gear of 3.6. Thus, by Trial and error method, we took the value of teeth on Sun gear as 20 and the value of teeth on planet gear 16, we get the value of teeth on annular ring gear as 52 by the formula stated below.

From the basic diagrams of the planetary gear train, we can say that,

$$d_r = 2 (d_p + d_s) \tag{37}$$

But $m = d / t$,
 Thus,

$$t_r = 2 (t_p + t_s) \tag{38}$$

Table 4: Assumed & Derived Total Number of Teeth of Planetary Gear-set

	No: of teeth
Sun gear	20
Planet gear	16
Annular ring gear	52

3) Selection of material

Gears are the important element of a mechanical system, which are used for variation of speed and power, failure of even a single tooth of a gear will make the machine to stop. Hence our aim is to strengthen the gear which is a key element of gear box. Thus, we need to select appropriate gear material by considering its strength, cost, hardenability and machinability. The material properties and costing of pinion and gear material were studied, and standard gear materials were identified from PSG Design Data Book. The material sorting is done on the basis of availability, cost and strength of the material.

From the below mentioned materials and their properties, the underlined material i.e. 30Ni4Cr1 or as per the British Standards, EN30A, has very high tensile and yield strength. Which is very useful for beam strength criterion. It also has very high BHN value which helps in wear strength criterion. Even from above mentioned advantages, the cost is also very economical and thus is used as our gear material.

Table 5: Different Types of Materials Available in the Market and their Properties [12]

Indian Standards	British Standards	S _u (N/mm ²)	S _y (N/mm ²)	Brinell Hardness (BHN)	Cost (Rs/Kg)
40Ni2Cr1 Mo28	EN24	950	600	27	80
C45	EN8	70	460	229	80
C55MN75	EN9	780	360	255	67
35Ni1Cr6 0	EN111	850	540	248	80
<u>30Ni4Cr1</u>	<u>EN30A</u>	<u>1550</u>	<u>1330</u>	<u>444</u>	<u>67</u>
40Cr1	EN18	850	540	248	-

4) Shaft design

Table 6: Given or Assumed Data used for the Calculations of the Shaft Design

Data obtained from the fundamental force equation of the helical gear:[]			
For planetray gear-set 1		For planetary gear-set 2	
P _t	5331.31 N	P _t	7649.2 N
P _r	2092.83 N	P _r	3002.75 N
P _a	2153.99 N	P _a	3090.5 N
Material for the shaft		45C8	
S _{ut}	600 N/mm ²		
S _{yt}	380 N/mm ²		
K _b	2		
K _t	1.5		

Here, P_t, P_r and P_a are the different type of the forces that are acting on the respective planetary gear-set while being in operation. P_r is the Radial component of the force, P_t is the Tangential component of the force and the P_a is the thrust component of the force. [13]

$$P_t = 2M_t / d_p \tag{39}$$

$$P_r = P_t [\tan \alpha_n / \cos \phi] \tag{40}$$

$$P_a = P_r \tan \phi \tag{41}$$

Step 1:

In this starting step, as we want to determine the shaft diameter, we have to calculate two values (0.30)S_{yt} and (0.18)S_{ut} on the basis of maximum shear stress theory and select the lower most value amongst the both. The selected value is multiplied by 0.75 which results into the τ_{max}. This value is the permissible maximum shear stress

Step 2:

Considering horizontal components of forces acting on the shaft,

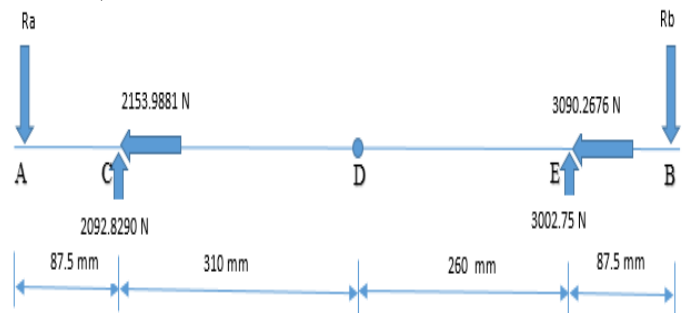


Figure 2: Horizontal Forces Equilibrium

Taking equilibrium of forces and then taking the moment @ A, we will get the values for R_a and R_b. Now, similarly we will consider all of the vertical components of forces acting on the shaft and similarly we will take equilibrium of the forces and take moment @ A, we will get the values for

