

Design and Control of a Quadcopter

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Abstract— Most of the quadcopters out there talk about controlling the roll, pitch, and yaw. We understand there was a need of focusing on attitude, altitude, and position control and help with required control systems. Our paper goes through the complete process of making a person understand the working of a quadcopter in 6 DOF even after having 4 actuators. Talking about the different available configurations of quadcopter and dynamics of handling each to achieve a required speed, position, and altitude control. We have gone through a general control strategy and explaining the process of tuning PID.

Keywords—Design; Control; Quadcopter; PID; Dynamics

I. INTRODUCTION

Quadrotor (Quadcopter) is a four-propeller helicopter, usually arranged in a square, with four independent rotors, reducing the need for a swashplate. A swashplate method was needed to allow the helicopter to use more degrees of freedom, but the same level of control could be achieved by adding multiple rotors.

The development of quadcopters has come to a standstill recently because controlling four independent rotors has proved very difficult and impossible without the help of electronics. The declining costs of current microprocessors have made quadcopter controls electrical and completely independent for commercial, military, and recreational purposes.

Quadcopter control is a particularly difficult and exciting problem, with 6 degrees of freedom (3 renders and 3 rotations) and only four independent inputs (rotor speed), quadcopters are severely underactuated. To achieve the six degrees of freedom, rotation and translation movement go hand in hand. The emergence potential is very nonlinear, especially after the calculation of complex aerodynamic effects. Lastly, unlike land vehicles, helicopters have very little friction to restrict their movement, so they should give their flexibility to stop moving and stay stable. Together, these factors create a problem of control that is very interesting. This paper will introduce a simplified model of Quadcopter dynamics and control.

II. DESIGN AND CONTROL

A. Design of Quadcopter

The standard quadcopter has four rotating angles and four input power, which is provided by each motor as shown in Fig 1. The quadcopter configuration can move and remain vertically which is a great advantage as it reduces the requirements for the landing platform. There are about 2 configurations for multiple quadcopter designs “+” and “x”.

The X-configuration quadcopter is considered more stable compared to the + configuration, which is an additional acrobatic type. Duplicates 2 and 4 rotate counterclockwise (CCW), 1 and 3 rotate clockwise (CW). For the quadcopter to be able to maintain forward movement by increasing the speed of the front rotors while reducing the speed of the rear rotor simultaneously, which means changing the pitch angle. This process is required to pay for the result of the action / response (3rd Law of Newton). Progresses 1 and 2 have a contrasting pitch towards 2 and 4, so all thrush has the same direction.

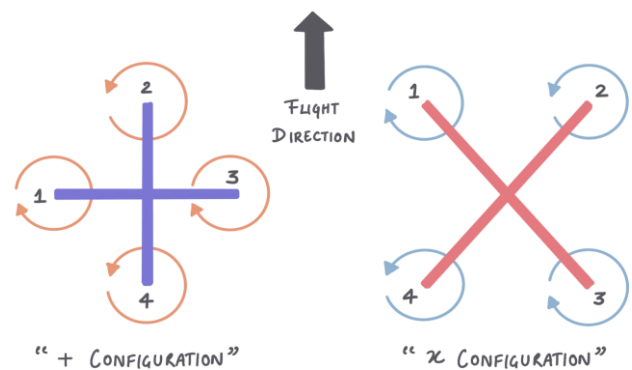


Fig. 1 Types of Quadcopter Configuration

B. Dynamics of Quadcopter

We will begin to discover the power of the quadcopter by introducing two frames that will work on it. The non-inserted frame is defined by the ground, with gravity pointing in the wrong z-direction. The body structure is defined by the shape of a quadcopter, with rotor axes pointing in the right z-direction and arms pointing in x and y directions.

