Decomposition of Supra M-Continuous Mappings

[1] M. Sathya Bama
Department of Mathematics,
Sree Sevugan Annamalai College,
Devakottai, Sivagangai

[2]O. Ravi
Department of Mathematics
P.M.T.College, Madurai
Tamil Nadu, India

***Abstract*:- In this paper, supra A-set, supra t-set, supra h-set and supra C-set and some new supra topological maps are introduced. Characterizations and properties of such new notions are studied. Also investigate the relationships with other mappings like supra α*-continuous.**

Keywords and Phrases:- Supra A-set, supra t-set, supra h-set and supra C-set, supra A-continuous map, supra M-continuous map, supra B-continuous map,supra α*-continuous map, supraA*-continuous map, supra B*-continuous map, supra topological space.

I. INTRODUCTION

Njastad [7] initiated the concept of nearly open sets in topological spaces. Following it many research papers were introduced by Tong[12, 13], Przemski [2] and Ganster[3] in the name of "Decomposition of Continuity" in topological spaces. In 1983,

Mashhour et al. [6] introduced the supra topological spaces and studied S–continuous maps and S* - continuous maps. In 2008, Devi et al. [1] introduced and studied a class of sets called supra α-open and a class of maps called sα-continuous maps between topological spaces, respectively. Ravi et al. [9] introduced and studied a class of sets called supra β-open and a class of maps called supra β-continuous, respectively. Kamaraj et al. [5] introduced and studied the concepts of supra regular-closed set. It is an effort based on them to bring out a paper in the name of "Decomposition of supra M-continuity" in supra topological spaces using the new sets like supra A-set, supra t-set, supra h-set and supra C-set and new mappings like supra A-continuous, supra B-continuous map, supra α*-continuous map, supra A*-continuous map, supra B*-continuous. In this paper, we obtain some important results in supra topological spaces. In most of the occasions, our ideas are illustrated and substantiated by suitable examples.

II. PRELIMINARIES:

Throughout this paper (X,τ), (Y, σ) and (Z, ν) (or simply, X, Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.1 [6, 10] : Let X be a non-empty set. The subfamily $\mu \subseteq P(X)$ where P(X) is the power set of X is said to be a supra topology on X if $X \in \mu$ and μ is closed under arbitrary unions. The pair (X, μ) is called a supra topological space.

The elements of μ are said to be supra open in (X, μ). Complements of supra open sets are called supra closed sets.

Definition 2.2 [10] : Let A be a subset of (X, μ). Then
i. supra closure of a set A is, denoted by $cl^{\mu}(A)$, defined as $cl^{\mu}(A) = \cap \{ B : B$ is a supra closed and $A \subseteq B\}$;

ii. supra interior of a set A is, denoted by $int^{\mu}(A)$, defined as $int^{\mu}(A) = \cup\{G : G$ is a supra open and $A \supseteq G\}$.

Definition 2.3 [6] : Let (X, τ) be a topological space and μ be a supra topology on X. We call μ is a supra topology associated with τ if $\tau \subseteq \mu$.

Definition 2.4 *:* Let (X, μ) be a supra topological space. A subset A of X is called

i. supra semi-open set [10] if $A \subseteq cl^{\mu}(int^{\mu}(A))$;
ii. supra α-open set [1, 10] if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$;
iii. supra regular-open [9] if $A = int^{\mu}(cl^{\mu}(A))$;
iv. supra pre-open set [11] if $A \subseteq int^{\mu}(cl^{\mu}(A))$.

The complements of the above mentioned open sets are called their respective closed sets. The family of all supra regular-closed sets of X is denoted by SRC(X).

III. SUPRA C-SETS

In this section we introduce a new type of set as follows:
Definition 3.1 : A subset S of X is said to be
i. supra A-set if $S = M \cap N$ where M is supra open set and N is SRC(X)
ii. supra t-set if $int^{\mu}(cl^{\mu}(S)) = int^{\mu}(S)$

iii. supra B-set if S = M∩N where M is supra open and N is a supra t-set
iv. supra C-set if S = M∩N where M is supra open and N is a supra h-set.
v. supra h-set if $int^{\mu}(cl^{\mu}(int^{\mu}(S))) = int^{\mu}(S)$

Theorem 3.2: Let (X, μ) be a supra topological space. If A is a supra t-set of X and B⊆X with A⊆B⊆cl^{μ}(A) then B is a supra t-set.

Proof: We note that $cl^{\mu}(B) \subseteq cl^{\mu}(A)$. So we have $int^{\mu}(B) \subseteq int^{\mu}(cl^{\mu}(B)) \subseteq int^{\mu}(cl^{\mu}(A)) = int^{\mu}(A) \subseteq int^{\mu}(B)$. Thus $int^{\mu}(B) = int^{\mu}(cl^{\mu}(B))$ and hence B is supra t-set.