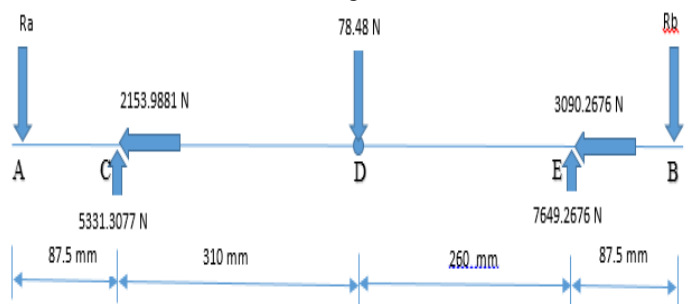


Figure 3: Vertical Forces Equilibrium

Step 3:

In this step, we will find the maximum bending moment for given scenario. As we can see from the figure, bending moment occurs at point C and point E. Among this points, the maximum value for the bending moment occurs at point E

Step 4:

Based on the ASME code made on theory of shear stress failure, the formula for the shaft diameter for different M_t can be given as follows: [13]

$$D^3 = (16 / \pi \cdot \tau_{max}) \{ \sqrt{(k_b \cdot M_b)^2 + (k_t \cdot M_t)^2} \} \tag{42}$$

Table 7: Results for the Shaft Calculations

τ_{max}	81 N/mm ²
$(M_b)_c$	394376.82 Nmm
$(M_b)_c - max$	913087.94 Nmm
d @ 111.8 Nm	48.67
d @ 230 Nm	48.89
d @ 330 Nm	50

C. Safety considerations

1) Beam & Wear strength of gears

We assumed value of module as 2.5, 3, 3.5 and 4 and out of this values, values other than 4 came unsafe so by trial and error method, the value of the module for the planetary gear-set was assumed 4mm.

a) Beam Strength

In order to determine the beam strength of the helical gear, it is considered to be equivalent to a formative spur gear i.e. an imaginary spur gear in a plane that is perpendicular to the tooth element. [14] The formula for finding maximum value of tangential force that can be transmitted by the gear without undergoing the bending failure i.e. beam strength is,

$$S_b = m_n b \sigma_b Y \tag{43}$$

Here, Y is the Lewis form factor which is dependent on the virtual number of teeth.

b) Wear Strength

In order to determine the wear strength of the helical gear, it is considered to be equivalent to a formative spur gear i.e. an imaginary spur gear in a plane that is perpendicular to the tooth element. [14] The formula for finding the wear strength of the helical gear is given by,

$$S_w = b Q d_p K / \cos \phi^2 \tag{44}$$

Where Q is ratio factor, K is the K factor and ϕ is the pressure angle.

$$Q = 2z_g / (z_g + z_p) \tag{45}$$

$$d_p = z_p m_n / \cos \phi \tag{46}$$

$$K = 0.16 [BHN / 100]^2 \tag{47}$$

c) Factor of Safety

Factor of Safety can be defined as the ratio of the structure's strength to that of the actually applied load. Higher the value of factor of safety, safer is the design in the working conditions. But as the economy is also a factor to keep in mind, usually the Factor of Safety lies between 1.5 and 4. The power can be calculated from the following formula: [14]

$$P = 2\pi N M_t / 60 \cdot 10^6 \tag{48}$$

For Beam Strength, effective load can be given as,

$$P_{eff} = (C_s P_t + P_d) \tag{49}$$

$$P_d = \{21v(C_{eb} \cos^2 \phi + P_t) \cos \phi\} / \{21v + \sqrt{(C_{eb} \cos^2 \phi + P_t)}\} \tag{50}$$

Where P_d is the Dynamic load.

