Design and Analysis of Inventory Model for Quadratic Trapezoidal Type Demand under Partial Backlogging

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Abstract: In this paper, we consider the inventory model for perishable items with quadratic trapezoidal type demand rate, that is, the demand rate is a piecewise quadratic function. The model consider allows for shortages and the demand is partially backlogged. The model is solved analytically by minimizing the total inventory cost. The result is illustrated with numerical example for the model.

Keywords: Quadratic trapezoidal demand. Deterioration. Shortages. Partial backlogging

1. INTRODUCTION

Deteriorating items are very common thing in our daily life situation. In recent years, many researchers have studied inventory models for deteriorating items, however, academia has not reached a consensus on the definition of the deteriorating items. According to the study of Wee (1993), deteriorating items refers to the items that become decayed, damaged, evaporative, expired, invalid, devaluation and so on through time. According to the definition, deteriorating items can be classified in to two categories. The first category refers to the items that become decayed, damaged, evaporative, or expired through time, like meat, vegetables, fruit, medicine, flowers and so on; the other category refers to the items that lose part or total value through time because of new technology or the introduction of alternatives, like computer chips, mobile phones, fashion and seasonal goods and so on. The inventory problem of deteriorating items was first studied by Whitin (1957), he studied fashion items deteriorating at the end of the storage period. Then Ghare and Schrader (1963) concluded in their study that the consumption of the deteriorating items was closely relative to a negative exponential function of time. Various authors (Deng et al. (2007), Cheng and Wang (2009), Cheng et al. (2011), Hung (2011)) studied inventory models for deteriorating items in various aspects.

In world business market, demand has been always one of the most key factors in the decisions relating to the inventory and production activities. There are mainly two categories demands in the present studies, one is deterministic demand and the other is stochastic demand. Various formations of consumption tendency have been studied, such as constant demand (Padmanabhan and Vrat (1990), Sukla (2012), Sukla and Sahu (2008) Chung and Lin (2001), Benkherouf et al. (2003), Chu et al (2004)), level-dependent demand (Giri and Choudhuri (1998), Chung et al. (2000), Bhattacharya (2005), Wu et al. (2006)), price dependent demand (Wee and Law (1999), Abad (1996, 2001)), time dependent demand (Rash et al. (1976), Henery (1979), Sachan (1984), Dave (1989), Teng (1996), Teng et al. (2002), Skouri and Papachristos (2002), Panda, Sahoo, and Sukla (2012), Sett et al. (2013), Shah, Chaudhari and Jani (2015), Shah, Chaudhari and Jani (2016), Mishra et al. (2013)) and time and price dependent demand (Wee (1995)). Among them, ramp type demand is a special type of time dependent demand. Hill (1995), one of the pioneers, developed an inventory model with ramp type demand that begins with a linear increasing demand until to the turning point, denoted as $\mu$, proposed by previous researchers, then it becomes a constant demand. There has been a movement towards developing this type of inventory system for minimum cost and maximum profit problems. Several authors: Mandal and Pal (1998) focused on deteriorating items. Wu et al. (1999) were concerned with backlog rates relative to the waiting time. Wu and Ouyang (2000) tried to build an inventory system under two replenishment policies: starting with shortage or without shortage. Panda, Sahoo and Sukla (2013), Wu (2001) considered the deteriorated items satisfying Weibull distribution. Giri et al (2003) dealt with more generalized three parameter Weibull deterioration distribution. Deng (2005) extended the inventory model of Wu et al. (1999) for the situation where the in-stock period is shorter than $\mu$. Manna and Chaudhuri (2006) set up a model where the deterioration is dependent on time. Panda et al. (2007) constructed an inventory model with a comprehensive ramp type demand. Deng et al. (2007)
contributes to the revision of Mandal and Pal (1998), and Wu and Ouyang (2000). Panda et al. (2008) examined the cyclic deterioration items. Wu et al. (2008) studied the maximum profit problem with the stock-dependent selling rate. They developed two inventory models all related to the conversion of the ramp type demand, and then examined the optimal solution for each case. However, in a realistic product life cycle, demand is increasing with time during the growth phase. Then, after reaching its peak, the demand becomes stable for a finite time period called the maturity phase. Thereafter, the demand starts decreasing with time and eventually reaching zero or constant. In this work, we extend Hill’s ramp type demand rate to quadratic trapezoidal type demand rate. Such type of demand pattern is generally seen in the case of any fad or seasonal goods coming to market. The demand rate for such items increases quadratic-ally with the time up to a certain time and then ultimately stabilizes and becomes constant, and finally the demand rate approximately decreases to a constant, and then begins the next replenishment cycle. We think that such type of demand rate is quite natural and useful in real world market situation. One can think that our work may provide a solid foundation for the future study of this kind of important inventory models with quadratic trapezoidal type demand rate and preservation technology.

