

# DCT based Channel Estimation in OFDM using MMSE and LS

Sowmya Sreenivasan

Department of Electronics and Communication  
Christ University Faculty Of Engineering,  
Bangalore- 560075, Bangalore, India

Sujatha. S

Department of Electronics and Communication  
Christ University Faculty Of Engineering,  
Bangalore- 560075, Bangalore, India

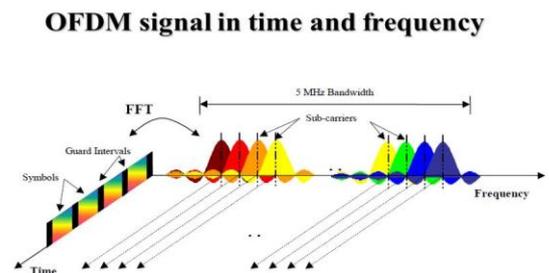
**Abstract**— Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier modulation technique and it has the ability to handle frequency-selective fading due to multi-path, without complex equalization filters. In this paper, a channel estimation scheme based on channel impulse response is presented. Channel estimation method based on the Discrete Cosine Transform (DCT) using phase shifted pilot sequences for MIMO-OFDM systems. With reasonable design and phase shifted pilot sequences for each transmitter antenna, the DCT-based channel estimation method can achieve highly efficient and accurate estimation without sacrificing much system overhead. When compared to DFT the channel is assessed using Minimum Mean-Square Error (MMSE) and Least-Squares (LS) channel estimators.

**Keywords**—Channel estimation, MIMO-OFDM, discrete cosine transform (DCT), phase shifted pilot sequences, discrete Fourier transform (DFT)

## I. INTRODUCTION

It is one of the main challenges which are faced in wireless OFDM systems. There are several approaches for channel estimation in OFDM system subcarriers. Channel estimation techniques for OFDM based systems can be divided into two main types: blind and non-blind. In the blind channel estimation approach it exploit the statistical behavior of the received signals and require a large amount of data. Then in the non-blind channel estimation methods the information of previous channel estimates or some portion of the transmitted signal are available to the receiver to be used for the channel estimation. But the issues of DCT based channel Estimation in OFDM is that the response of the channels in the network rapidly changes due to the mobility of receivers, transmitters or due to the scattering objects. [1].Yen-Hui Yeh et al., [2] have proposed two discrete cosine transform (DCT)-based pilot-symbol-aided channel estimators, which can mitigate the aliasing error and high-frequency distortion of the direct discrete Fourier transform (DFT)-based channel estimators when the multipath fading channels have non-sample-spaced path delays. These estimators are based on the property of channel frequency response and the concept of interpolation in transform domain. Both proposed estimators outperform the conventional DFT based channel estimators. The advantage of these estimators is that they have the advantage of easy and convenient realization by existing efficient DCT/IDCT hardware and software. Feifei Gao et al., [3] have proposed two maximum-likelihood joint frequency offset and phase offset estimators considering non-circular transmission of DCT-OFDM. The phase offset can be estimated only if the transmitted symbols are non-circular. The advantages of these

offset estimators are the frequency offset estimation range increases from only one subcarrier spacing to its maximum. Second, the frequency offset and phase offset estimation is not only more accurate, but also more robust to the amount of the redundancy per block. Bin Jiang et al., [4] have proposed a 2-D DCT-based channel estimator for OFDM systems with virtual subcarriers in mobile wireless channels. This approach can well approximate the optimal MMSE channel estimation with the low-complexity implementation. The advantage of this approach is that it can deal with the spectral leakage, which results in an irreducible error floor and also the DFT-based filtering or interpolation in time-domain which has performance degradation for system with high Doppler frequency. OFDM is a promising candidate for high data rate wireless communications for its many advantages, notably, its high spectral efficiency, robustness to frequency selective fading, as well as the feasibility of low-cost transceiver implementations. The multi-input multi-output (MIMO) system, has the potential to obtain a diversity gain to mitigate the fading effect and to improve the system capacity, in order to meet the high data rate requirements from wireless applications. Hence, the combination of DCT with OFDM technology provides a promising candidate for next generation fixed and mobile wireless systems [5]. For wideband wireless communication, it is necessary to dynamically estimate the channel before demodulating the MIMO-OFDM signals, since radio channel is frequency selective and time-dependent. For OFDM channel estimation based on pilot, it can be classified as preamble method and PSAM method (Pilot Symbol Assisted Modulation or comb-type pilot method) in accordance with the difference of insertion position of pilots [6]. Since the conventional preamble assisted channel.



Please note that signals on different subcarriers (frequency axis) carry modulation symbols (time axis). Modulation symbols (QPSK or QAM) are separated by short breaks (guard intervals)

Figure 1: OFDM

## II. CHANNEL ESTIMATION

To examine the state of channel, we consider the following assumptions. Let  $s(x)$  be the transmitted input symbols and  $s'(x)$  be the received output symbols. Assume,  $g(x)$  as the channel impulse response and  $N(x)$  represents the white complex Gaussian channel noise. From a multi-amplitude signal constellation, the transmitted symbols ( $s(x)$ ) are considered. Supposing that the D/A and A/D converters comprise ideal low-pass filters along with bandwidth  $1/T_s$ . Here,  $T_s$  symbolizes the sampling interval and  $T_C$  denotes the time length of a cyclic extension, which is used to eradicate inter-block interference and to maintain the orthogonality of tones.

The channel impulse response  $g(x)$  can also be described as the function of a time limited pulse train. It takes the form as follows, [3]

$$g(x) = \sum_k \beta_k \eta(x - \gamma_k T_s) \quad (1)$$

The amplitudes ( $\beta_k$ ) are complex valued and take value between  $0 \leq \gamma_k T_s \leq T_C$ .

