Depth of Intuitionistic I-Fuzzy EDGE and Height of Intuitionistic I-Fuzzy EDGE of Intuitionistic I-Fuzzy Graph

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***Abstract*: In this paper, depth of intuitionistic I-fuzzy edge and height of intuitionistic I-fuzzy edge of intuitionistic I-fuzzy graph are defined and introduced. Using this concept, some more theorems and results are given.**

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Key Words: Fuzzy subset, I-fuzzy subset, intuitionistic I-fuzzy subset, intuitionistic I-fuzzy relation, strong intuitionistic I-fuzzy relation, intuitionisticI-fuzzy graph, intuitionistic I-fuzzy loop, intuitionistic I-fuzzy pseudo graph, degree of intuitionistic I-fuzzy vertex, total degree of intuitionistic I-fuzzy vertex, order of the intuitionistic I-fuzzy graph, size of the intuitionistic I-fuzzy graph, intuitionistic I-fuzzy regular graph, intuitionistic I-fuzzy totally regular graph, intuitionistic I-fuzzy complete graph, depth of intuitionistic I-fuzzy edge, height of intuitionistic I-fuzzy edge.

INTRODUCTION:

In 1965, Zadeh [14] introduced the notation of fuzzy set as a method of presenting uncertainty. Since complete information in science and technology is not always available. Thus we need mathematical models to handle various types of systems containing elements of uncertainty. Intuitionistic fuzzy set was introduced by Atanassov. K.T[4]. After that Rosenfeld[8] introduced fuzzy graphs. Yeh and Bang[13] also introduced fuzzy graphs independently. Fuzzy graphs are useful to represent relationships which deal with uncertainty and it differs greatly from classical graph. It has numerous applications to problems in computer science, electrical engineering system analysis, operations research, economics, networking routing, transportation, etc. Ramakrishnan P.V and Lakshmi . T [7] introduced depth of μ, height of μ and fuzzy spanning super graphs. Arjunan. K & Subramani.C [2, 3] introduced a new structure of fuzzy graph and I-Fuzzy graph. I-fuzzy spanning supergraphs and intuitionistic fuzzy spanning supergraphs have been defined and introduced by Vasudevan.B et al.[11, 12]. In this paper, depth of intuitionistic I-fuzzy edge and height of intuitionistic I-fuzzy edge of intuitionistic I-fuzzy graph are defined and introduced.

PRELIMINARIES:

Definition 1.1[14]. Let X be any nonempty set. A mapping M: X $\rightarrow$ [0,1] is called a fuzzy subset of X.

Definition 1.2[14]. Let X be any nonempty set. A mapping [M] : X $\rightarrow$ D[0, 1] is called a I-fuzzy subset (interval valued fuzzy subset) of X, where D[0,1] denotes the family of all closed subintervals of [0,1] and $[M](x) = [M^-(x), M^+(x)]$, for all x in X, where M^- and M^+ are fuzzy subsets of X such that $M^-(x) \leq M^+(x)$, for all x in X. Thus $M^-(x)$ is an interval (a closed subset of [0,1]) and not a number from the interval [0,1] as in the case of fuzzy subset.

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Definition 1.3[4]. An **intuitionistic fuzzy subset** (IFS) A in X is defined as an object of the form [A] = { < x, $\mu_A(x)$, $\nu_A(x)$ > / x in X } where μ_A:X→[0, 1] and ν_A: X→[0, 1] define the degree of membership and the degree of non-membership of the element x∈X respectively and for every x∈X satisfying $\mu_A(x) + \nu_A(x) \le 1$.

Definition 1.4. An **intuitionistic I-fuzzy subset** (IIFS) [A] in X is defined as an object of the form [A] = { < x, $\mu_{[A]}(x)$, $\nu_{[A]}(x)$ > / x in X } where $\mu_{[A]}$:X→D[0, 1] and $\nu_{[A]}$: X→D[0, 1] define the degree of membership and the degree of non-membership of the element x∈X respectively and for every x∈X satisfying $\mu_{[A]}^{+}(x) + \nu_{[A]}^{+}(x) \le 1$.

Example 1.5[4]. [A] = { < a, [0.4, 0.7], [0.2, 0.3] >, < b, [0.1, 0.5], [0.2, 0.5] >, < c, [0.5, 0.8], [0.1, 0.2] > } is an intuitionistic I-fuzzy subset of X = { a, b, c }.