2. ASSUMPTION AND NOTATIONS

The fundamental assumption and notations used in this paper are given as follows: The demand rate, $R(t)$, which is positive and consecutive, is assumed to be a quadratic trapezoidal type function of time, that is

$$R(t) = \begin{cases} 
 b_1 t + c_1 t^2, & t \leq \mu_1, \\
 R_0, & \mu_1 \leq t \leq \mu_2, \\
 b_2 t - c_2 t^2, & \mu_2 \leq t \leq T 
\end{cases}$$

(1)

Chose $b_1$, $c_1$, $b_2$ and $c_2$ such a way that $b_2 t - c_2 t^2$ should not be negative for $\mu_2 \leq t \leq T$. Where $\mu_1$ is the time point changing from the increasing quadratic demand to constant demand, and $\mu_2$ is the time point changing from the constant demand to the decreasing demand.

- Replenishment rate is infinite, thus replenishment is instantaneous.
- $R(t)$ is the inventory level at any time $t$, $0 \leq t \leq T$.
- $T$ is the fixed length of each ordering cycle.
- $\theta$ is the constant rate of deterioration, $0 < \theta < 1$.
- $t$ is the time when the inventory level reaches zero.
- $t^*$ is an optimal point.
- $k_0$ is the fixed ordering cost per order.
- $k_i$ is the cost of each deteriorated item.

3. MATHEMATICAL AND THEORETICAL RESULTS

Here, we consider the deteriorating inventory model with demand rate is trapezoidal type quadratic function. Replenishment occurs at time $t=0$ when the inventory level attains its maximum. For $t \in [0, t_1]$, the inventory level reduces due to both demand and deterioration. At time $t_1$, the inventory level reaches zero, then shortage is allowed to occur during the interval $(t_1, T)$, and all of the demand during the shortage period $(t_1, T)$ is completely backlogged. The total amount of backlogged items is replaced by the next replenishment. The rate of change of the inventory during the stock period $[0, t_1]$ and shortage period $(t_1, T)$ is governed by the following differential equations:

$$\frac{dl(t)}{dt} + \theta l(t) + R(t) = 0, 0 < t < t_1,$$  

(2)

$$\frac{dl(t)}{dt} + R(t) = 0, t_1 < t < T,$$  

(3)

with boundary condition $l(0)=S$ and $l(t_1)=0$. One can think about $t_1$, $t_2$ may occur within $[0, \mu_1]$ or $[\mu_1, \mu_2]$ or $[\mu_2, T]$. Hence in this paper we are going to discuss all three possible cases.

Case I: $0 < t_1 \leq \mu_1$. The quadratic trapezoidal type market demand and constant rate of deterioration, the inventory level gradually diminishes during the period $[0, t_1]$ and ultimately reaches to zero at time $t=t_1$. Then, from equations (2) and (3), we have

$$\frac{dl(t)}{dt} + \theta l(t) + b_1 t + c_1 t^2 = 0, 0 < t < t_1,$$  

(4)

$$\frac{dl(t)}{dt} + b_2 t + c_2 t^2 = 0, t_1 < t < \mu_1,$$  

(5)

$$\frac{dl(t)}{dt} + R_0 = 0, \mu_1 < t < \mu_2,$$  

(6)

$$\frac{dl(t)}{dt} + b_2 t - c_2 t^2 = 0, \mu_2 < t < T.$$  

(7)

