

# Design and Analysis of Flex Spline with Involute Teeth Profile for Harmonic Drive Mechanism

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**Abstract**— Harmonic Drive, also known as Strain wave gear is a non-conventional power transmission element used in various applications where precision positioning mechanisms and higher torque density actuator drives are required. This device is compact and lightweight by which large torques can be produced by relatively small motors because of its velocity reduction and torque amplification properties. This capability permits the use of small motors and harmonic drives in applications where much larger motors and bulky gearboxes would be otherwise required. This makes them ideal for space and robotic applications since they can be directly mounted at the joint along with the motor.

This paper outlines the design considerations and methodology, FE Analysis and optimization of Flex Spline, the most critical element in the Harmonic drive, for a given specifications. It is a cup shaped flexible elastic member having external teeth, which undergoes deformation while assembling and when transmitting the torque. A comparative study has been done between the various Flex Spline configurations for choosing the optimal one in the Harmonic drive assembly. The number of teeth on the Flex Spline in contact with the Circular Spline is optimized considering the load sharing of fully engaged teeth. Among the different teeth profiles are available; the conventional involute teeth profile is selected by considering manufacturing easiness. A parametric study on load sharing on teeth and flexibility in radial and axial directions are conducted for finalizing the Flex Spline's geometry. A global model of this Flex Spline has been generated and analyzed by Finite Element Method to verify the stress patterns are within the elastic limits and to study the flexibility effect.

## I. INTRODUCTION

The theory underlying the operation of harmonic drive gears was developed during the mid-1950s. Invented in 1959 by Walt Musser in the United States, the Harmonic Drive gear was first applied in aircraft and defense applications [1]. The first major space application was in 1971 as the Wheel drive actuator transmission element of the Appollo 15, the lunar roving vehicle [2]. In 1972, the Harmonic drive was used for the Telescopic drive actuator for the NASA's Pioneer 10 planetary probe. In the 1980s Harmonic drive have been used in Airbus aircraft for the position pick up of flaps. Other applications include Antenna pointing mechanism, Actuators for surgical robotics, Vibration control measurements etc. [3]

Compared to conventional gearing, the Harmonic Drive gear offers the user a number of significant advantages. They are (a) High positioning accuracy better than 30 arcs seconds. (b) Due to very low hysteresis losses, a repeatability of  $\pm 5$  arcs seconds is possible. (c) Zero backlashes: due to the natural pre-loading in the region of tooth engagement, the gear operates without backlash. (d) Since power is transmitted through multiple tooth engagement, Harmonic drive gears offer a very high torque capacity. (e) It exhibits very high torsional stiffness with almost linear stiffness characteristics. (f) Compared to the conventional gears, efficiency of the Harmonic drive is very high to the order of 80%. (g) Tooth friction losses and wear are negligible and hence have very long life. (h) Harmonic Drive gears are reversible[4,5].

The Harmonic drive is the subject of continuous development, due to the versatile applications. The design criticalities and manufacturing difficulties make the system very costly one. The major development requirements are higher power density with reduced size and weight, higher torsional stiffness and lower gear reduction ratios. The key element in these new developments is the Flex Spline. The number of teeth, teeth profile, rim thickness and axial length of the Flex Spline cup are the subjects of design improvements. In this paper, the design methodology of the Flex Spline using conventional straight involute spline with  $30^\circ$  pressure angle is explained. An optimized design is evolved for a given specification of 100 Nm torque and 100:1 speed reduction ratio. The Finite Element Analysis informs the strength and flexibility requirements.

## II. PRINCIPLE OF OPERATION

The Harmonic drive gear is unique in transmitting high torque through an elastically deformable component. The gear has just three concentric elements.