For Wear Strength, effective load can be given as,

$$P_{eff} = C_s P_t / C_v \tag{51}$$

Factor of Safety can be determined by the following formula,

$$S_b \text{ or } S_w = C_s P_t (FoS) / C_v \tag{52}$$

The programming was based on python. The code for helical gear design is given in the Appendix I. The program will calculate the value of constant such as lewis' form factor, sum of errors between two meshing teeth etc. automatically from input parameters, i.e. we do not need to refer to the standard tables in order to find constant values. We have developed an algorithm in such a way that it will automatically extrapolate the standard value of module and will ascertain the other dimensions based on that value. In addition to determining dimensions, it is also able to determine whether design is safe for beam and wear strength. Dynamic condition between two meshing teeth are also considered. The results are mentioned in the Table (9).

Table 8: Given & Assumed Data used for the Beam & Wear Strength Calculations

Data that is taken from above calculations or that is assumed are mentioned below:	
Z_s	20
Z_p	16
P	100 KW
N	3000 rpm
S_{ut}	1550 N/mm ²
BHN	450
ϕ	22°
Assume:	
B	60
m_n	4mm
C_s	1.25
α_n	20°

Table 9: Results of the Beam & Wear Strength Calculations

S_b	41002.67 N
S_w	17932.767 N
P_{eff} (considering Dynamic loads)	15842.18 N
P_{eff} (not considering Dynamic loads)	15742.33 N
FoS of Beam Strength	2.589

D. Modelling of the Transmission

The transmission model was made with the results of various calculations done till now. The model was prepared using various platforms as such as Dassault systems' SolidWorks 2017 & 2018 and NX 10.

Due to the time constraints, we were unable to model the friction clutch plate but in the fig the detailed location of the friction clutch plate is shown.

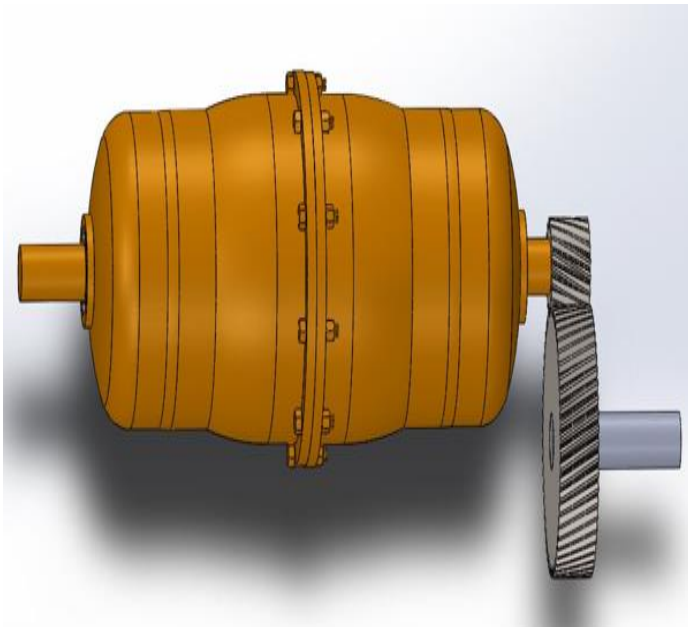


Figure 4: Solidworks Model of the Transmission Gear-box (Covered)

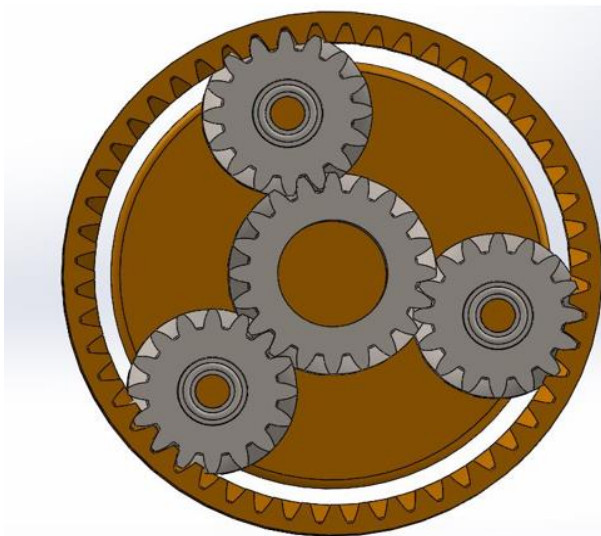


Figure 5: SolidWorks Model of the Transmission Gear-box (Internal Side-view)