Now solving the differential equations (4) – (7) with the condition $l(t_1)=0$ and continuous property of $l(t)$, we get...
The total amounts of inventory carried during the interval \([0, t_1]\), say \(C_T\), is

\[
C_T = \int_0^{t_1} I(t) dt
\]

\[
= \left[ \left( b_i t_i + c_i t_i^2 \right) - \frac{b_i + 2c_i t_i}{\theta^2} \frac{2c_i}{\theta^3} \right] e^{\theta(t_1 - t)}
\]

\[
- \left( b_i t_i + c_i t_i^2 \right) + \frac{b_i + 2c_i t_i}{\theta^2} - \frac{2c_i}{\theta^3}
\]

\[
, \quad 0 \leq t \leq t_1
\]

\[
I(t) = \left( t_i^2 - t_i^3 \right) \frac{b_i}{2} + \left( t_i^3 - t_i^4 \right) \frac{c_i}{3}, \quad t_i \leq t \leq \mu_1
\]

\[
I(t) = -R_i t + \left( t_i^3 + \mu_i \right) \frac{b_i}{2} + \left( t_i^2 + 2\mu_i \right) \frac{c_i}{3}
\]

\[
, \quad \mu_1 \leq t \leq \mu_2
\]

\[
I(t) = \left( t_i^2 + \mu_i^2 \right) \frac{b_i}{2} + \left( t_i^3 + 2\mu_i^3 \right) \frac{c_i}{3}
\]

\[
, \quad \mu_2 \leq t \leq T
\]

The total shortage quantity during the interval \([t_1, T]\), say \(B_T\), is

\[
B_T = -\int_{t_1}^{T} I(t) dt
\]

\[
= \left[ \left( b_i t_i + c_i t_i^2 \right) - \frac{b_i + 2c_i t_i}{\theta^2} \frac{2c_i}{\theta^3} \right] e^{\theta(t_1 - t)}
\]

\[
- \left( b_i t_i + c_i t_i^2 \right) + \frac{b_i + 2c_i t_i}{\theta^2} - \frac{2c_i}{\theta^3}
\]

\[
, \quad 0 \leq t \leq t_1
\]

\[
I(t) = \left( t_i^2 - t_i^3 \right) \frac{b_i}{2} + \left( t_i^3 - t_i^4 \right) \frac{c_i}{3}, \quad t_i \leq t \leq \mu_1
\]

\[
I(t) = -R_i t + \left( t_i^3 + \mu_i \right) \frac{b_i}{2} + \left( t_i^2 + 2\mu_i \right) \frac{c_i}{3}
\]

\[
, \quad \mu_1 \leq t \leq \mu_2
\]

\[
I(t) = \left( t_i^2 + \mu_i^2 \right) \frac{b_i}{2} + \left( t_i^3 + 2\mu_i^3 \right) \frac{c_i}{3}
\]

\[
, \quad \mu_2 \leq t \leq T
\]

The beginning inventory level can be computed as

\[
S = I(0) = \left( - \frac{b_i}{\theta^2} + \frac{2c_i}{\theta^3} \right) e^{\theta t_1} - 1
\]

\[
+ \left( \frac{b_i t_i + c_i t_i^2}{\theta^2} - \frac{2c_i t_i}{\theta^3} \right) e^{\theta t_i}
\]

\[
(13)
\]
The average total cost per unit time for \(0 < t_1 \leq \mu_1\) is given by

\[
A_1(t_1) = \frac{1}{T} [k_0 + k_1 D_T + k_2 C_T + k_3 B_T] \tag{16}
\]

The first order derivative of \(A_1(t_1)\) with respect to \(t_1\) is as follows:

\[
\frac{dA_1(t_1)}{dt} = \frac{1}{T} \left[ \left( k_1 + \frac{k_2}{\theta} \right) (e^{\alpha_1} - 1) + k_3 (t_1 - T) \right]
\]

\[
(b_1 t_1 + c_1 t_1^2) = 0
\]

The necessary condition for \(A_1(t_1)\) to be minimized, is

\[
\frac{dA_1(t_1)}{dt} = 0,
\]

that is

\[
\frac{1}{T} \left[ \left( k_1 + \frac{k_2}{\theta} \right) (e^{\alpha_1} - 1) + k_3 (t_1 - T) \right] = 0 \tag{18}
\]

\[
(b_1 t_1 + c_1 t_1^2) = 0
\]

This implies that

\[
\left[ \left( k_1 + \frac{k_2}{\theta} \right) (e^{\alpha_1} - 1) + k_3 (t_1 - T) \right] = 0 \tag{19}
\]

Let \(p(t_1) = \left( k_1 + \frac{k_2}{\theta} \right) (e^{\alpha_1} - 1) + k_3 (t_1 - T) \tag{20}\)