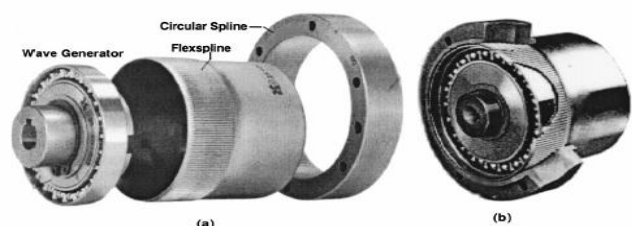


Fig. 1 (a) Exploded and (b) Assembled view of Harmonic Drive

- The Circular Spline (CS) is a solid cylindrical ring with internal gear teeth.
- The Flex Spline (FS) is a non-rigid, thin cylindrical cup with external teeth at the open end of the cup. The closed end of the cup is provided with a flange connection to following machine elements.
- The Wave Generator (WG) comprises a thin raced ball bearing fitted onto an elliptical plug, serving as a high efficiency torque converter.

These three basic components function in the following way:

- The Flex Spline is slightly smaller in diameter than the Circular Spline and usually has two fewer teeth than the CS. The elliptical shape of the Wave generator causes the teeth of the FS to engage the CS at two regions at opposite ends of the major axis of the ellipse.
- As the WG (input) rotates, the zone of tooth engagement travels with the major axis of the ellipse.
- For each 180° clockwise movement of the WG, the FS (output) moves counter clockwise by one tooth relative to the CS (fixed).
- Each complete clockwise rotation of the WG results in the FS moving counter clockwise by two teeth from its previous position relative to the CS.

The reduction ratio is therefore not a function of the relative sizes of the toothed components, as in the case of spur gears or planetary gears, but simply of the number of teeth. Using this principle of operation, reduction ratios of 30:1 to 320:1 can be achieved with just three basic components. Gears are available with outer diameters from 20 to 330 mm with a peak torque capacity from 0.5 to 9000 Nm respectively.

### III. DESIGN PARAMETERS AND ITS CRITICALITIES

The Harmonic Drive Mechanism is designed for transmitting a torque of 100 Nm with a speed reduction ratio of 100:1.

Module is the most important parameter as far as the design of Flex Spline is concerned. Since the component has to transmit a very high load, a larger force will act on the gear teeth. So during the operation, the number of teeth engaged shall be maximum. This issue can be solved by taking the maximum number of teeth for the given configuration. As far as in Robotics and space applications are concerned, the component should be very compact. Since the number of teeth is larger, the only parameter which can be reduced to make the component small is the module. Therefore, the Module must be as small as possible. When the module is reduced, the fabrication difficulties are raised up. In order to make a compromise between these two and considering the fabrication feasibility, a module of 0.176 mm is selected for the present analysis [6].

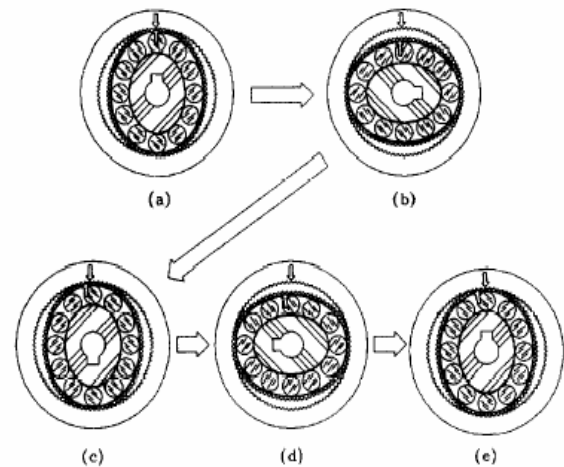


Fig. 2 Principle of motion of the harmonic drive: (a) Initial position; (b) After 90° revolution; (c) After 180° revolution; (d) After 270° revolution; (e) After 360° revolution.