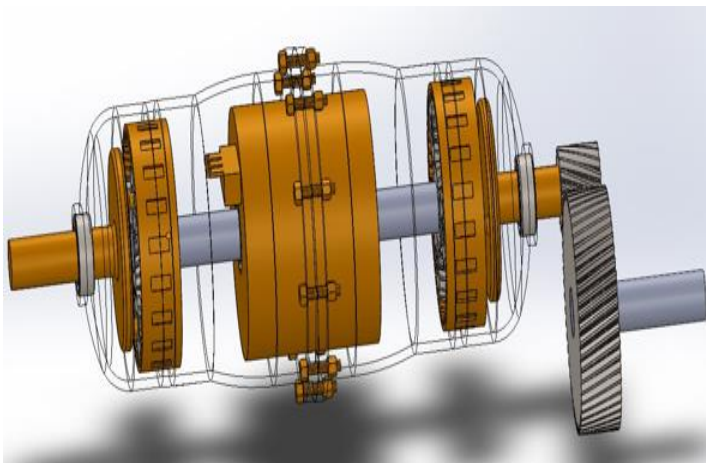


Figure 6: SolidWorks Model of the Transmission Gear-box (Uncovered)

E. Safety Analysis

For the analysis part, we used ANSYS software. This software is very famous and performs very complex analysis and thus is able to give very accurate results in comparison to that of the results from other similar software. Thus, it's very common software that is used in various industries and engineering companies.

ANSYS develops and markets finite element analysis software used to simulate engineering problems. ANSYS is used to determine how a product will function with different specifications, without building test products or conducting crash tests. For example, ANSYS software simulate how a bridge will hold up after years of traffic.

Basically, ANSYS users break down larger structures into small components by a process called meshing that are each modelled and tested individually [2]. A user may start by defining the dimensions of an object and then adding weight, pressure, temperature and other physical properties. Finally, the ANSYS software simulates and analyses movement, fatigue, fractures, fluid flow, temperature distribution, electromagnetic efficiency and other effects over time.

For the analysis, the 3-Dimensional solid model of gear-set is imported in ANSYS as .igs file and analysis is performed by finite element program ANSYS Workbench 15.0. Firstly, the 3-D solid model is assembly of two mating gear as shown in figures below. The number of elements and nodes generated are 30996 and 142192 respectively. The FE model of gear is shown in figure 8 showing this mesh formed on the gears.

The load is applied in the form of moment. The moment of 330 N-m applied on the faces of the pinion. The frictionless support is applied on the bore of pinion and gear. Frictionless support places a normal constraint on an entire surface. Translational displacement is allowed in all directions except in and out of the supported plane.

The ANSYS version used for the analysis is R15.0 and the input parameters used for it is as follows:

- Material – EN30A
- Moment – 330 N/mm
- Module – 4mm
- Thickness – 60mm
- Teeth on pinion – 16
- Teeth on gear – 20
- Environmental Temperature – 32 C
- Helix angle – 22
- Normal pressure angle – 20