Definition 1.6. Let [A] = { 〈 x, $\mu_{[A]}(x)$, $\nu_{[A]}(x)$ 〉 / x∈X }, [B] = { 〈 x, $\mu_{[B]}(x)$, $\nu_{[B]}(x)$ 〉 / x∈X} be any two intuitionistic I-fuzzy subsets of X. We define the following relations and operations:

(i) [A] ⊆ [B] if and only if $\mu_{[A]}(x) \le \mu_{[B]}(x)$ and $\nu_{[B]}(x) \le \nu_{[A]}(x)$ for all x in X.

(ii) [A] = [B] if and only if $\mu_{[A]}(x) = \mu_{[B]}(x)$ and $\nu_{[B]}(x) = \nu_{[A]}(x)$ for all x in X.

(iii) [A]∩[B] = { 〈 x, rmin { $\mu_{[A]}(x)$, $\mu_{[B]}(x)$ }, rmax { $\nu_{[A]}(x)$, $\nu_{[B]}(x)$ } 〉 / x∈X }

where rmin {$\mu_{[A]}(x)$, $\mu_{[B]}(x)$} = [min {$\mu_{[A]}^{-}(x)$, $\mu_{[B]}^{-}(x)$}, min{ $\mu_{[A]}^{+}(x)$, $\mu_{[B]}^{+}(x)$}] and rmax{$\nu_{[A]}(x)$, $\nu_{[B]}(x)$} = [max{ $\nu_{[A]}^{-}(x)$, $\nu_{[B]}^{-}(x)$ }, max{ $\nu_{[A]}^{+}(x)$, $\nu_{[B]}^{+}(x)$ }].

(iv) [A]∪[B] = { 〈 x, rmax { $\mu_{[A]}(x)$, $\mu_{[B]}(x)$ }, rmin { $\nu_{[A]}(x)$, $\nu_{[B]}(x)$ } 〉 / x∈X }

where rmax{ $\mu_{[A]}(x)$, $\mu_{[B]}(x)$ } = [max{$\mu_{[A]}^{-}(x)$, $\mu_{[B]}^{-}(x)$ }, max{$\mu_{[A]}^{+}(x)$, $\mu_{[B]}^{+}(x)$ }] and rmin { $\nu_{[A]}(x)$, $\nu_{[B]}(x)$ } = [min { $\nu_{[A]}^{-}(x)$, $\nu_{[B]}^{-}(x)$ }, min { $\nu_{[A]}^{+}(x)$, $\nu_{[B]}^{+}(x)$ }].

(v) $[A]^{C}$ = { 〈 x, $\nu_{[A]}(x)$, $\mu_{[A]}(x)$ 〉 / x∈X }.

Definition 1.7. Let [M] = < $\mu_{[M]}$, $\nu_{[M]}$ > be an intuitionistic I-fuzzy subset in a set S, the **strongest intuitionistic I-fuzzy relation** on S, that is an intuitionistic I-fuzzy relation [V] = < $\mu_{[V]}$, $\nu_{[V]}$ > with respect to [M] given by $\mu_{[V]}(x,y)$ = rmin { $\mu_{[M]}(x)$, $\mu_{[M]}(y)$ } and $\nu_{[V]}(x,y)$ = rmax{ $\nu_{[M]}(x)$, $\nu_{[M]}(y)$} for all x and y in S.

Definition 1.8. Let V be any nonempty set, E be any set and f: E ⟶ V×V be any function. Then [A] = < $\mu_{[A]}$, $\nu_{[A]}$ > is an Interval-valued intuitionistic subset of V, [S] = < $\mu_{[S]}$, $\nu_{[S]}$ > is an intuitionistic I-fuzzy relation on V with respect to [A] and [B] = < $\mu_{[B]}$, $\nu_{[B]}$ > is an intuitionistic I-fuzzy subset of E such that $\mu_{[B]}(e) \le \underset{e\in f^{-1}(x,y)}{\mu_{[S]}}(x,y)$ and $\nu_{[B]}(e) \ge \underset{e\in f^{-1}(x,y)}{\nu_{[S]}}(x,y)$.