### IV. FLEX SPLINE CONFIGURATIONS

Transmission ratio of the Harmonic Drive is very high in the order of 100:1, 50:1 etc...And it is achieved by the relative difference in number of teeth. The different types of configurations are available for different transmission ratios. They are shown in Figure 3. In Case 1, only one end of the external gear is in contact with the Internal Gear. Therefore, the load shared by fully engaged teeth is very high. Also the teeth engagement and disengagement will make lots of vibrations to the system. For a transmission ratio of 100:1, the teeth combination in this case is 100:101.

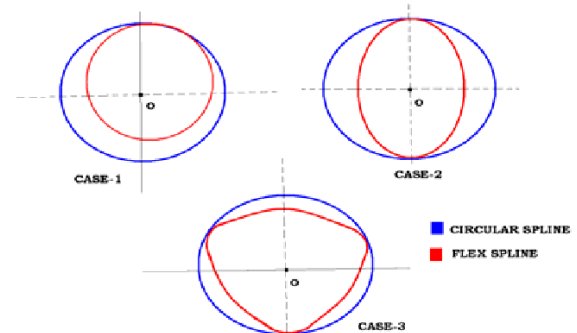


Fig 3 Types of configurations

In Case 2, two diametrically opposite ends of the External Gear is in contact with the Internal Gear through the major axis of an ellipse. So the Load shared by the fully engaged teeth will be halved when compared to the Case 1.

Here it is balanced compared to Case 1 and the vibration effects are lower as compared to the former. In this case, for a transmission ratio of 100:1, the teeth combinations are 200:202. In case 3, three lobes of the internal spline are in contact with the external spline. Therefore, for the same transmission Ratio, the load shared by the fully engaged teeth will be reduced. But the area of contact for a given size will be lesser than Case 2 and hence the torque carrying capability is less. So for the present design, Case 2 has been taken because of its high torque transmission capability.

The interference of the tooth tips of the circular gear and flexible gear during engagement and disengagement restricts the minimum tooth difference to a higher value. For teeth having involute profile and Pressure angle of 30°, the tip interference can be easily avoided with the tooth difference of two, which is an essential feature of the Harmonic Drive [7]. So the number of teeth 202 and 200 are finalized for Circular Spline Flex Spline respectively for the transmission ratio of 100:1.

V. DESIGN OF THE FLEX SPLINE

In Harmonic Drive, the teeth in contact will not equally share the total load. It depends upon the area of contact between the teeth pair. Therefore, a study regarding the load sharing with the number of teeth in contact is conducted.

Flex Spline is a flexible elastic member which has to flex (elastically deform) a certain amount while operation. Even though it is flexing, the tooth has to take the load coming on to it. Flex Gear when assembled with the wave generator its load is transferred through the tooth. So the load carrying capabilities of the teeth are assumed similar with a normal rigid gear. Therefore, the Flex gear tooth is designed as considering it as a rigid gear.

In order to find the actual engagement in gear teeth, the Addendum Circle meeting points are noted in locations A, B, C and D. The area A to B, the Flex Gear is fully engaged with the Fixed Internal gear. The portion B to C shows the area of disengagement. The shared area will make an angle of 112° with the Addendum Circle Diameter of the two mating points. Thus, an initial estimation of 124 teeth has been found out.

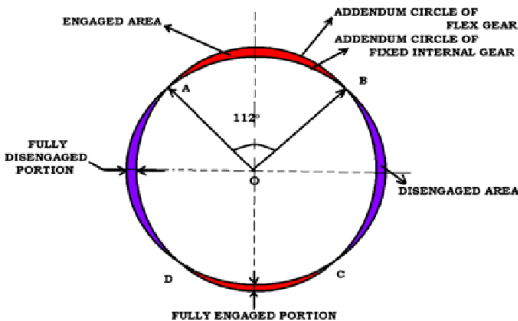


Fig. 4 Number of Teeth shares the Total Load

The variation of the load sharing and number of teeth are shown in Fig. 5.