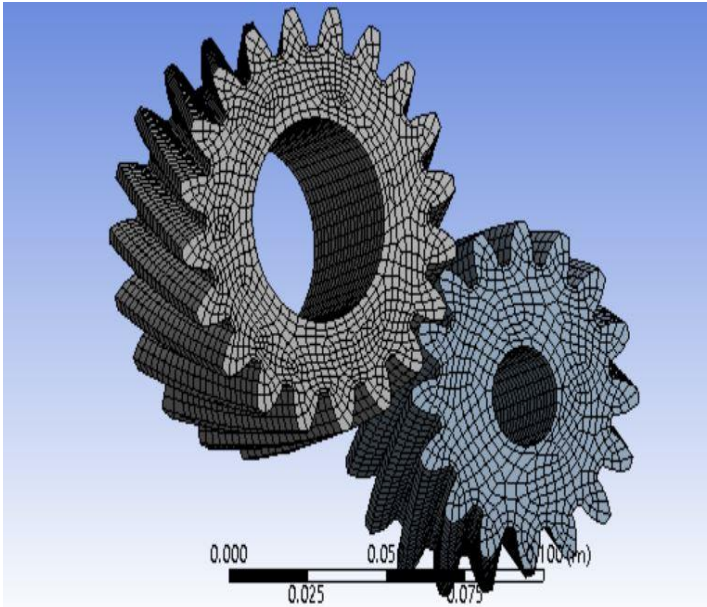


Figure 7: Meshing Results for the Prepared Helical Gears in the ANSYS

1) Total deformation

As it's shown in the figure below, the deformation occurs maximum at the tip of the tooth. And minimum at the base of the tooth. The red section marked on the tip of the tooth signifies the maximum deformation on the tooth i.e. 0.0335 mm. the maximum deformation of the gear is negligible and thus there is no problem of gear deformation for given material and environment.

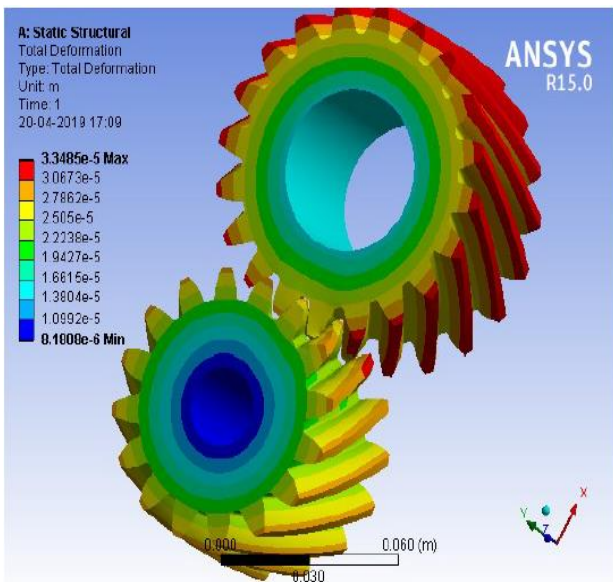


Figure 8: ANSYS Results for the Total Deformation of the Gear

2) Equivalent Stress

Engineering stress is the applied load divided by the original cross-sectional area of a material and is also known as nominal stress. The value of Stress determines the fatigue failure for the gear-set. As it's shown in the figure above, the value of the Stress is maximum at the base of the tooth i.e.

624.6 MPa and minimum at tip. The material has the strength of 1550 MPa and thus it's clear that,

$$\sigma < S_{ut}$$

$$FoS = S_{ut} / \sigma = 2.48 \quad (52)$$

Thus,

$$\%E = FoS_{theo} - FoS_{ANSYS}100 = 0.011\% \quad (53)$$

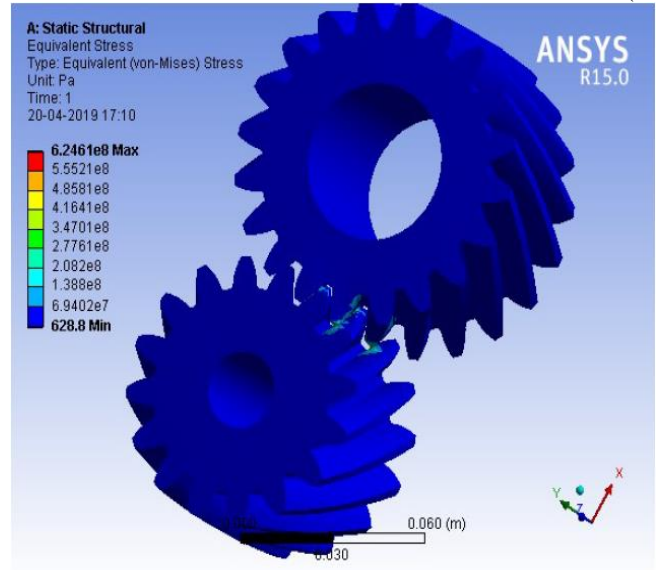


Figure 9: ANSYS Results for the Equivalent Stress

3) Equivalent elastic Strain

The elastic Strain is pretty much self-explanatory as we all are familiar with the following formula:

$$E = \sigma / \epsilon ; \sigma \propto \epsilon \quad (54)$$

Where E signifies Young's Modulus. From the formula the fact is clear that the Stress and Strain are directly proportional to each other. Thus, Strain will also be maximum at the base of the tooth i.e. 0.003 m/m which is under control. The results from the ANSYS are shown in the figure below.