Remark 3.3: i. The union of two supra h-set need not be a supra h-set.
ii. The union of two supra t-set need not be a supra t-set.

Example 3.4 : Let X = {a, b, c} with μ = {X, φ, {a, b} {a, c} {b, c}}. Here {a} and {b} are both supra t-set and supra h set but their union {a, b} is not both supra t-and supra h-sets.

IV. COMPARISONS

Theorem 4.1 : Any supra open set is a supra A-set.

Proof : S = X∩S where X ∊ SRC(X) and S is supra open. The proof is completed.

The converse of the above theorem is not true as can be seen from the following examples.

Example 4.2 : Let X = {a, b, c, d}with μ = {X, φ, {a}, {a,d},{b,c,d}}.Here {d} is supra A-set but not supra open.

Theorem 4.3 :Any supra closed set is a supra t-set but not converse.

Proof: Since A = cl^{μ}(A), $int^{\mu}(A) = int^{\mu}(cl^{\mu}(A))$. The proof is completed.

Example 4.4: Consider Example 4.2, {b} is supra t-set but not supra closed.

Theorem 4.5: A supra regular-open set is a supra t-set but not converse.

Proof: Since S = $int^{\mu}(cl^{\mu}(S))$, $int^{\mu}(S) = int^{\mu}(cl^{\mu}(S))$. The proof is completed.

Example 4.5: Consider Example 4.2, {b} is supra t-set but not supra regular-open.

Theorem 4.6: A supra regular-open set is supra open but not converse.

Proof:: Suppose S is supra regular-open set then S = $int^{\mu}(cl^{\mu}(S))$. Then $int^{\mu}(S) = int^{\mu}(cl^{\mu}(S))$. Since S is supra regular-open, we have $int^{\mu}(S)$ = S. Thus S is supra open. The proof is completed.

Example 4.7: Consider the Example 4.2, {a, d} is supra open set but not supra regular open.

Theorem 4.8: Every supra t-set is supra B-set.

Proof: Let S be any supra t-set S = X∩S where X is supra open and S is supra t-set. The proof is completed.

The converse of the above theorem is not true as can be seen from the following example.

Example 4.9: Consider Example 4.2, {d} is supra B-set but not supra t-set.

Theorem 4.10: Any supra open set is a supra B-set.

Proof: Since S = X∩S where S is supra open and X is supra regular open, by Theorem 4.5, X is supra t-set. The proof is completed.

The converse of the above theorem is not true as can be seen the following example.

Example 4.11: Consider Example 4.2, {c} is supra B-set but not supra open set.

Theorem 4.12: Any supra closed is a supra B-set.

Proof: It follows from Theorem 4.3 and Theorem 4.5

Theorem 4.13: Every supra A-set is a supra B-set.

Proof: S = X∩S where X is supra open and S is supra regular-closed. Since S is supra closed, by Theorem 4.3, S is supra t-set. The proof is completed.

The converse of the above Theorem is not true as can be seen from the following example.

Example 4.14: Consider Example 4.2, {c} is supra B-set but not supra A-set.

Theorem 4.15: Any supra t-set is supra h-set but not converse.

Proof: Let S be supra t-set, then $int^{\mu}(S) = int^{\mu}(cl^{\mu}(S))$, $cl^{\mu}(int^{\mu}(S)) = cl^{\mu}(int^{\mu}(cl^{\mu}(S)))$ implies $int^{\mu}(cl^{\mu}(int^{\mu}(S))) = int^{\mu}(cl^{\mu}(S)) = int^{\mu}(S)$. The proof is completed.

The converse of the above theorem is not true as can seen from the following example.

Example 4.16: Let X = {a, b, c} withμ = {X, φ, {a, b}, {b, c}}. Here {b} is supra h-set but not supra t-set.

Theorem 4.17: Let (X, μ) be the supra topological Space.
(a) Any supra A-set is supra C-set.
(b) Any supra open set is supra C-set

Proof: **(a)** S = X∩S where X is supra open and S is supra h-set. The proof is completed.
(b) S = X∩S where X is supra h-set and S is supra open set. The proof is completed.

The converse of the above Theorem is not true as can be seen from the following example.

Example 4.18: Let X = {a, b, c} withμ = {X, φ, {a, b},{b, c}}. Here {b, c} is supra C-set but not supra h-set. Also {a} is supra C-set but not supra open set.

Theorem 4.19: Every supra B-set is supra C-set.

Proof: S = X∩S where X is supra open and S is supra t-set. By Theorem 4.15, S is supra h-set. The proof is completed.