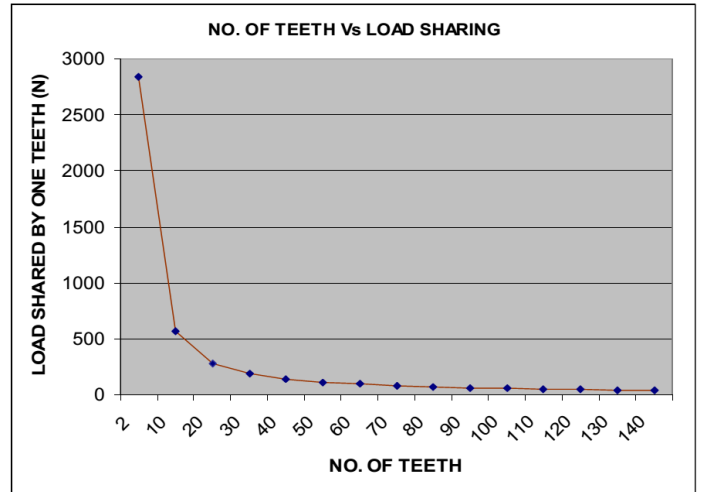


Fig. 5 Variation of Load shared by one tooth with the Number of teeth in contact.

The pressure angle plays very important role in power transmission. In harmonic drive, the transmission is occurred due to radial and axial motions of flex spine. Therefore, a higher-pressure angle is recommended [8]. For Harmonic drives with involute gears, the tip interference is easily avoided with an increase in pressure angle of 30°. The tooth design is done for Dynamic, Endurance and Wear loads using Lewis Formula and Buckingham’s Equation.

The Flex Spline is deformed in a cyclic manner while the gear drives are in operation. Therefore, its fatigue strength must be very higher. Material that is having a high strength is required for designing this type of gear system. So Cr-Ni steel, about 0.45% heat-treated whose configuration is 40Ni 2Cr 1Mo 28 is selected for the gears.

Considering the 30% of teeth on each side only contribute the load sharing [8], tooth design is done and the dimensions are shown in Fig. 6. The overall gear dimensions are shown in Fig. 7.

The face width of the flex Spline gear is obtained as 13 mm.

The thickness of the Flex Spline tube is directly related to its flexibility in bending and torsion. Using the bending moment and the torsion Equations, the thickness of flex tube is obtained as 0.4358 mm.