Since

\[
p(0) = -k_3 T < 0, \quad p(T) = \left( k_1 + \frac{k_2}{\theta} \right) (e^{\alpha_1} - 1) > 0 \quad \text{and}
\]

\[
p'(t_1) = \left( k_1 + \frac{k_2}{\theta} \right) e^{\alpha_1} + k_3 > 0,
\]

it implies that \(p(t_1)\) is a strictly monotonically increasing function and equation (19) has a unique solution at \(t_1^*\), for \(t_1^* \in (0, T)\). Therefore, we have

Property-1

The constant deteriorating rate of an inventory model with quadratic trapezoidal type demand rate under the time interval \(0 < t_1 \leq \mu_1\), \(A_1(t_1)\) attains its minimum at \(t_1 = t_1^*\), where \(p(t_1^*) = 0\) if \(t_1^* < \mu_1\). On the other hand, \(A_1(t_1)\) attains its minimum at \(t_1^* = \mu_1\) if \(t_1^* \geq \mu_1\).

The total back order amount at the end of the cycle is

\[
\Delta_1 = \frac{b_1}{2} (t_1^2 + \mu_1^2) - \frac{c_1}{3} (t_1^3 + 2\mu_1^3) \tag{21}
\]

\[
+ \frac{b_2}{2} (T^2 + \mu_1^2) - \frac{c_2}{3} (T^3 + 2\mu_1^2)
\]

Therefore, the optimal order quantity, denoted by \(Q^*\), is \(Q^* = S^* + \Delta_1\), where \(S^*\) denote the optimal value of \(S\).

Case-II, \(\mu_1 \leq t_1 \leq \mu_2\)

For the time period \(t_1 \in [\mu_1, \mu_2]\), then, the differential equations governing the inventory model can be expressed as follows:

\[
\frac{dI(t)}{dt} + Q(t) + b_1 t + c_1 t^2 = 0, \quad 0 < t < \mu_1 \tag{22}
\]

\[
\frac{dI(t)}{dt} + Q(t) + R_0 = 0, \quad \mu_1 < t < \mu_2 \tag{23}
\]

\[
\frac{dI(t)}{dt} + R_0 = 0, \quad \mu_1 < t < T \tag{24}
\]

\[
\frac{dI(t)}{dt} + b_2 t - c_2 t^2 = 0, \quad \mu_2 < t < T \tag{25}
\]

Solving differential equations (22) to (25), using \(I(t=0) = 0\), we get

\[
I(t) = \frac{R_0}{\theta} e^{\alpha_1} - \frac{b_1}{\theta^2} e^{\alpha_1} - \frac{b_1 t + c_1 t^2}{\theta} + \frac{b_1 + 2c_1 t}{\theta^2} - \frac{2c_1}{\theta^3} e^{\alpha_1} + \frac{2c_1}{\theta^3} e^{\alpha_1}, \quad 0 \leq t \leq \mu_1
\]

\[
I(t) = \frac{R_0}{\theta} (e^{\alpha_1} - 1), \quad \mu_1 \leq t \leq \mu_2 \tag{27}
\]

\[
I(t) = R_0 (t - t_1), \quad t_1 \leq t \leq \mu_2 \tag{28}
\]

\[
I(t) = R_0 (t_1 - t), \quad t_1 \leq t \leq \mu_2 \tag{29}
\]

The beginning inventory can be computed as

\[
S = I(0) = \frac{R_0}{\theta} e^{\alpha_1} - \frac{b_1}{\theta^2} e^{\alpha_1} + \frac{b_1}{\theta^2} + \frac{2c_1}{\theta^3} e^{\alpha_1} + \frac{2c_1}{\theta^3} e^{\alpha_1} \tag{30}
\]
The total amount of items which is perish within the time interval \([0, t^*_1]\) is
\[
D_T = S - \int_0^{t_1} R(t)\,dt
= S - \left( \int_0^{t_1} b_1 t + c_1 t^2 \,dt - \int_0^{t_1} R_0 \,dt \right)
= R_0 \frac{\theta_1}{\theta^2} - \left( \frac{b_1}{\theta^2} + \frac{2c_1 \mu_1}{\theta^3} - \frac{2c_1}{\theta^3} \right) e^{\theta t} - R_0 (t_1 - \mu_1)
+ \frac{b_1}{2 \theta^2} - c_1 \left( \frac{2}{3} \frac{\mu_1}{\theta^3} + \frac{2c_1}{\theta^3} e^{\theta (\mu_1 - t)} \right)
\]  
(31)