Then the ordered triple [F] = ([A], [B], f) is called an intuitionistic I-fuzzy graph, where the elements of [A] are called **intuitionistic I-fuzzy points** or **intuitionistic I-fuzzy vertices** and the elements of [B] are called **intuitionistic I-fuzzy lines** or **intuitionistic I-fuzzy edges** of the intuitionistic I-fuzzy graph [F]. If f(e) = (x, y), then the intuitionistic I-fuzzy points (x, $\mu_{[A]}(x)$, $\nu_{[A]}(x)$), (y, $\mu_{[A]}(y)$, $\nu_{[A]}(y)$) are called **intuitionistic I-fuzzy adjacent points** and intuitionistic I-fuzzy points (x, $\mu_{[A]}(x)$, $\nu_{[A]}(x)$), intuitionistic I-fuzzy line (e, $\mu_{[B]}(e)$, $\nu_{[B]}(e)$) are called **incident** with each other. If two district intuitionistic I-fuzzy lines (e_1, $\mu_{[B]}(e_1)$, $\nu_{[B]}(e_1)$) and (e_2, $\mu_{[B]}(e_2)$, $\nu_{[B]}(e_2)$) are incident with a common intuitionistic I-fuzzy point, then they are called **intuitionistic I-fuzzy adjacent lines.**

Definition 1.9. An intuitionistic I-fuzzy line joining an intuitionistic I-fuzzy point to itself is called an **intuitionistic I-fuzzy loop**.

Definition 1.10. Let [F] = ([A], [B], f) be an intuitionistic I-fuzzy graph. If more than one intuitionistic I-fuzzy line joining two intuitionistic I-fuzzy vertices is allowed, then the intuitionistic I-fuzzy graph [F] is called an **intuitionistic I-fuzzy pseudo graph**.

Definition 1.11. [F] = ([A], [B], f) is called an **intuitionistic I-fuzzy simple graph** if it has neither intuitionistic I-fuzzy multiple lines nor intuitionistic I-fuzzy loops.

Example 1.12. F = ([A], [B], f), where V = {v_1, v_2, v_3, v_4, v_5}, E = {a, b, c, d, e, h, g } and f : E $\rightarrow$ V×V is defined by f(a) = (v_1, v_2) , f(b) = (v_2, v_2), f(c) = (v_2, v_3), f(d) = (v_3, v_4), f(e) = (v_3, v_4), f(h) = (v_4, v_5), f(g) = (v_1, v_5). An intuitionistic I-fuzzy subset [A] = { (v_1, [0.5, 0.7], [0.2, 0.3]), (v_2, [0.4, 0.6], [0.1, 0.3]), (v_3, [0.4, 0.8], [0.2, 0.2]), (v_4, [0.3, 0.5], [0.2, 0.3]), (v_5, [0.3, 0.7], [0.2, 0.2]) } of V. An intuitionistic I-fuzzy relation [S] = { ((v_1, v_1), [0.5, 0.7], [0.2, 0.3]), ((v_1, v_2), [0.4, 0.6], [0.2, 0.3]), ((v_1, v_3), [0.4, 0.7], [0.2, 0.3]), ((v_1, v_4), [0.3, 0.5], [0.2, 0.3]), ((v_1, v_5), [0.3, 0.7], [0.2, 0.3]), ((v_2, v_1), [0.4, 0.6], [0.2, 0.3]), ((v_2, v_2), [0.4, 0.6], [0.1, 0.3]), ((v_2, v_3), [0.4, 0.6], [0.2, 0.3]), ((v_2, v_4), [0.3, 0.5], [0.2, 0.3]), ((v_2, v_5), [0.3, 0.6], [0.2, 0.3]), ((v_3, v_1), [0.4, 0.7], [0.2, 0.3]), ((v_3, v_2), [0.4, 0.6], [0.2, 0.3]), ((v_3, v_3), [0.4, 0.8], [0.2, 0.2]), ((v_3, v_4), [0.3, 0.5], [0.2, 0.3]), ((v_3, v_5), [0.3, 0.7], [0.2, 0.2]), ((v_4, v_1), [0.3, 0.5], [0.2, 0.3]), ((v_4, v_2), [0.3, 0.5], [0.2, 0.3]), ((v_4, v_3), [0.3, 0.5], [0.2, 0.3]), ((v_4, v_4), [0.3, 0.5], [0.2, 0.3]), ((v_4, v_5), [0.3, 0.5], [0.2, 0.3]), ((v_5, v_1), [0.3, 0.7], [0.2, 0.3]), ((v_5, v_2), [0.3, 0.6], [0.2, 0.3]), ((v_5, v_3), [0.3, 0.7], [0.2, 0.2]), ((v_5, v_4), [0.3, 0.5], [0.2, 0.3]), ((v_5, v_5), [0.3, 0.7], [0.2, 0.3]) } on V with respect to [A] and an intuitionistic I-fuzzy subset [B] = {(a, [0.4, 0.5], [0.2, 0.4]), (b, [0.3, 0.5], [0.2, 0.3]), (c, [0.4, 0.6], [0.2, 0.4]), (d, [0.2, 0.5], [0.3, 0.4]), (e, [0.3, 0.5], [0.2, 0.3]), (h, [0.3, 0.5], [0.3, 0.4]), (g, [0.3, 0.6], [0.2, 0.4]) } of E.