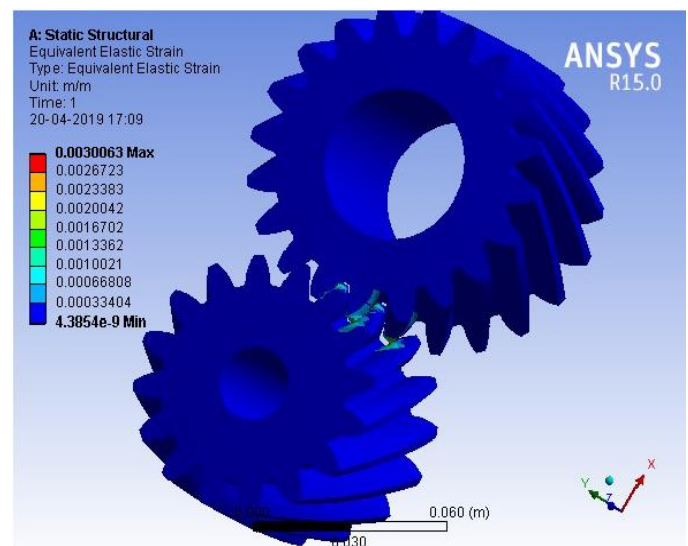


Figure 10: ANSYS Results for Equivalent Elastic Strain

III. RESULTS

As we have discussed in the sections before, there will be two schedules on which the hybrid car would be driven. Those modes are Performance schedule and the economy schedule. The above mentioned graph is for the Performance schedule. This graph shows the maximum speed and maximum acceleration for the vehicle at different modes in Performance schedule. This mode will give more performance characteristics while reducing the fuel efficiency figure. The X-axis defines various modes and Y-axis defines values for maximum velocity and maximum acceleration in km/h and m/s^2 respectively.

The following graph has similar detailing on X and Y axis and units. The graph below is for economy schedule. This schedule won't give high performance as in Performance schedule but the fuel consumption efficiency would be better than the Performance schedule. That's the reason why the acceleration is less than that in the Performance schedule.