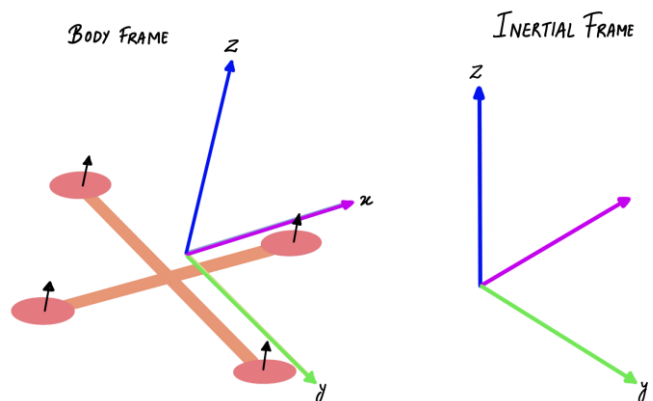


Fig. 2 Quadcopter frame and Inertial frame

The power of a quadcopter can be defined in many different ways such as quaternion, Euler angle, and directional matrix. However, in building the reference stability control height at the axis angle is necessary, so the designed controller can reach a stable plane. In height control, all angle indicators on each axis should be approximately zero especially when taking, landing or moving upwards. The position of the quadcopter can be defined by the three Euler angles of roll angle (Φ), pitch angle (θ), and yaw angle (ϕ). In figure 2 $\omega_1, \omega_2, \omega_3, \omega_4$ (angular velocity) of the propellers T_1, T_2, T_3, T_4 : the forces produced by the propellers: $F_i \propto \omega_i^2$ depending on the propeller position, air density, etc. M : quadcopter size, mg : quadcopter weight, Φ, θ, ϕ : roll, pitch and yaw angles.

The position of the quadcopter is defined by the inertial frame x, y, z -axes with ξ . The shape, e.g., angular position is defined in the inner circle with three angles of Euler η . The pitch angle θ determines the rotation of the quadcopter around the y -axis. Roll angle ϕ determines the rotation around the x -axis and the yaw angle ψ around the z -axis. Vector q contains vectors of line and angular shape

$$\xi = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \eta = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, q = \begin{bmatrix} \xi \\ \eta \end{bmatrix}$$

The reference source of the body is in the middle of the quadcopter multiplicity area. In the body structure, the direct velocities are determined by Jacobian and angular velocity ω .

$$JB = \begin{bmatrix} J_x, B \\ J_y, B \\ J_z, B \end{bmatrix} \omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

The rotation matrix from the body frame to the inertial frame is

$$R = \begin{bmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\ s\psi s\theta & s\psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi \\ -s\theta & c\theta c\phi & c\theta s\phi \end{bmatrix}$$

When $Sx = \sin(x)$ and $Cx = \cos(x)$. The orthogonal rotation matrix of $R^{-1} = R^T$ which is the rotating matrix from the inert frame to the body frame. There are 3 types of angular velocity that can be defined as the output of (ψ, θ, ϕ) in terms of time,

$$\begin{aligned} \Phi^* &= \text{Roll rate,} \\ \Theta^* &= \text{Pitch rate,} \\ \Psi^* &= \text{Yaw rate} \end{aligned}$$

When you consider the quadcopter rotation mode it provides 4 calculations of power, times, and rotational speed. Defined by:

$$\begin{aligned} \text{Equilibrium of forces: } \sum_{i=1}^4 T_i &= -mg \\ \text{Equilibrium of directions: } T_{1,2,3,4} &\parallel g \\ \text{Equilibrium of moments: } \sum_{i=1}^4 M_i &= 0 \\ \text{Equilibrium of rotation speeds: } (\omega_1 + \omega_3) - (\omega_2 + \omega_4) &= 0 \end{aligned}$$

And the consequence is : $\Phi^* = 0, \Theta^* = 0, \Psi^* = 0$

By increasing / decreasing the rotational speed of all the shelters, the quadcopter can make the movement fly up and down.

$$\text{Flying up: } \sum_{i=1}^4 T_i > -mg,$$

$$\text{Flying down: } \sum_{i=1}^4 T_i < -mg, \text{ Euler angles and rates remain } 0.$$

Changing the speed of propellers gives us equations of yaw, roll, and pitch quadcopter:

$$\begin{aligned} \text{Yaw: } \dot{\Psi}^* &= k_y((\omega_1 + \omega_3) - (\omega_2 + \omega_4)) \quad \phi = \int \Psi^* dt \\ \text{Roll } \dot{\Phi}^* &= k_R((\omega_1 + \omega_3) - (\omega_2 + \omega_4)) \quad \theta = \int \Phi^* dt \\ \text{Pitch } \dot{\Theta}^* &= k_p((\omega_1 + \omega_3) - (\omega_2 + \omega_4)) \quad \psi = \int \Theta^* dt \end{aligned}$$

Therefore, a decrease of 2 rotor velocity and an increase in rotor velocity 4 acquires roll movement. Similarly, reducing 1 rotor velocity and increasing 3 rotor velocities to detect pitch movement. Increasing the angular velocities of the two opposing rotors and reducing the velocities the other two achieves yaw movements.