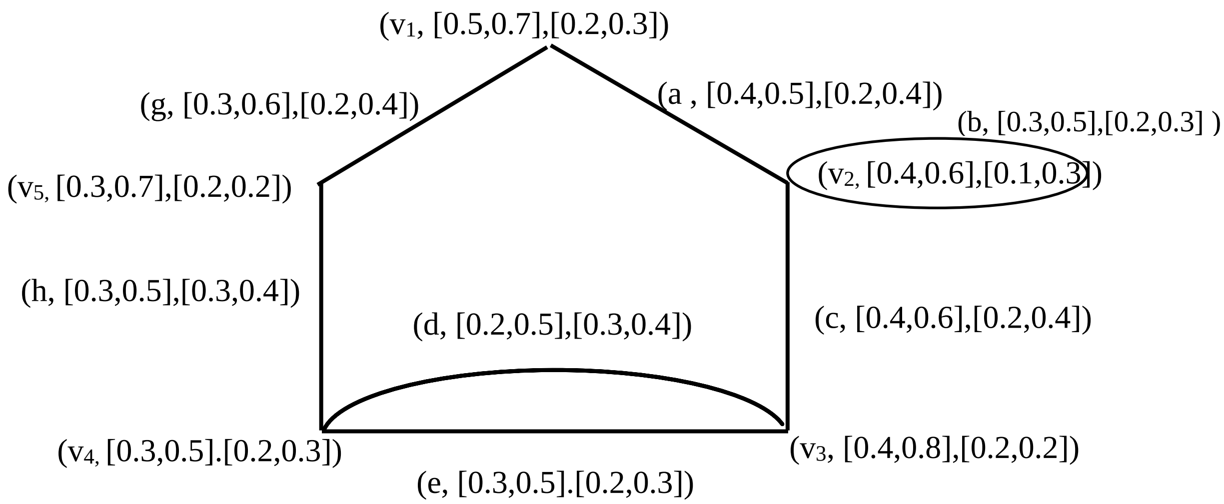

Fig 1.1

In figure 1.1, (i) (v_1, [0.5, 0.7], [0.2, 0.3]) is an intuitionistic I-fuzzy point. (ii) (a, [0.4, 0.5], [0.2, 0.4]) is an intuitionistic I-fuzzy edge. (iii) (v_1, [0.5, 0.7], [0.2, 0.3]) and (v_2, [0.4, 0.6], [0.1, 0.3]) are intuitionistic I-fuzzy adjacent points. (iv) (a, [0.4, 0.5], [0.2, 0.4]) join with (v_1, [0.5, 0.7], [0.2, 0.3]) and (v_2, [0.4, 0.6], [0.1, 0.3]) and therefore it is incident with (v_1, [0.5, 0.7], [0.2, 0.3]) and (v_2, [0.4, 0.6], [0.1, 0.3]). (v) (a, [0.4, 0.5], [0.2, 0.4]) and (g, [0.3, 0.6], [0.2, 0.4]) are intuitionistic I-fuzzy adjacent lines. (vi) (b, [0.3, 0.5], [0.2, 0.3]) is an intuitionistic I-fuzzy loop. (vii) (d, [0.2, 0.5], [0.3, 0.4]) and (e, [0.3, 0.5]. [0.2, 0.3]) are intuitionistic I-fuzzy multiple edges. (viii) It is not an intuitionistic I-fuzzy simple graph. (ix) It is an intuitionistic I-fuzzy pseudo graph.

Definition 1.13. The fuzzy graph [H] = ([C], [D], f) where [C] = < $\mu_{[C]}$, $\nu_{[C]}$ > and [D] = < $\mu_{[D]}$, $\nu_{[D]}$ > is called an **intuitionistic I-fuzzy subgraph** of [F] = ([A], [B], f) if [C] $\subseteq$ [A] and [D] $\subseteq$ [B].