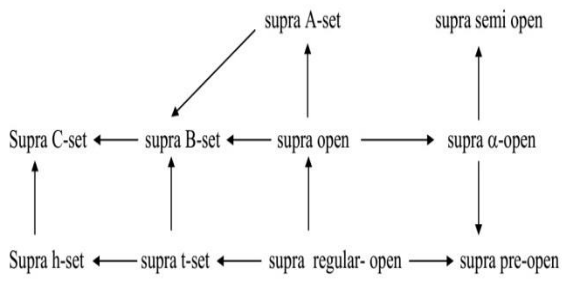
The converse of the above theorem is not true as can be seen from following Example.

Example 4.20: Consider Example 4.18, {b} is supra C-set but not supra B-set.

Remark 4.21: Supra A-set and supra semi open-sets are independent.

Consider Example 4.2. Here {d} is supra A-set but not supra semi-open set. Also {a, b, d} is supra semi open set but not supra A-set.

Remark 4.22: From the above discussions we have the following diagram of implications



None of the above implications is reversible.

V. DECOMPOSITION OF SUPRA M-CONTINUITY

Definition 5.1: Let (X,τ) and (Y, σ) be two topological spaces with τ⊆μ and σ⊆λ. A map f: (X, μ) → (Y, λ) is said to be

i. supra M-continuous (supra-irresolute [9]) if $f^{-1}(V)$ is supra open in X for every supra open V of Y

ii. supra α-continuous [1] if $f^{-1}(V)$ is supra α-open in X for every open V of Y.

We introduce a new class of mappings as follows.

Definition 5.2: Let (X, τ) and (Y, σ) be two topological spaces with τ⊆μ and σ⊆λ. A map f: (X, μ)→(Y, λ) is said to be

i. supra α*-continuous if $f^{-1}(V)$ is supra α-open in X for every supra open set V of Y

ii. supra A-continuous if $f^{-1}(V)$ is supra A set in X for every open set V of Y

iii. supra A*-continuous if $f^{-1}(V)$ is supra A set in X for every supra open set V of Y;

iv. supra B-continuous if $f^{-1}(V)$ is supra B-set in X for every open set V of Y;

v. supra B*-continuous if $f^{-1}(V)$ is supra B-set in X for every supra open set V of Y;

vi. supra C-continuous if $f^{-1}(V)$ is supra C-set in X for every open set V of Y;

vii. supra C*-continuous if $f^{-1}(V)$ is supra C-set in X for every supra open set V of Y.

Theorem 5.3: A set S of X is supra regular-open if and only if S is supra pre-open and supra t-set.

Proof: Let S be supra regular-open. By theorem 4.5, S is supra t-set. Also By Theorem 4.6, S is supra open. Thus S is supra pre-open.

Conversely, Let S be supra pre-open and supra t-set. Since $int^{\mu}(S) \subseteq S \subseteq int^{\mu}(cl^{\mu}(S)) = int^{\mu}(S)$, $S = int^{\mu}(cl^{\mu}(S))$. Hence, S is supra regular open.

Theorem 5.4: A subset S of X is supra open if and only if it is both supra α-open and supra A-set.

Proof: Let S be supra open. Then S is supra α-open and by Theorem 4.1, S is supra A-set. Conversely, Let S be supra α-open and supra A-set. Since S is supra A-set, S = X∩S where X is supra open and S ∈ SRC(X). Since S is supra α-open,

$X\cap S \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(X\cap S)))$
$= int^{\mu}(cl^{\mu}(int^{\mu}(X)\cap int^{\mu}(S)))$
$= int^{\mu}(cl^{\mu}(X\cap int^{\mu}(S)))$ (as X is supra open)
$\subseteq int^{\mu}(cl^{\mu}(X)\cap cl^{\mu}(int^{\mu}(S)))$
$= int^{\mu}(cl^{\mu}(X)\cap S)$ as S ∈ SRC(X)
$\subseteq int^{\mu}(cl^{\mu}(X)\cap int^{\mu}(S)$ ---------- (1)

Now since $X \subseteq int^{\mu}(cl^{\mu}(X)$, by(1)

S $= X\cap S = (X\cap S)\cap X$
$\subseteq (int^{\mu}(cl^{\mu}(X) \cap int^{\mu}(S)) \cap X$
$\subseteq X\cap int^{\mu}((S))\cap X$
$= X\cap int^{\mu}(S)$
$= int^{\mu}(S)$

Therefore $S\subseteq int^{\mu}(S)$ But $int^{\mu}(S)\subseteq S$. Hence S is supra-open.

Theorem 5.5: A subset S of X is supra open if and only if supra α-open and supra B-set.

