Abstract— Antenna is an important block in any wireless system, as it transforms electrical signals to radio signal and vice versa. How well it does this job is a determining factor in how well a wireless system will operate. The performance characteristics of antenna array largely depend upon the spacing between the antenna array elements. This paper presents a systematic approach to the optimum placement of elements of a dipole array for effective estimation of arrival angle. It focuses on Circular array consisting of 8 omnidirectional dipole elements for Direction of Arrival estimation in the azimuthal plane using Multiple Signal Classification algorithm. The element positions are optimized by the use of a Genetic Algorithm. Performance parameters like root mean square error and side lobe level have been evaluated initially for nonuniform circular array after varying its inter-element positions. Ten iterations of Genetic Algorithm were carried out, where each iteration consisted of forty chromosomes. Genetic algorithm showed substantial improvement of 5dB in side lobe level.

Index Terms— Direction Of Arrival (DOA) estimation, Genetic Algorithm (GA), Multiple Signal Classification (MUSIC), Root Mean Square Error (RMSE), Side lobe Level (SLL).

I. INTRODUCTION

Wireless environment is filled with signals Not-Of-Interest (SNOI). Omnidirectional antenna arrays tend to receive all the signals along with the Signal-Of-Interest (SOI). These SNOI cause interference which mandates use of complex filtering and equalization techniques. Direction of arrival (DOA) estimation is a technique used for finding (estimating) the arrival angle of SOI. The Direction-of-Arrival estimation techniques using antenna arrays are applied in large areas such as radar, sonar, medicine, satellite and communication systems. Almost all of the aforementioned applications require highly directive radiation pattern which cannot be achieved by a single antenna element. Antenna arrays applied to overcome such problems give different radiation patterns with respect to its geometrical and electrical configurations. The choice of apt array element position to give the required radiations pattern is the principal goal of array designing [1]. Optimization of array element position for direction of arrival estimation using classical methods face a major drawback of getting into local minima/maxima and are sensitive to initialization. Meta-heuristic optimizations like Genetic Algorithm (GA) were used to optimize the array manifolds.

Recently circular arrays are preferred over other configurations due to their ability to perform scan all directions without substantial change in radiation pattern [2]. DOA estimation using antenna array largely depends upon the spacing between the elements [3]. The ambiguity in estimation increases as the element spacing becomes more than half wavelength due to grating lobes. A narrow spacing on the other hand also degrades the performance due to mutual coupling effects. However, mutual coupling has comparatively lesser effect on circular arrays than linear or circular arrays due to absence of edge elements [4].

II. GEOMETRY AND ARRAY FACTOR

Consider a circular antenna array of \( N \) antenna elements non-uniformly spaced on a circle of radius \( a \) in the \( x-y \) plane in Fig. 1. The elements in the circular antenna array are taken to be isotropic sources; so the radiation pattern of this array can be described by its array factor. In the \( x-y \) plane, the array factor for the circular array shown in Fig. 1 is given by [5]:

\[
AF(\phi) = \sum_{i=1}^{N} I_i e^{jka \cos(\phi - \phi_i) + \alpha_i}
\]  

(1)

\[
ka = \frac{2\pi}{\lambda} = \sum_{i=1}^{N} d_i
\]  

(2)
\[ \phi_n = \frac{2\pi}{\lambda a} = \sum_{i=1}^{N} d_i \] (2)

In the above equations, \( I_o \) and \( \phi_n \) represent the excitation amplitude and phase of the \( n \)-th element, and \( d_i \) represents the arc separation (in terms of wavelength) between element \( n \) and element \( n-1 \) (\( d_i \) being the arc distance between the first and the last \( n = N \) elements i.e. \( n = 1 \) to \( N \)). \( \phi_i \) is the angular position of the \( n \)-th element in \( x-y \) plane. To direct the peak of the main beam in the \( \phi_i \) direction, the excitation phase of the \( n \)-th element is chosen to be

\[ \alpha_n = -ka\cos(\phi_i - \phi_n) \] (3)

In this case, the array factor can be written as:

\[ AF(\phi) = \sum_{i=1}^{N} I_i e^{j[ka\sin(\phi_i - \phi_n) - k\alpha_n]} \] (4)

In this design \( \alpha_0 \) is chosen to be 0, i.e., the peak of the main beam is in the \( x \) direction.

The paper is organized as follows:

Section II gives brief idea about Genetic and MUSIC algorithm. Section III explains the concept of DOA. Section IV assimilates results for different scenarios. Section V presents observations and comparative results. Section VI concludes the paper.

III. GENETIC ALGORITHM OPTIMIZATION

Genetic Algorithm (GA) is a global optimization algorithm derived from evolution and natural selection. GA techniques are becoming widely used to solve electromagnetic problems due to their robustness, wide range of applications and readiness in their implementation [6]. Although Genetic Algorithm cannot always provide optimal solution, it has its own advantages and is a powerful tool for solving complex problems [7]. Fig. 2 shows the process flow of genetic algorithm followed by DOA estimation. Initial chromosomes are binary representations of the angular element positions. These positions are randomly selected for the first iteration. 10-bit binary representation of chromosomes helps in achieving a resolution 0.35156°. These binary positions are coded as antenna element positions and fed to FEKO to get the radiation pattern. MATLAB then calculates the SLL for each population. The strongest chromosomes (highest SLL) are selected for the next iteration. 20 more chromosomes are generated using the 50% mutation and 10% crossover. After 25 iterations best 20 chromosomes are used for DOA estimation.