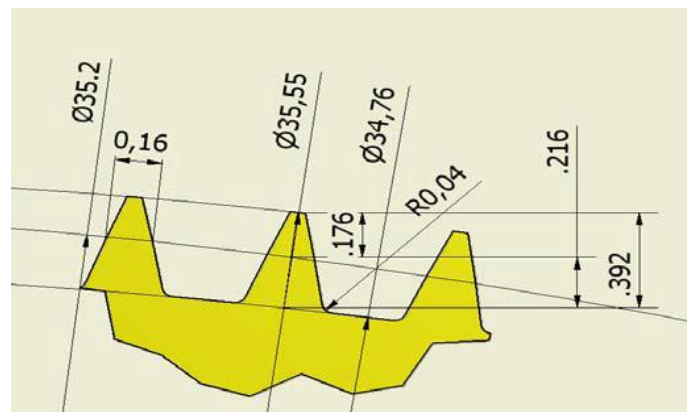


Fig. 6 Tooth dimensions of the Flex Spline Gear

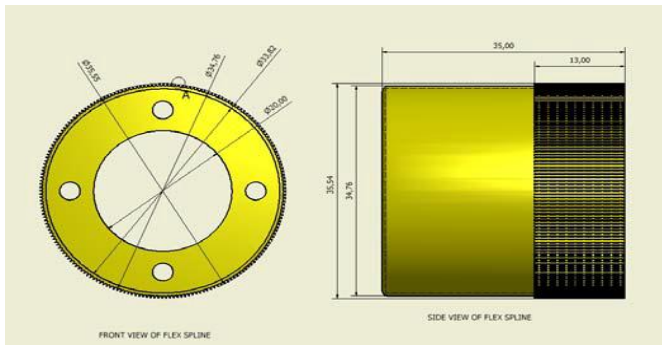


Fig. 7 Front and side views of the Flex Spline tube before deformation

When the wave generator is assembled to the circular Flex Spline, it becomes an elliptical shape. For the exact meshing of the Flex Spline and the Circular Spline, a radial elongation and compression of 0.18 mm is required at the major and minor axes of the Flex gear. Therefore, the length of the Flex Spline tube depends upon several factors like angle of twist, deformation produced, and force applied to get the deformation etc... The length is to be found by considering all these parameters.

The angle of twist and the deformation produced are the critical parameters for the determination of length of the Flex Spline tube. The angle of twist vary with respect to the length has been calculated by simple torsion equation by considering the Flex Spline thickness as 0.43mm and is plotted in Fig. 8.

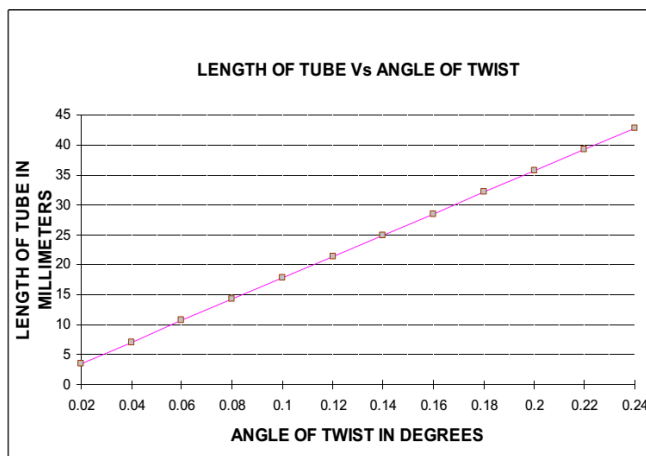


Fig. 8 Variation of Length of tube with Angle of Twist

### VI. FE ANALYSIS OF THE FLEX SPLINE

The radial flexibility of the Flex Spline tube while mating with Wave generator is found by Finite Element Analysis. A general purpose FE software ANSYS has been introduced to solve this problem. For a minimum load of 10N, the deflections and Von-Mises stress distributions are noted for varying the length. The Length Vs Deformation, Length Vs Von Mises stress distribution are shown in Fig. 9 and Fig. 10. And from these results an optimum value of 30 mm which produces 0.18 mm deflection which permits a 0.15° twist is taken for the present design. And for a perfect mating, the deformation of the 30 mm lengthed Flex Spline under different load conditions are studied and is shown in Fig. 11 and its Von Mises stress distribution are also plotted in Fig. 12.

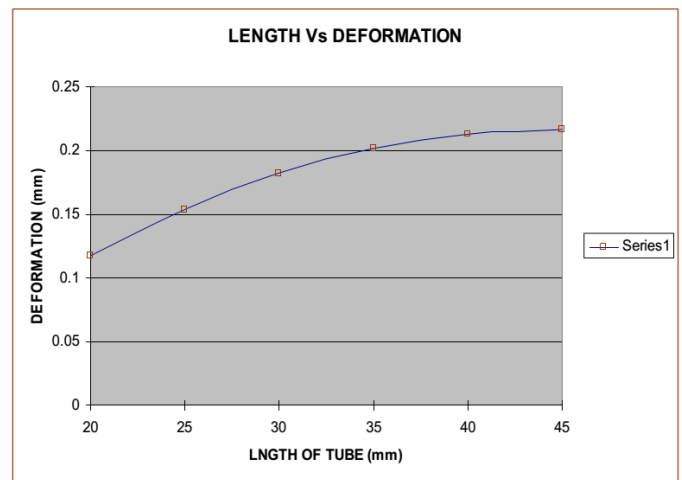


Fig. 9. Variation of deformation with Length of Flex Spline tube for a force of 10N