The total amount of inventory carried during the time interval \([0, t^*_1]\) is
\[
C_T = \int_0^{t_1} I(t)\,dt
= \int_0^{t_1} I(t)\,dt + \int_{t_1}^{t_1} I(t)\,dt
= \left[ \frac{R_0}{\theta} \left( e^{\theta t} - \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3} e^{\theta (\mu_1 - t)} \right) + \frac{b_1}{\theta} + \frac{2c_1 \mu_1}{\theta^3} - \frac{2c_1}{\theta^3} \right] dt
\]
(32)

The total amount of shortage during the interval \([t_1, T]\)
\[
B_T = -\int_{t_1}^{T} I(t)\,dt
= -\left[ \int_{t_1}^{t_1} I(t)\,dt - \int_{t_1}^{T} I(t)\,dt \right]
\]
(33)

The required necessary condition for \(A_2(t_1)\) to be minimized is \(\frac{dA_2(t_1)}{dt_1} = 0\), that is
\[
\left[ \left( k_1 + \frac{k_2}{\theta} \right) (e^{\theta t_1} - 1) + k_3 (t_1 - T) \right] = 0
\]
(36)

Let \(p(t_1) = \left( k_1 + \frac{k_2}{\theta} \right) (e^{\theta t_1} - 1) + k_3 (t_1 - T)\),
(37)

since \(p'(t_1) = \left( k_1 + \frac{k_2}{\theta} \right) e^{\theta t_1} \theta + k_3 > 0\), which implies that \(p(t_1)\) is strictly monotonically increasing function during the interval \(\mu_1 \leq t_1 < \mu_2\).

**Property-2**

The constant deteriorating rate of an inventory model with quadratic trapezoidal type demand function during the time interval \(\mu_1 \leq t_1 \leq \mu_2\), \(A_2(t_1)\) attains its minimum at \(t^*_1 = \mu_1\) if \(t_1^* < \mu_1\) and \(A_2(t_1)\) attains its minimum at \(t^*_1 = \mu_2\) if \(\mu_2 < t_1^*\).

Now, we can calculate the total amount of back-order quantity at the end of the cycle is
\[ \Delta_2 = -R_0 + \frac{b_t}{2} (T^2 + \mu_2^2) - \frac{c_2}{3} (T^3 + 2\mu_3^3) \]  
(38)

Therefore, the optimal order quantity denoted by \( Q^* \) is \( Q^* = S^* + \Delta_2 \), where \( S^* \) denotes the optimal value of \( S \).

**Case-III \( \mu_2 \leq t_1 < T \)**

For the time interval \( t_1 \in [\mu_2, T] \), then, the differential equations governing the inventory model can be expressed as follows:

\[ \frac{dI(t)}{dt} + \theta I(t) + b_t t + c_1 t^2 = 0, \quad 0 < t < \mu_1 \]  
(39)

\[ \frac{dI(t)}{dt} + \theta I(t) + R_0 = 0, \quad \mu_1 < t < \mu_2 \]  
(40)

\[ \frac{dI(t)}{dt} + \theta I(t) + b_t t - c_2 t^2 = 0, \quad \mu_2 < t < t_1 \]  
(41)

\[ \frac{dI(t)}{dt} + b_t t - c_2 t^2 = 0, \quad t_1 < t < T \]  
(42)

Solving the differential equations (39)-(42) with \( I(t_i)=0 \), we can get

\[ I(t) = -R_0 + \frac{b_t t_1 - c_2 t_1^2}{\theta} + \frac{b_t}{2} \frac{c_1}{\theta^2} + \frac{c_1^2}{\theta^3} + \frac{b_1 + 2c_1 \mu_1}{\theta^2} + \frac{2c_1}{\theta^3} e^{\theta(t_1-\mu_1)} \]  
(43)

\[ I(t) = -R_0 + \frac{b_t t_1 - c_2 t_1^2}{\theta} + \frac{b_t}{2} \frac{c_1}{\theta^2} + \frac{c_1^2}{\theta^3} + \frac{b_1 + 2c_1 \mu_1}{\theta^2} + \frac{2c_1}{\theta^3} e^{\theta(t_1-\mu_1)} \]  
(44)

\[ I(t) = \frac{b_t}{2}(t^2 - t^2) + \frac{c_1^2}{3}(t^3 - t_1^3), \quad t_1 \leq t < T \]  
(45)