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Definition 1.14. Let [F] = ([A], [B], f) be an intuitionistic I-fuzzy graph. Then the **degree of an intuitionistic I-fuzzy vertex** is defined by d(v) = (d_μ(v),d_ν(v)) where $d_\mu(v) = \sum_{e \in f^{-1}(u,v)} \mu_{[B]}(e) + 2 \sum_{e \in f^{-1}(v,v)} \mu_{[B]}(e)$ and

$d_\nu(v) = \sum_{e \in f^{-1}(u,v)} \nu_{[B]}(e) + 2 \sum_{e \in f^{-1}(v,v)} \nu_{[B]}(e)$.

Definition 1.15. Let [F] = ([A], [B], f) be an intuitionistic I-fuzzy graph. The **total degree of intuitionistic I-fuzzy vertex v** is defined by $d_T(v) = (d_{T_\mu}(v), d_{T_\nu}(v))$ where $d_{T_\mu}(v) = \sum_{e \in f^{-1}(u,v)} \mu_{[B]}(e) + 2 \sum_{e \in f^{-1}(v,v)} \mu_{[B]}(e) + \mu_{[A]}(v) = d_\mu(v) + \mu_{[A]}(v)$ and

$d_{T_\nu}(v) = \sum_{e \in f^{-1}(u,v)} \nu_{[B]}(e) + 2 \sum_{e \in f^{-1}(v,v)} \nu_{[B]}(e) + \nu_{[A]}(v) = d_\nu(v) + \nu_{[A]}(v)$ for al v in V.

Definition 1.16. The **minimum degree** of the intuitionistic I-fuzzy graph [F] = ([A], [B], f) is δ[F] = (δ_μ[F] , δ_ν[F]) where δ_μ[F] = rmin{ d_μ(v) / v∈V } and δ_ν[F] = rmax{ d_ν(v) / v∈V} and the **maximum degree** of [F] is Δ[F] = (Δ_μ[F], Δ_ν[F]) whereΔ_μ[F] = rmax{ d_μ(v) / v∈V }andΔ_ν[F] = rmin{ d_ν(v) / v∈V }.

Definition 1.17. Let [F] = ([A], [B], f) be an intuitionistic I-fuzzy graph. Then the **order of intuitionistic I-fuzzy graph** [F] is defined to be O[F] = (O_μ[F] , O_ν[F]) where $O_\mu[F] = \sum_{v \in V} \mu_{[A]}(v)$ and $O_\nu[F] = \sum_{v \in V} \nu_{[A]}(v)$

Definition 1.18. Let [F] = ([A], [B], f) be an intuitionistic I-fuzzy graph. Then the **size of the intuitionistic I-fuzzy graph [F]** is defined to be S[F] = (S_μ[F] , S_ν[F]) where $S_\mu[F] = \sum_{e \in f^{-1}(x,y)} \mu_{[B]}(e)$ and $S_\nu[F] = \sum_{e \in f^{-1}(x,y)} \nu_{[B]}(e)$.

Definition 1.19. An intuitionistic I-fuzzy graph [F] = ([A], [B], f) is called **intuitionistic I-fuzzy regular graph** if d(v) = [m, n] for all v in V. It is also called **intuitionistic I-fuzzy [m, n]-regular graph.**

Definition 1.20. An intuitionistic I-fuzzy graph [F] is an **intuitionistic I-fuzzy [m, n]-totally regular graph** if each intuitionistic I-fuzzy vertex of [F] has the same total degree [m, n].

Theorem 1.21. The sum of the degree of all intuitionistic I-fuzzy vertices in a intuitionistic I-fuzzy graph [F] = ([A], [B], f) is equal to twice the sum of the membership value of all intuitionistic I-fuzzy edges. That is $\sum_{v \in V} d(v) = 2S([F])$.

2. DEPTH OF INTUITIONISTIC I-FUZZY EDGE AND HEIGHT OF INTUITIONISTIC I-FUZZY EDGE OF INTUITIONISTIC I-FUZZY GRAPH

Definition 2.1. Let $[F] = ([A], [B], f)$ be an intuitionistic I-fuzzy graph. Then the depth of intuitionistic I-fuzzy edge $[B]$ is defined by D([B]) = ($\mu_{[d]}$([B]), $\nu_{[d]}$([B])) = (rmin { $\mu_{[B]}$(e) / e∈E) }, rmax { $\nu_{[B]}$(e) / e∈E) }).