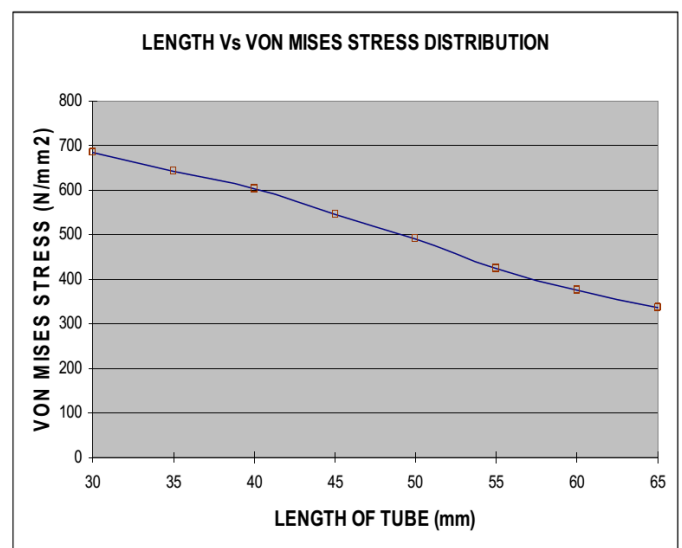


Fig. 10 Variation of Von Mises Stress distribution Vs Length for an applied force of 10N

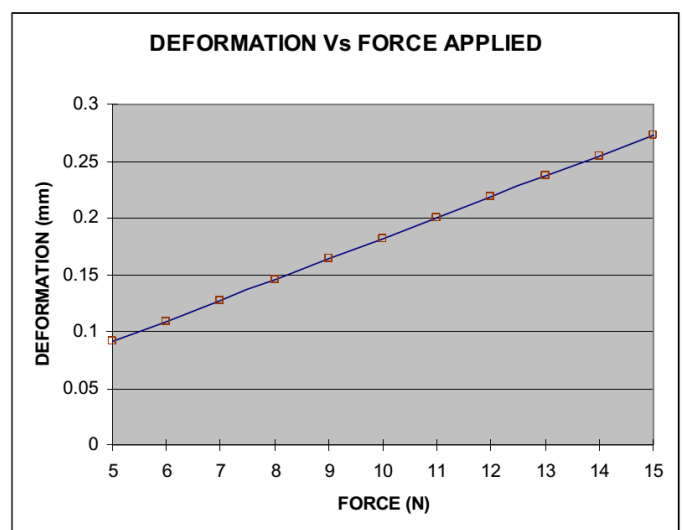


Fig. 11. Variation of Deformation with Force applied for a fixed length of 30 mm

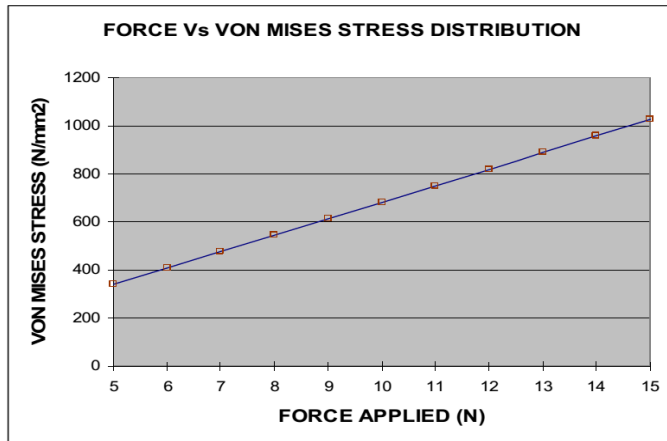


Fig. 12 Variation of Von Mises stress distribution Vs Force applied for a length of 30 mm

But for a length of 30 mm and an applied force of 10N, the maximum shear stress on the Flex spline tube is observed as larger than the Yield strength of the material. To solve this problem, the shell thickness taken as 0.45 mm and analysis is done for the new dimensions. From which an optimized length of 37 mm is obtained for the Flex Spline tube.