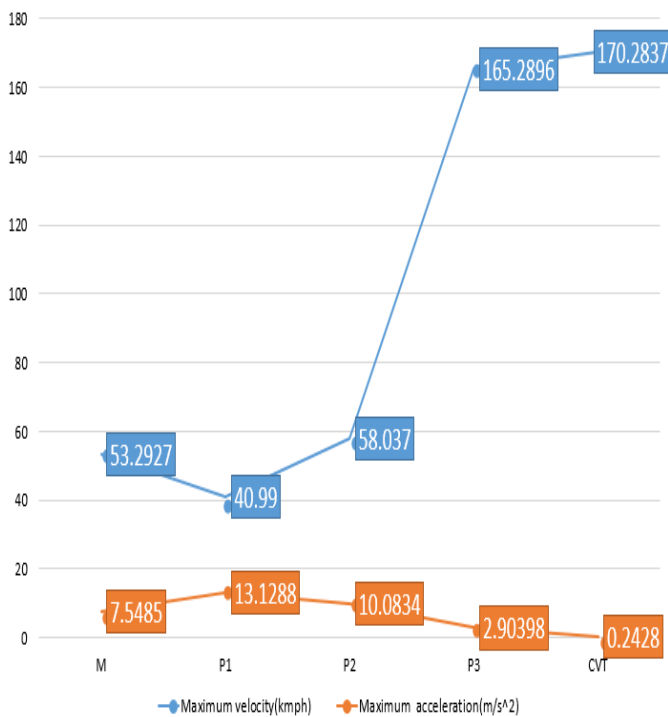


Figure 11: Graphical Representation of the Results for Performance Schedule

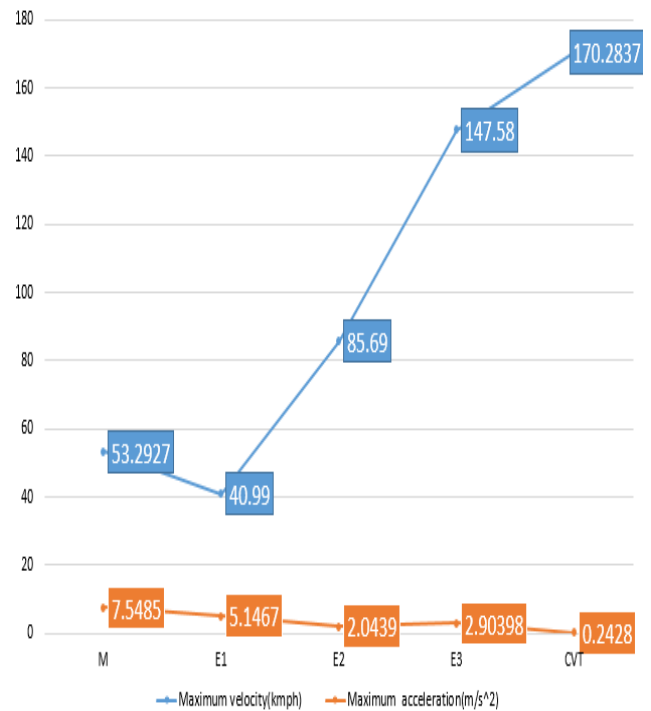


Figure 12: Graphical Representation of the Results for Economy Schedule

The following graph is the combination of both of the above graphs. Thus, it shows performance as well as the economy schedule and their maximum acceleration and maximum velocity at each modes of operation.

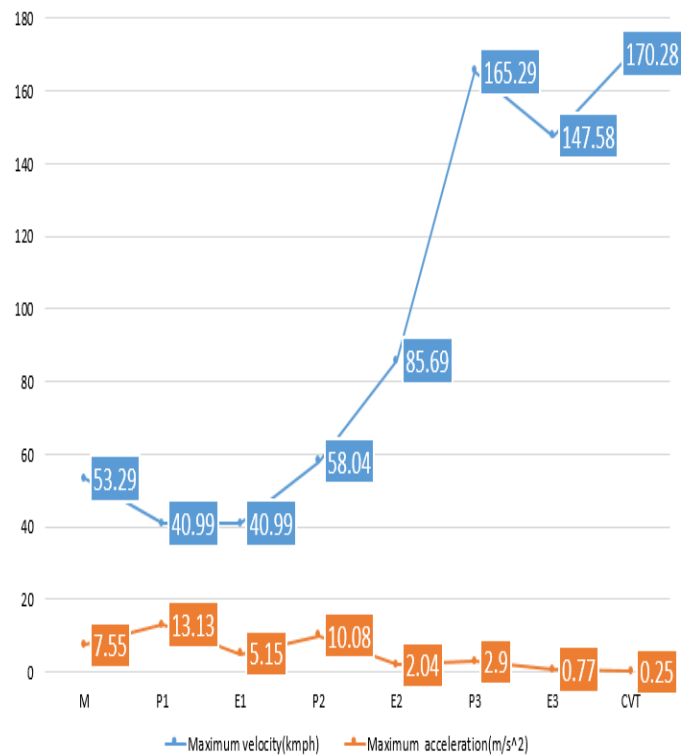


Figure 13: Combined Graphical Representation of the Results

IV. SUMMARY

There are 2 planetary gear-set and 4 clutches used in this transmission for which feasible number of teeth as well as shaft is designed as per the requirements. This design is feasible even in terms of safety considerations, for this the beam and wear strength was calculated theoretically as well as with the help of the software, namely Python and ANSYS. The values of the factor of safety for Python and theoretical calculations were nearly same whereas the value of the same in theoretical calculations and ANSYS had error of mere 0.011%.

This design can be used for RWD as well as FWD vehicles. In case of the RWD vehicles, transmission is in-line to the final reduction drive whereas in FWD vehicles, the transmission is co-axial to the final differential unit.

This transmission design has 15 in total modes of operation under 2 schedules. This 2 schedules are used according to the demand of the situation. The name of this schedules explains the output itself, Performance schedule & Economic schedule.

This transmission design has many advantages & features. Some of which are compactness due to the integrated motor design, high efficiency, high reliability & availability of different modes of operation.

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