Definition 2.2. Let $[F] = ([A], [B], f)$ be an intuitionistic I-fuzzy graph. Then the height of intuitionistic I-fuzzy edge [B] is defined by H([B]) = ($\mu_{[d]}$([B]), $\nu_{[d]}$([B])) = (rmax { $\mu_{[B]}$(e) / e∈E) }, rmin { $\nu_{[B]}$(e) / e∈E) }).

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Example 2.3.

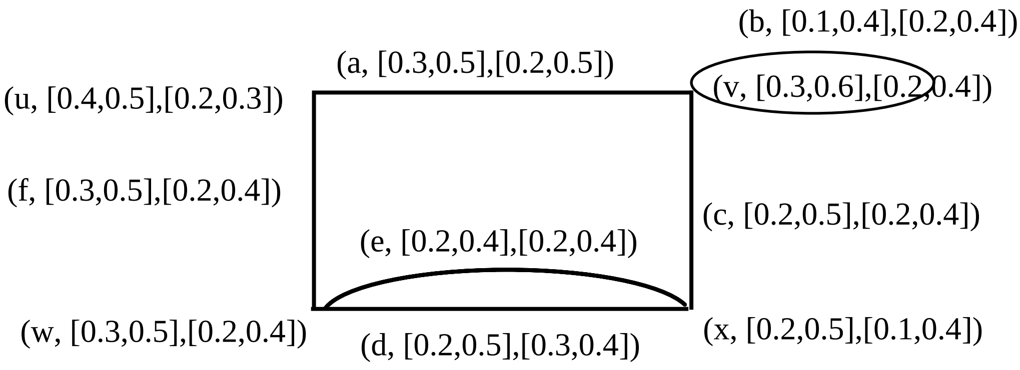

Fig 2.1 Intuitionistic I-fuzzy graph [F]

Here D(B) = ([0.1,0.4],[0.3,0.5]) and H(B) = ([0.3,0.5], [0.2,0.4]).

Remark 2.4. Clearly $D([B]) \leq [B](e) \leq H([B])$, since $\mu_{[D]}([B]) \leq \mu_{[B]}(e) \leq \mu_{[H]}([B])$ and $\nu_{[D]}([B]) \geq \nu_{[B]}(e) \geq \nu_{[H]}([B])$.

Theorem 2.5. Let [F] = ([A], [B], f) be any intuitionistic I-fuzzy graph with $|V| = p$ and $|E| = q$. Then $D([B]) \leq \frac{S([F])}{q} \leq H([B])$.

Proof. Suppose [F] = ([A], [B], f) is any intuitionistic I-fuzzy graph with p-intuitionistic I-fuzzy vertices.

Obviously, $\mu_{[D]}([B]) \leq [B](e) \leq H([B]) \Rightarrow \sum_{e\in E} D([B]) \leq \sum_{e\in E} [B](e) \leq \sum_{e\in E} H([B])$

$\Rightarrow qD([B]) \leq S([F]) \leq qH([B]) \Rightarrow D([B]) \leq \frac{S([F])}{q} \leq H([B])$.

Theorem 2.6. Let[F] = ([A], [B], f) be any intuitionistic I-fuzzy simple graph with p-intuitionistic I-fuzzy vertices.Then $\frac{2S([F])}{p(p-1)} \leq H([B])$.

Proof. By Theorem 2.5, $\frac{S([F])}{q} \leq H([B]) \Rightarrow S([F]) \leq qH([B]) \Rightarrow \frac{2S([F])}{p(p-1)} \leq H([B])$.

Theorem 2.7. Let [F] = ([A], [B], f) be a intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices.

Then $D([B]) \leq \frac{2S([F])}{p(p-1)} \leq H([B])$.

Proof. By Theorem 2.5, $D([B]) \leq \frac{S([F])}{q} \leq H([B]) \Rightarrow qD([B]) \leq S([F]) \leq qH([B])$

Since F is intuitionistic I-fuzzy complete graph,

$\frac{p(p-1)}{2} D([B]) \leq S([F]) \leq \frac{p(p-1)}{2} H([B])$. Which implies that

$D([B]) \leq \frac{2S([F])}{p(p-1)} \leq H([B])$.

Theorem 2.8. Let [F] = ([A], [B], f) be a intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices and $[A]$ be [s,t]-constant function.

Then $D([B]) = \frac{2S([F])}{p(p-1)} = H([B])$.