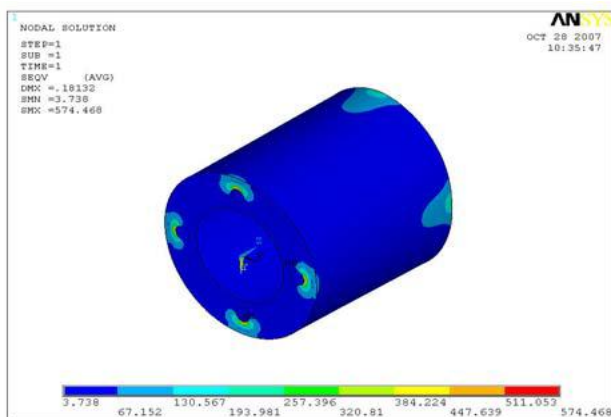


Fig. 13 Von mises stress distribution of flex spline tube under loading

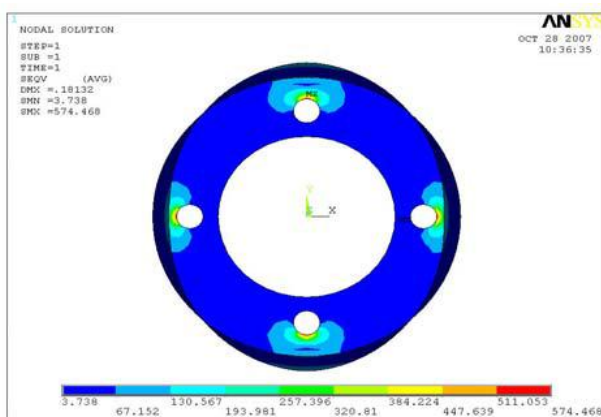


Fig. 14. Deformation of the flex spline tube under load

The Von Mises Stress distribution and the Deformation on the Flex Spline tube is shown in Figure 13 and Figure 14 respectively.

The model of assembly of Flex Spline in the Harmonic drive mechanism is shown in Fig 15.

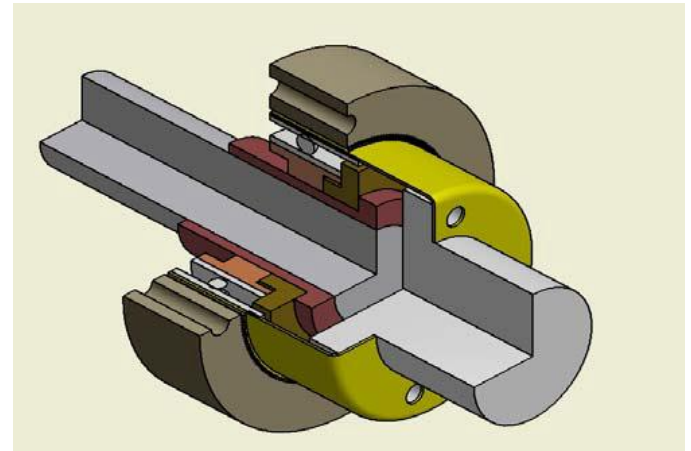


Fig.15 Isometric view of Harmonic Drive Assembly

## VII. RESULTS AND CONCLUSION

Through this paper, an attempt of implementation of the traditional involute tooth profile in the design of the Flex Spline of the Harmonic Drive is conducted. The effect of load sharing on the teeth dimensions is done. A parametric study on flexibility of Flex Spline with axial and radial motions of the wave generator is done. A Finite element analysis was conducted to optimize the length of the Flex Spline tube and plotted the variations of deformations.

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