C. Equations of Motion

In a quadcopter control system, it is important to consider how powerful the system should be in relation to external forces such as wind. It is important to consider the response time of the quadcopter w.t to external forces operating in the short term. The control strategy will generally depend on the systems under 3 controls. The quadcopter mounting sensors measure angular speed and translation acceleration. These values are compared to the desired attitude and positions to calculate the error signal included in the control system.

The PID control algorithm is used to control the height of the quadcopter. The PID controller acts as a closed loop feedback system. The release of a PID control system is a control number that will drive the system closer to the desired position. The structure of the PID controller is:

$$\begin{aligned} u &= \text{control input} \\ P &= \text{Proportional control term} \\ I &= \text{Integral control term} \\ D &= \text{Derivative control term} \\ k_p &= \text{Proportional control gain} \\ k_I &= \text{Integral control gain} \\ k_D &= \text{Derivative control gain} \\ t &= \text{instantaneous time} \\ x_d(t) &= \text{desired value of state } x \text{ at time } t \\ x(t) &= \text{measured value of state } x \text{ at time } t \\ e(t) &= \text{Error value at time } t \\ \dot{e}(t) &= \text{Derivative of error at time } t \end{aligned}$$

$$u = P + I + D$$

$$e(t) = x_d(t) - x(t)$$

$$P = k_p e(t)$$

$$I = k_I \int_0^t e(t-1) dt$$

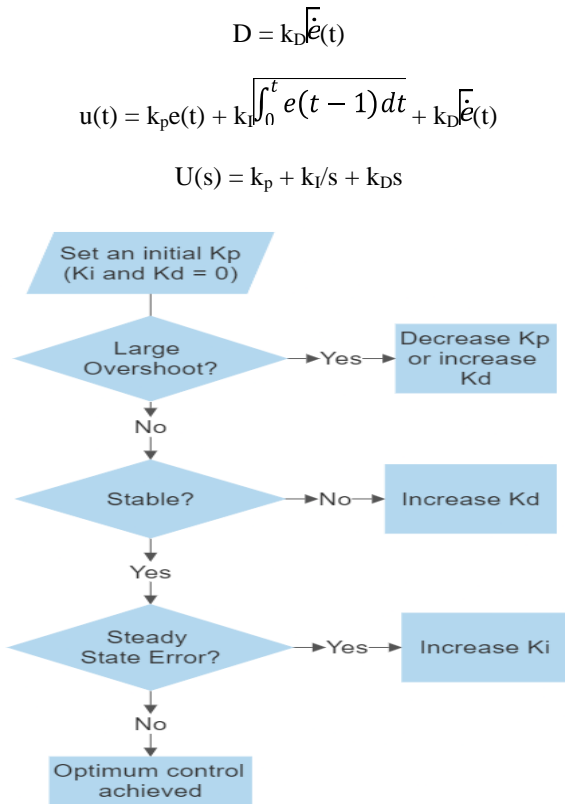


Fig. 3 Methodology to obtain control parameters

D. General Control Strategy

The quadrotor can travel up to six degrees of freedom (3 renderings and 3 rotations) but has only 4 inputs (speed for each car). While the quadrotor is able to move directly on a vertical z-axis without changing any other state, it must change its position in the x and y axis. Since it is not possible to control six degrees of freedom with just 4 controls, we instead create controllers to settle in x, y, z, and the subject you want. The quadrotor should be able to move safely in the desired location while keeping rolls and angles stable. Modern quadrotors have ESCs that control the angular velocities of each rotor.