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Proof. Assume that $[F]$ is a intuitionistic I-fuzzy complete graph with p-intuitionistic

I-fuzzy vertices and $[A](v) = [s,t]$ for all v in V.That is $\mu_{[B]}(e) = \mu_{[S]_{e\epsilon f^{-1}(x,y)}}(x,y)$ and $\gamma_{[B]}(e) = \gamma_{[S]_{e\epsilon f^{-1}(x,y)}}(x,y)$ for all x, y

inV. Then implies that $\mu_{[B]}(e) = \mu_{[A]}(x) \cap \mu_{[A]}(y)$ and $\gamma_{[B]}(e) = \gamma_{[A]}(x) \cup \gamma_{[A]}(y)$ = [s,t] for all x and y in V,
so $D([B]) = [B](e) = H([B])$

$$\Rightarrow \sum_{e\in E} D([B]) = \sum_{e\in E}[B](e) = \sum_{e\in E} H([B]) \Rightarrow qD([B]) = S([F]) = qH([B])$$

which implies $\frac{p(p-1)}{2}D([B]) = S([F]) = \frac{p(p-1)}{2}H([B])$.

Hence$D([B]) = \frac{2S([F])}{p(p-1)} = H([B])$.

Corollary 2.9. Let [F] = ([A], [B], f) be an intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices and $[A]$ be a [s, t]-constant function. Then

$$\sum_{v\in V} d(v) = p(p-1)H([B]) = p(p-1)D([B])$$

Theorem 2.10. If $[F]$ is an intuitionistic I-fuzzy [m,n]- regular graph with

p-intuitionistic I-fuzzy vertices. Then $H([B]) \geq \frac{[m,n]}{p-1}$.

Proof. Suppose $[F]$ is an intuitionistic I-fuzzy [m,n]- regular graph with p-intuitionistic I-fuzzy vertices. Here $d(v) = [\text{m}, \text{n}]$ for all v in V, $\sum_{v\in V} d(v) = \sum_{v\in V}[m,n] = p[m,n]$. We get $2S([F]) = p[m,n]$ implies that $S([F]) = \frac{p[m,n]}{2}$. By 2.6 Theorem, $\frac{p[m,n]}{2} \leq \frac{p(p-1)}{2}H([B]) \Rightarrow \frac{[m,n]}{p-1} \leq H([B])$ which implies that $H([B]) \geq \frac{[m,n]}{p-1}$.

Theorem 2.11. Let [F] = ([A], [B], f) be a intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices and $[A]$ be [s, t] -constant function.
Then $H([B]) = [\text{s}, \text{t}] = D([B])$.
Proof. Assume that $[F]$ is a intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices and $[A](v) = [\text{s}, \text{t}]$ for all v in V. That is $\mu_{[B]}(e) = \mu_{[S]_{e\epsilon f^{-1}(x,y)}}(x,y)$

and $\gamma_{[B]}(e) = \gamma_{[S]_{e\epsilon f^{-1}(x,y)}}(x,y)$ for all x, y inV. Then implies that $\mu_{[B]}(e) = \mu_{[A]}(x) \cap \mu_{[A]}(y)$ and $\gamma_{[B]}(e) = \gamma_{[A]}(x) \cup \gamma_{[A]}(y)$ = [s,t] for all x and y in V. Therefore $d(v) = (p-1)[\text{s}, \text{t}]$ for all v in V. Which implies that $\sum_{v\in V} d(v) = \sum_{v\in V}(p-1)[\text{s},\text{t}] = p(p-1)[\text{s},\text{t}]$. By Corollary 2.9, $\sum_{v\in V} d(v) = p(p-1)H([B]) = p(p-1)D([B])$.
Hence$H([B]) = [\text{s}, \text{t}] = D([B])$.

Theorem 2.12. Let [F] = ([A], [B], f) be any intuitionistic I-fuzzy simple graph with p-intuitionistic I-fuzzy vertices . Then $\delta([F]) \leq (p-1)H([B])$.

Proof. For any intuitionistic I-fuzzy graph, $\delta([F]) \leq \frac{2S([F])}{p}$. By Theorem 2.6,$\frac{2S([F])}{p} \leq (p-1)H([B])$ which implies that $\delta([F]) \leq (p-1)H([B])$.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 8 Issue 08, August-2019

Theorem 2.13. Let[F] $=$ ([A], [B], f) be an intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices and $[A]$ be [s, t]-constant function.