The total amount of inventory level at the beginning can be computed as

\[ S = I(0) = -R_0 + \frac{b_t}{2} \frac{c_1}{\theta^3} + \frac{b_t t_1 - c_2 t_1^2}{\theta} - \frac{b_t}{2} \frac{c_1}{\theta^2} + \frac{c_1^2}{\theta^3} \]  
(46)

\[ S = I(0) = -R_0 + \frac{b_t}{2} \frac{c_1}{\theta^3} + \frac{b_t t_1 - c_2 t_1^2}{\theta} - \frac{b_t}{2} \frac{c_1}{\theta^2} + \frac{c_1^2}{\theta^3} \]  
(47)

\[ D_T = S - \int_0^{t_i} R(t) dt \]

\[ = S - \int_0^{\mu_2} R(t) dt - \int_{\mu_2}^{t_i} R(t) dt \]

\[ I(t) = \frac{b_t + 2c_1 \mu_1}{\theta^2} + \frac{2c_1}{\theta^3} e^{\theta(t_1-\mu_1)} \]

\[ + \frac{b_t t_1 - c_2 t_1^2}{\theta} - \frac{b_t}{2} \frac{c_1}{\theta^2} + \frac{c_1^2}{\theta^3} \]  
(48)

\[ + \frac{b_t t_1 - c_2 t_1^2}{\theta} - \frac{b_t}{2} \frac{c_1}{\theta^2} + \frac{c_1^2}{\theta^3} \]  
(49)

\[ + \frac{b_t t_1 - c_2 t_1^2}{\theta} - \frac{b_t}{2} \frac{c_1}{\theta^2} + \frac{c_1^2}{\theta^3} \]  
(50)

\[ + \frac{b_t t_1 - c_2 t_1^2}{\theta} - \frac{b_t}{2} \frac{c_1}{\theta^2} + \frac{c_1^2}{\theta^3} \]  
(51)

\[ + \frac{b_t t_1 - c_2 t_1^2}{\theta} - \frac{b_t}{2} \frac{c_1}{\theta^2} + \frac{c_1^2}{\theta^3} \]  
(52)

The total amount of inventory carried during the time interval \([0, t_i]\) is

\[ C_T = \int_0^{t_i} I(t) dt \]

\[ = \int_0^{\mu_2} I(t) dt + \int_{\mu_2}^{t_i} I(t) dt \]

\[ = \int_0^{\mu_2} I(t) dt + \int_{\mu_2}^{t_i} I(t) dt \]

\[ = \frac{b_t}{2}(t^2 - t^2) + \frac{c_1^2}{3}(t^3 - t_1^3), \quad t_1 \leq t < T \]  
(53)

\[ \frac{b_t}{2}(t^2 - t^2) + \frac{c_1^2}{3}(t^3 - t_1^3), \quad t_1 \leq t < T \]  
(54)
\[
\begin{align*}
\frac{b_1 - c_2 t^2}{\theta} + \frac{2c_2}{\theta^2} \left( e^{\theta t} - 1 \right) + \left( b_2 - 2c_2 \mu_2 \right) \frac{-2c_2}{\theta^2} \left( e^{\theta t} - 1 \right) + \frac{2c_1}{\theta^2} \left( 1 - e^{\theta t} \right)/\theta \phi_1(t_1) \\
&= -\int_{t_i}^{T} \left[ \frac{b_2}{2} (t_1^2 - t^2) + \frac{c_2}{3} (t_3^3 - t_3^2) \right] dt \\
&= -\frac{b_2}{2} t_1^2 (T - t_i) + \frac{b_2}{6} (T^3 - t_1^3) - \frac{c_2}{12} (T^4 - t_1^4) \\
&\quad + \frac{c_2 t_1^2}{3} (T - t_i)
\end{align*}
\]

Then, the total average cost per unit time under the time interval $\mu_2 \leq t_i \leq T$, can be written as

\[
A_3(t_i) = \frac{1}{T} \left[ k_0 + k_1 D_T + k_2 C_T + k_3 B_T \right]
\]

The first order derivative of $\dot{A}_3(t_i)$ with respect to $t_i$ is as follows:

\[
\frac{dA_3(t_i)}{dt_i} = \frac{1}{T} \left[ \left( k_1 + \frac{k_2}{\theta} \right) (e^{\theta t_1} - 1) + k_3 (t_3 - T) \right] (b_1 t_1 - c_2 t_1^2)
\]