- u_1 – the resulting thrust of the four rotors
- u_2 – the difference of thrust between the motors on the x-axis which results in roll angle changes and subsequent movement in the lateral x-direction.
- u_3 – the difference of thrust between the motors on the y axis which results in pitch angle changes and subsequent movement in the lateral y-direction.
- u_4 – the difference of torque between the CW and CC rotors which results in the moment that rotates the quadrotor around the vertical z-axis.

We will use the equations for the net thrust and moment equations to define the control inputs to the control outputs of quadrotor system. The control inputs are defined below.

F_1^b = total force of thrust in the body coordinate frame (N)
 w = motor angular velocity (rads/sec)

$\tau_{\phi, \theta, \psi}$ = torques along the body aes (N-m)
 K_d = drag torque proportionality constant
 K_T = thrust coefficient
 l = length of arm (m)
 u_1 = total thrust of all motors
 u_2 = roll control input
 u_3 = pitch control input
 u_4 = yaw control input

$$\begin{bmatrix} F_b^t \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} K_T & K_T & K_T & K_T \\ 0 & -lK_T & 0 & lK_T \\ K_T & 0 & K_T & 0 \\ K_d & -K_d & K_d & -K_d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

E. Attitude and Altitude Control

We want to use this controller to stabilize the quadrotor around the stable hover. This means that all the desired angles are 0 and we set the desired height to a certain minimum value for redemption purposes. Controlling altitude is similar to controlling the total amount of a quadcopter. This is important because these control inserts influence and movement on the x and y axis. Therefore, high throttle values lead to very strong movements in the x and y directions. In our simplified model, the direct acceleration is separated from the positive. However, this does not hold when the quadrotor rotates as vertical acceleration components are also present in x and y planes. Therefore, the height control compensates for both the gravitational force and the deviation of the normal vector due to the inclination.

- u_1 = total thrust of all motors
- p_d = Desired angular velocity along x^b (radians/second)
- q_d = Desired angular velocity along y^b (radians/second)
- r_d = Desired angular velocity along z^b (radians/second)
- k_P = Proportional control gain
- k_I = Integral control gain
- k_D = Derivative control gain
- ϕ_d = Desired roll angle (radians)
- θ_d = Desired pitch angle (radians)
- ψ_d = Desired yaw angle (radians)
- Z_d^G = Desired altitude(m)

m = mass of the quadrotor (kg)
 g = acceleration due to gravity(m/s²)
 $e(t)$ = error value at time t

$$u_1(t) = \frac{1}{\cos(\phi)\cos(\theta)} \left(k_P e_z(t) + k_I \int_0^t e_z(t) dt + k_D \frac{e_z(t) - e_z(t-1)}{dt} + mg \right)$$

$$p_d(t) = k_P e_\phi(t) + k_I \int_0^t e_\phi(t) dt + k_D \frac{e_\phi(t) - e_\phi(t-1)}{dt}$$

$$q_d(t) = k_P e_\theta(t) + k_I \int_0^t e_\theta(t) dt + k_D \frac{e_\theta(t) - e_\theta(t-1)}{dt}$$

$$r_d(t) = k_p e_{\psi}(t) + k_i \int_0^t e_{\psi}(t) dt + k_D \frac{e_{\psi}(t) - e_{\psi}(t-1)}{dt}$$

Note that yaw/heading control is often accomplished sufficiently with only PD control so the integration gain would not be required.

F. Position Control

While the mood / height controller can maintain a stable quadrotor movement and be followed by the input / height input from the RC, it cannot control the quadcopter translation movement. Smaller angles will lead to movement and translation. To prevent this flooding method of translation, it is used to control the location. Input to the position controller are position measurements and the results are required by roll and pitch angle to move the quadrotor to the desired position x and y. This controller also performs a search where a series of consecutive waypoints can be fed to the position control so that the quadrotor follows a certain three-dimensional path.