Then$\delta([F]) = \Delta([F]) = (p-1)H([B]) = (p-1)D([B])$.

Proof. By Theorem 2.11, $d(v) = (p-1)[s,t]$ for all v in V and $H(B) = D(B) = [s,t]$ also $\delta([F]) = \Delta([F]) = (p-1)[s,t]$ implies that $\frac{\delta([F])}{p-1} = \frac{\Delta[(F])}{p-1} = [s,t] \Rightarrow H([B]) = D([B]) = \frac{\delta([F])}{p-1} = \frac{\Delta([F])}{p-1}$ implies that $\delta([F]) = \Delta([F]) = (p-1)H([B]) = (p-1)D([B])$.

Theorem 2.14. If [F] = ([A], [B], f) is a intuitionistic I-fuzzy [s,t] -totally regular graph with p-intuitionistic I-fuzzy vertices. Then $d_T(v) \le$ (p-1) H([B]) + $\frac{O([F])}{p}$.

Proof. For any intuitionistic I-fuzzy graph, 2S([F]) + O([F]) = p[s, t].

By Theorem 2.6,$S([F]) \le \frac{p(p-1)}{2}H \Rightarrow p(p-1)H([B]) + \text{O([F])} \ge p[s,t]$

$\Rightarrow p(p-1)H([B]) + \text{O([F])} = pd_T(v)$. Hence $d_T(v) \le$ (p-1) H([B]) + $\frac{O([F])}{p}$.

Theorem 2.15. If [F] = ([A], [B], f) is both intuitionistic I-fuzzy [m,n]-regular graph and intuitionistic I-fuzzy [s,t]-totally regular graph with p-intuitionistic I-fuzzy vertices. Then $d_T(v) \le$ (p-1) H([B]) + $\frac{O([F])}{p}$.

Proof. By Theorem 2.10, $H([B]) \ge \frac{[m,n]}{p-1}$. By hypothesis,[m, n] + $\frac{O([F])}{p}$ = [s, t]

$\Rightarrow$ (p-1)H([B]) +$\frac{O([F])}{p}$ $\ge$[s, t] $\Rightarrow d_T(v) \le$ (p-1) H([B]) + $\frac{O([F])}{p}$.

Theorem 2.16. If[F] $=$ ([A], [B], f) is a intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices and $[A]$ is a [s, t]-constant function.

Then $O([F]) = pH([B]) = pD([B])$.

Theorem 2.17. Let [F] = ([A], [B], f) be any intuitionistic I-fuzzy graph with respect to set V and E where $|V| = p$ and $|E| = q$. Then $q\ D([B]) \le \frac{\sum_{v\in V} d(v)}{2} \le qH([B])$.

Theorem 2.18. Let [F] = ([A], [B], f) be any intuitionistic I-fuzzy simple graph with p-intuitionistic I-fuzzy vertices. Then $\sum_{v\in V} d(v) \le p(p-1)H([B])$.

Theorem 2.19. Let [F] = ([A], [B], f) be a intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices. Then$p(p-1)D([B]) \le \sum_{v\in V} d(v) \le p(p-1)H([B])$.

Theorem 2.20. Let [F] = ([A], [B], f) be a intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices and $[A]$ be $[k]$-constant function.

Then $p(p-1)D([B]) = \sum_{v\in V} d(v) = p(p-1)H([B])$.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 8 Issue 08, August-2019

Proof. By Theorem 2.8, $D([B]) = \frac{2S([F])}{p(p-1)} = H([B])$. Since [F] is a intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices and by Theorem 2.19, so $p(p-1)\, D([B]) = \sum_{v\in V} d(v) = p(p-1)H([B])$.

Theorem 2.21. Let [F] = ([A], [B], f) be a intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices and $[A]$ be [s,t]-constant function. Then $\sum_{v\in V} d_T(v) = p^2 H([B]) = p^2 D([B])$.

Proof. By Theorem 2.20, $\sum_{v\in V} d(v) = p(p-1)\, H([B]) = p(p-1)\, D([B])$.

$\Rightarrow \sum_{v\in V} d_T(v) = \sum_{v\in V} d(v) + \sum_{v\in V} [A](v)$, since $[A]$ is [s,t]-constant function,

$= p(p-1)\, H([B]) + pH([B]) = p^2 H([B])$

Similarly $\sum_{v\in V} d_T(v) = p^2 D([B])$.

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