Using  $n$ -point discrete-time Fourier transform (DFT <sub>$n$</sub> ), the system is modeled as follows,

$$S' = DFT_n \left( IDFT_n(S) \otimes \frac{g}{\sqrt{n}} + N \right) \quad (2)$$

Where,  $S = [S_0, S_1, \dots, S_{n-1}]^T$ ,  
 $S' = [S'_0, S'_1, \dots, S'_{n-1}]^T$  and  
 $N = [N_0, N_1, \dots, N_{n-1}]^T$  are the vectors of i.i.d complex Gaussian variables. Another vector  $g = [g_0, g_1, \dots, g_{n-1}]^T$  is well thought out by the cyclic equivalent of sinc-functions.  $g / \sqrt{n}$  symbolizes the response of observed channel impulse after sampling the frequency response of  $g(x)$  and it is given as,

$$g(x) = \frac{1}{\sqrt{n}} \sum_k \beta_k e^{-j\frac{\pi}{n}(x+(n-1)\gamma_m)} \frac{\sin(\pi\gamma_k)}{\sin\left(\frac{\pi}{n}(\gamma_k - x)\right)} \quad (3)$$

A set of  $n$  independent Gaussian channels of the system can be described as,

$$s'(x) = h(x)s(x) + N(x) \quad (4)$$

where,  $x = 0, 1, \dots, n-1$

In the above equation,  $h(x)$  represents the complex channel attenuation produced by the vector

$$h = [h_0, h_1, \dots, h_{n-1}]^T = DFT_n(g) \quad (5)$$

The vector  $N$  is equivalent to,

$$N = [N_0, N_1, \dots, N_{n-1}]^T = DFT_n(N) \quad (6)$$

Where  $N$  is an i.i.d complex zero-mean Gaussian noise vector, The above descriptions can be written in the form of matrix as follows,

$$s' = XQg + N \quad (7)$$

Here,  $X$  is the matrix that contains the elements of  $x$  on its diagonal.

## III. PROPOSED PROBLEM

In this paper, the channel is assessed using Minimum Mean-Square Error (MMSE) and Least-Squares (LS) channel estimators. Consider the channel vector  $g$  is Gaussian and uncorrelated with the channel noise  $N$ , then the MMSE estimate of channel  $g$  can be given as follows, [8]

$$\hat{g}_{MMSE} = R_{gs} R_{ss}^{-1} S' \quad (8)$$

The terms in equation (8) can be derived as,

$$R_{gs} = E\{g s'^{(H)}\} = R_{gg} Q^H X^H \quad (9)$$

$$R_{ss} = E\{s' s'^{(H)}\} = XQR_{gg}Q^H X^H + \sigma_N^2 I_n \quad (10)$$

Where,  $R_{gs}$  and  $R_{ss}$  are the cross covariance matrix between  $g$  and  $s'$  and the auto-covariance matrix of  $s'$  respectively. And then,  $R_{gg}$  denotes the noise variance (i.e)  $E\{N(x)|^2\}$ . We presume that both auto covariance matrix ( $R_{gg}$ ) and noise variance are known values.

The estimation of MMSE in frequency domain is computed as follows,

$$\hat{h}_{MMSE} = Q \hat{g}_{MMSE} = Q O_{MMSE} Q^H X^H s' \quad (11)$$

Where, the value of  $O_{MMSE}$  can be given as,

$$O_{MMSE} = R_{gg} \left[ (Q^H X^H X Q)^{-1} \sigma_N^2 + R_{gg} \right]^{-1} (Q^H X^H X Q)^{-1} \quad (12)$$

For the cyclic impulse  $g$ , the LS estimator minimizes  $(s' - X Q g)^H (s' - X Q g)$  and produces,

$$\hat{h}_{LS} = Q O_{LS} Q^H X^H s' \quad (13)$$

The value of  $O_{LS}$  can be derived as,

$$O_{LS} = (Q^H X^H X Q)^{-1} \quad (14)$$

The equation for  $\hat{h}_{LS}$  is further reduced as,

$$\hat{h}_{LS} = X^{-1} s' \quad (15)$$

The estimation value obtained from LS estimator is also equivalent to zero-forcing estimator.

#### IV. SIMULATION RESULTS

In the proposed part, we first implement the channel estimation of the OFDM by using the LS and MMSE using DCT [3]

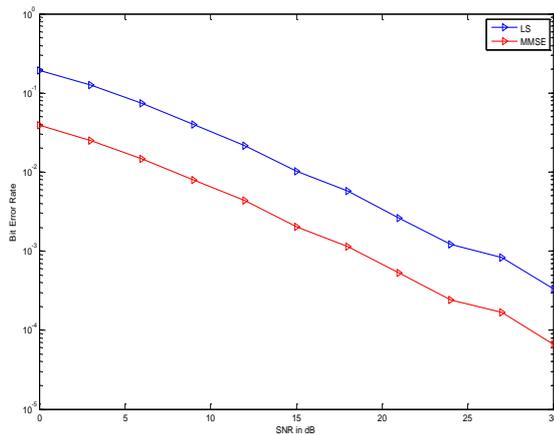


Figure 2. BER Vs SNR for Channel estimation of OFDM using LS and MMSE.

#### V. CONCLUSION

In this paper, a channel estimation scheme is based on pilot aided block type training symbols using LS and MMSE algorithm. The Channel estimation is one of the fundamental issues of OFDM system design. The MMSE is compared with LS and the MMSE performs better than the LS. The channel is assessed using Minimum Mean-Square Error (MMSE) and Least-Square (LS) channel estimators.

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