- K_p = Proportional control gain
- k_i = Integral control gain
- k_D = Derivative control gain
- X_d^P = Desired global X position (m)
- Y_d^P = Desired global Y position (m)
- Φ_d = Desired roll angle (radians)
- θ_d = Desired pitch angle (radians)

$$\theta_d(t) = k_p(X_d^G - X^G) + k_i \int_0^t (X_d^G - X^G) dt + k_D(\dot{X}_d^G - \dot{X}^G)$$

$$\phi_d(t) = k_p(Y_d^G - Y^G) + k_i \int_0^t (Y_d^G - Y^G) dt + k_D(\dot{Y}_d^G - \dot{Y}^G)$$

III. IMPLEMENTATION

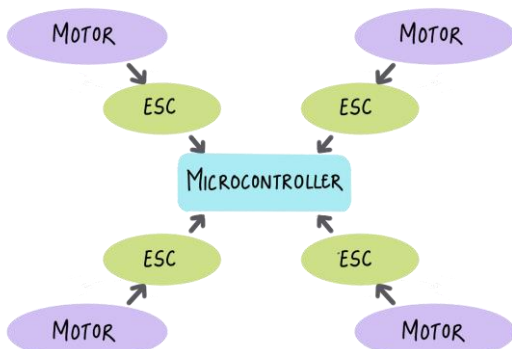


Fig. 4 Block Diagram

Accelerometers and gyroscopes are sensors used to determine posture. The first measures the speed at each side of the connection system (x, y, z), and the second measures the angular velocity. According to our IMU data, accelerometers are very sensitive to vibrations and their response time is not as fast as gyroscopes. Gyroscopes tend to withstand vibrations but turn up certain degrees over time. With this implementation, a combination of data collected by both

sensors will help us to determine the exact position of the quadcopter.

When the radio controller (or a pre-configured function) provides a command, the quadcopter controller must respond as quickly and accurately as possible. The first method of control values is given to system simulations. However, manual manipulation of the controller values is required to synchronize the controller and obtain the desired performance of the system.



Fig. 5 Assembly

In order to tune the controllers, a function is implemented capable to change PID values.

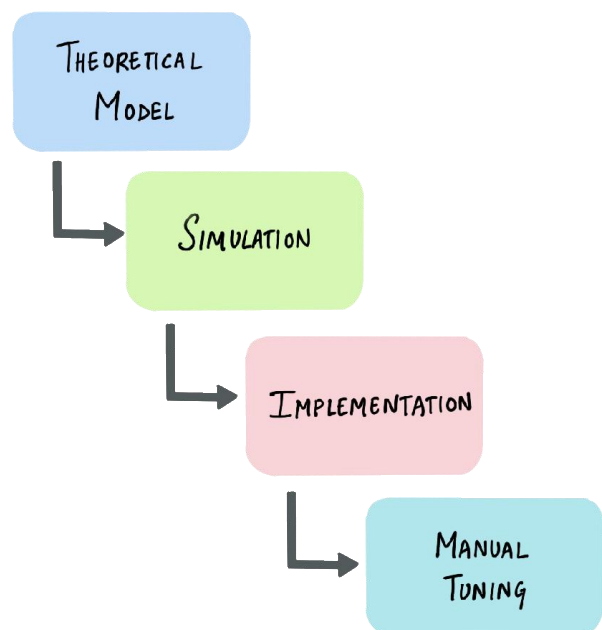


Fig 6 Processes of Implementation

IV. CONCLUSION

We are introducing a quadcopter instruction model to answer the question "How to use attitude, altitude and position controller in a quadcopter". Based on this model and strength tests in safe controlled environments, real world stability was tested for free flights indoors and outdoors. In the development of the research phase, theoretical and artistic methods were used. These approaches were the basic basis for demonstrating the direct relationship between the theoretical model and the behavior of the "transformation effect" given the artistic approach; therefore, the experimental results obtained have been shown to be similar to the hypotheses studied.

Due to the simplified considerations of the model, the control structure was divided into two phases: the theory phase, corresponding to the mathematical model and the structure of the control, and the actual implementation of the ideas in the visual model. The theoretical model was used to quantify the quadcopter's capacity and the theoretical control system, while in the experimental phase, it was proved that the simplification of the theory led to the creation of a plant that did not meet all aspects of the real system, and therefore experimental adjustments were required.

On outdoor, the quadcopter has deviated from its original position due to the influence of existing air, however, this can be manually controlled by remote. The future addition of a satellite position controller is proposed to avoid position deviation by wind while using the built-in GPS module installed in the quadcopter. However, the system remains stable for roll ϕ , pitch θ , yaw angles, and resistance to wind and other external forces.

V. REFERENCES

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