Proof: Let S be a supra open set. Then S is supra α-open. Also, by Theorem 4.10, S is supra B-set.

Conversely let S be supra α-open and supra B-set. Since S is supra B-set, S = X∩S where X is supra open and S is supra t-set.Then $S = X\cap S \subseteq X \cap int^{\mu}(cl^{\mu}(S))$ (as S is supra pre-open) $= X\cap int^{\mu}(S)$ (as S is supra t-set). We have $S \subseteq X\cap int^{\mu}(S)$ implies $S \subseteq int^{\mu}(S)$. But always $int^{\mu}(S) \subseteq S$. Thus $S = int^{\mu}(S)$ and S is supra open.

Theorem 5.6: A subset S is supra open in X if and only if S is supra α-open set and supra C-set.

Proof: Let S be supra open in X. Then S is supra α-open set and by Theorem 4.17, S is supra C-set.

Conversely, let S be a supra α-open set and supra C-set. Since S is supra C-set, S = X∩S where X is supra open and S is supra h-set. Since S is supra α-open and S is supra h-set. Since S is supra α-open set, $S \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(S))) = int^{\mu}(cl^{\mu}(int^{\mu}(X\cap S))) \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(X))) \cap int^{\mu}(cl^{\mu}(int^{\mu}(S))) = int^{\mu}(cl^{\mu}(X)) \cap int^{\mu}(S)$ (as X is supra open and S is supra h-set). Now $S = X\cap S = X\cap(X\cap S) = X\cap S \subseteq X\cap(int^{\mu}(cl^{\mu}(X))\cap int^{\mu}(S)) \subseteq X\cap int^{\mu}(S)$ (as $X \subseteq int^{\mu}(cl^{\mu}(X))$. $S\subseteq int^{\mu}(S)$. but $int^{\mu}(S)\subseteq S$. Thus $S=int^{\mu}(S)$ and S is supra open.

Theorem 5.7: Let (X, τ) and (Y, σ) be two topological space with τ⊆μ, σ⊆λ. Let f: (X, μ)→(Y, σ) be a mapping. Then f is supra M-continuity if and only if

i. f is supra α*-continuous and supra A-continuous.

ii. f is supraα*-continuous and supra B-continuous.

iii. f is supra α*-continuous and supra C-continuous.

Proof: It is the decompositions of supra M-continuity from Theorem 5.4, 5.5, 5.6.

REFERENCES

[1] R. Devi, S. Sampathkumar and M. Caldas, On supra α-open sets and sα-continuous maps, General Mathematics, 16(2) (2008), 77-84.

[2] J. Dontchev and M. Przemski,On the various decomposition of continuous and some weakly continuous function, Acta Math. Hungar.71(1996)(1-2), 109-120.

[3] M. Gansterand I. L. Reilly, A Decomposition of continuity, Acta Math. Hungar., 56(3-4)(1990), 299-301.

[4] E. Hatir, T. Noiri and S. Yuksel, A Decomposition of continuity, Acta Math. Hungar., 94(1-2)(1996), 145-150.

[5] M. Kamaraj, G. Ramkumar, O. Ravi and M. L. Thivagar, Mildly supra normal spaces and some maps, International Journal of Advances in Pure and Applied Mathematics, 1(4)(2011), 68-85.

[6] A. S. Mashhour, A. A. Allam, F. S Mahmoud and F. H. Khedr, On supra topological spaces, Indian J. Pure and Appl. Math., 14(4) (1983), 502-510.

[7] O. Njastad, On some classes of nearly open sets, Pacific J. Math., 15(1965), 961-970.

[8] O. Ravi, M. L. Thivagar and ErdalEkici, On (1, 2)* -sets and Decompositions Bitopological (1, 2)*- continuous mappings, Kochi J. Math., 3(2008), 181-189.

[9] O. Ravi, G. Ramkumar and M. Kamaraj, On supra β-open sets and supra β-continuity on topological spaces, Proceed. National Seminar held at Sivakasi, India, (2011), 22-31.

[10] O. R. Sayed and T. Noiri, On supra b-open sets and supra b-continuity on topological spaces, European J.Pure and Applied Math.,(3) (2) (2010), 295-302.

[11] O. R. Sayed, Supra pre-open sets and supra pre-continuity on topological spaces, Vasile Alecsandri, University of Bacau, Faculty of Sciences, Scientific Studies and Research Series Mathematics and Informatics., 20(2)(2010), 79-88.

[12] J. Tong, A decomposition of continuity, Acta Math. Hungar., 48(1-2)(1986), 11-15.

[13] J. Tong, On Decomposition of continuity in topological spaces, Acta Math. Hungar., 54(1-2)(1989), 51-55.