IV. DIRECTION OF ARRIVAL (DOA) ESTIMATION

Estimation of DOA is performed using MUSIC (MUltiple Signal Clasification) which provides unbiased estimate of more than one signal. MUSIC, introduced by R. O. Schmidt [8], is a popular subspace based method that is rather computationally intensive (compared to conventional estimation methods) [9]. It works under the presumption that the signals are uncorrelated. Compared to ESPRIT and ROOT MUSIC, MUSIC is the most stable and accurate algorithm, which provides high resolution despite lower value of SNR [10]. The signals in the azimuthal direction for \( M \) element array with \( D \) number of arrival signals estimated by the signal-subspace music algorithm that is given by:

\[ P_{\text{music}}(\phi) = \frac{1}{\left[\mathbf{S}^H(\phi)\mathbf{E}_n\mathbf{E}_n^H\mathbf{S}(\phi)\right]} \] (5)

Where noise subspace eigen vector \( \mathbf{E}_n = [e_1, e_2, \ldots, e_{M-D}] \) is orthogonal to array steering vectors at angles of arrival \( \theta_1, \theta_2, \ldots, \theta_D \).

The simulation results in Fig. 3 through 6 show the performance of MUSIC for Uniform Circular Array (UCA). The SNR is 20dB with 200 snapshots and 8 elements unless otherwise changed to check the dependence of respective parameters. Fig. 3 shows that the estimated angles of DOA are precise irrespective of the number of array elements, assuming the sources located at 20° and 60°. A sharper response is obtained with the increase in number of elements. The variation in MUSIC spectrum as a function of SNR is illustrated in Fig. 4, while Fig. 5 displays the effect of increasing the number of snapshots. It can be seen that the estimation improves with increase in number of elements, SNR and snapshots. Fig. 6 shows the response of MUSIC for
angle of arrival separation of $5^\circ$, $10^\circ$ and $20^\circ$ respectively. It can be observed that MUSIC estimation is ambiguous below angular separation of $10^\circ$.

Fig. 3: Estimation with different number of elements

Fig. 4: Estimation with different SNR

V. RESULTS

The results were computed using Matlab and FEKO simulation softwares. The execution for GA for 10 iterations on a core i3, quadcore processor with 4GB of RAM took 3 hours. Fig. 7 shows the improvement in SLL over the iterations of GA. The result was computed for a frequency of 200 MHz and radius of $1\lambda$. GA converges at SLL of 25.5dB. Fig. 8 depicts normalized radiation pattern for an 8-element array at 200 MHz for radius of $1\lambda$. Both the figures clearly show an improvement in SLL of around 5dB.

Fig. 9 indicates the change in SLL with radius of array $0.5\lambda$ and $1\lambda$. Fig. 10 shows the MUSIC estimation of angle of arrival of $50^\circ$ and $100^\circ$ with SNR of 0 dB. It is evident that the quality of estimation improves as the side lobes are reduced using GA. Table 1 shows the element position after 10 iterations of GA. Array with these element positions provide highest SLL.

VI. OBSERVATIONS AND COMPARISONS

It is observed from fig. 7 and fig. 8 that SLL is improved on an average of 5dB with the increase in iteration of GA. However GA converges after iterations leaving little scope for further improvement. The maximum SLL is 25.5674 dB as compared to 5.23 dB improvement in [11] for 30-element linear array. In fig. 9 the SLL is also improved as the radius is increased. As the radius increases the effect of mutual coupling reduces which results in SLL improvement by more than 6 dB. Fig. 10 proves that the MUSIC estimation becomes more effective as the directivity of the array increases.
VII. CONCLUSION
The GA is evolutionary based search algorithm which provides maximum random search and improvement of the parameters. The performance and convergence rate of the GA depends upon its mutation and crossover rate. It is also highly dependent on initialization. MUSIC provides accurate estimation of more than one signal. MUSIC estimation improves with increase in number of elements, SNR and number of snapshots. It also provides better estimation with increase in SLL with GA.

VIII. FUTURE SCOPE
Other optimization algorithms like Particle Swarm Optimization (PSO) for faster convergence. Other Estimation algorithms such as ROOT MUSIC AND ESPRIT can be evaluated for different frequencies in UHF-VHF range. Reduction in length of dipole elements (electrically small dipole) with optimized SLL also has potential for future improvement.

IX. REFERENCES.

Table 1: Element positions

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<thead>
<tr>
<th>Element</th>
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<tr>
<td>3</td>
<td>95.2734°</td>
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<td>4</td>
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<td>5</td>
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<td>6</td>
<td>231.3281°</td>
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<tr>
<td>7</td>
<td>291.4453°</td>
</tr>
<tr>
<td>8</td>
<td>354.7266°</td>
</tr>
</tbody>
</table>

Table 1: Element positions

Fig. 8: Normalized radiation pattern

Fig. 9: SLL with radius of 0.5λ and 1 λ.

Fig. 10: MUSIC Estimation