The required necessary condition for $\dot{A}_3(t_i)$ to be minimized is

\[
\frac{dA_3(t_i)}{dt_i} = 0, \quad \text{that is}
\]

\[
\frac{1}{T} \left[ \left( k_1 + \frac{k_2}{\theta} \right) (e^{\theta t_1} - 1) + k_3 (t_3 - T) \right]
\]

\[
(b_1 t_1 + c_1 t_1^2) = 0
\]

This implies that

\[
\left[ k_1 + \frac{k_2}{\theta} \right] (e^{\theta t_1} - 1) + k_3 (t_3 - T) = 0
\]

Let $p(t_i) = \left[ k_1 + \frac{k_2}{\theta} \right] (e^{\theta t_i} - 1) + k_3 (t_3 - T), \quad (55)$

since $p'(t_i) = \left( k_1 + \frac{k_1}{\theta} \right) e^{\theta t_i} + k_3 > 0$, which implies that $p(t_i)$ is strictly monotonically increasing function within the interval $t_1 \in [\mu_2, T]$. 

**Property-3**

In this case, the inventory model under the condition $\mu_2 \leq t_i < T$. $A_3(t_i)$ attains its minimum at $t_i = t_1^\ast$, where $p(t_1^\ast) = 0$ if $\mu_2 < t_1^\ast$. On the other hand, $A_3(t_1)$ attains its minimum at $t_1 = \mu_2$ if $t_1^\ast < \mu_2$. Now, we can calculate the total back-order quantity at the end of the cycle is

\[
\Delta_3 = \frac{b_2}{2} (T^2 - t_1^3) + \frac{c_2}{3} (t_1^3 - T^3)
\]

Therefore, the optimal order quantity, denoted by $Q$, is $Q^\ast = S^\ast + \Delta_3$, where $S^\ast$ denotes the optimal value of $S$. From the above three cases, we can derive the following results

**Result-1**

An inventory model having constant deteriorating rate with quadratic trapezoidal type demand, the optimal replenishment time is $t_1^\ast$ and $\dot{A}_3(t_i)$ attains its minimum at $t_i = t_1^\ast$ if and only if $t_1^\ast < \mu_2$. On the other hand, $A_2(t_i)$ attains its minimum at $t_i = t_1^\ast$ if and only if $\mu_2 < t_1^\ast$. Also, $A_2(t_i)$ attains its minimum at $t_i = t_1^\ast$ if and only if $\mu_2 < t_1^\ast$, where $t_1^\ast$ is the unique solution of equation $p(t_i) = 0$.

**Example 1**

We can consider suitable values of the following parameters as follows: $T = 20$ weeks, $\mu_1 = 6$ weeks, $\mu_2 = 15$ weeks, $b_1=10$ unit, $c_1=5$ unit, $b_2=20$ unit, $c_2=2$ unit, $\theta = 0.1$, $k_0=$220, $k_2=$3 per unit, $k_3=$$12 per unit, $k_4=$$4 per unit. By Using MATHEMATICA 8.0 the above data, we can find $p(\mu_2) = 168.1206 > 0$, the optimal replenishment time $t_1^\ast = 3.41$ weeks, the optimal order quantity $Q^\ast$ for each ordering cycle, is 3576.478 unit and the minimum cost $A_3(t_1^\ast) =$ $4688.2
In the above table some sensitivity analysis of the model is performed by changing the parameter -50%, -25%, -20, -10, 10%, 20%, 25%, and 50%, taking one at time and keeping the remaining parameters unaltered.
CONCLUSION 4

In a realistic product life cycle, demand is increasing with time during the growth phase. Then, after reaching its peak, the demand becomes stable for a finite time period called the maturity phase. Thereafter, the demand starts decreasing with time. Therefore, in this paper, we study the inventory model for constant deteriorating items with quadratic trapezoidal demand rate. We proposed an inventory replenishment policy for this type of inventory model. From the market information, we find that the quadratic trapezoidal type demand rate is more realistic than ramp type demand rate, constant demand rate and other time dependent demand rate. Our paper provides an interesting topic for the future study of such kind of important inventory models, and at the same time, the following problems can be considered for future research work:

1. How about the inventory model starting with shortages?
2. How about the inventory model with time dependent deteriorating rate instead of constant deteriorating rate?

5. REFERENCES:


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