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Damped Vibrations of an Isotropic Circular Plate of Parabolically Varying Thickness Resting on Elastic Foundation

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***Abstract:-* Damped vibrations of a circular plate of parabolically varying thickness resting on elastic foundation have been studied on the basis of classical plate theory. The fourth order differential equation of motion is solved by the method of frobenius. Using high speed digital computer, frequencies, deflection functions and moments corresponding to the first two modes of vibrations are computed for circular plate with clamped and simply supported edge conditions for various values of taper constants, damping parameter and elastic foundation. These results have been presented both in tabular and graphical forms.**

INTRODUCTION

In the research work the focus has been laid down the effect of taper constant, damping and elastic foundation on frequencies of an Isotropic circular plate of linearly varying thickness has been studied. The object of the work presented here is to study the damped vibration of a circular plate of parabolically varying thickness resting on elastic foundation.

Here the fourth order differential equation of motion is solved by the method of Frobenius. The transverse displacement is expressed as an infinite series in terms of radial coordinates. The frequencies, deflection functions and moment parameters corresponding to the first two modes of vibrations are computed for the circular plate with clamped and simply supported edge conditions for various values

of taper constant, damping parameter, and elastic foundation.

Equation of Transverse Motion

$$D\frac{\partial^4 W}{\partial r^4}+2\left[\frac{\partial D}{\partial r}+\frac{D}{r}\right]\frac{\partial^3 W}{\partial r^3}+\left[\frac{(2+\nu)}{r}\frac{\partial D}{\partial r}+\frac{\partial^2 D}{\partial r^2}-\frac{D}{r^2}\right]\frac{\partial^2 W}{\partial r^2}$$
$$+\left[\frac{\nu}{r}.\frac{\partial^2 D}{\partial r^2}-\frac{1}{r^2}\frac{\partial D}{\partial r}+\frac{D}{r^3}\right]\frac{\partial W}{\partial r}+\rho h\frac{\partial^2 W}{\partial t^2}+K_d\frac{\partial W}{\partial t}+K_f W=0 \qquad (1)$$

Also $D=\frac{E(r)h^3(r)}{12(1-\nu^2)}$, hence introducing the non dimensional variables, $H=\frac{h}{a}, R=\frac{r}{a}, w=\frac{W}{a}$

where 'a' is the radius of the Plate and $\bar{E}=\frac{E}{a}$ and $\bar{\rho}=\frac{\rho}{a}$. Here the thickness of plate H is assumed to vary in the form $H = H_o(1-\alpha R^2)$, where $H_0 = H|_{R=0}$, α = taper constant.

In the light of these assumptions equation (1) takes the form,

$$\left(1-\alpha R^2\right)^3\frac{\partial^4 w}{\partial R^4}+\left[-12\alpha\left(1-\alpha R^2\right)R+2\left(1-\alpha R^2\right)^3 R^{-1}\right]\frac{\partial^3 w}{\partial R^3}$$
$$+\left[-6\alpha\left(1-\alpha R^2\right)^2+24\alpha^2\left(1-\alpha R^2\right)R^2-\left(1-\alpha R^2\right)^3 R^{-2}-12\alpha\left(1-\alpha R^2\right)^2-6\nu\alpha\left(1-\alpha R^2\right)^2\right]\frac{\partial^2 w}{\partial R^2}+\left[-6\nu\alpha\left(1-\alpha R^2\right)^2 R^{-1}\right.$$
$$\left.+24\nu\alpha^2\left(1-\alpha R^2\right)R+6\alpha\left(1-\alpha R^2\right)^2 R^{-1}+\left(1-\alpha R^2\right)^3 R^{-3}\right]$$
$$\frac{\partial w}{\partial R}+\left[\frac{12\left(1-\alpha R^2\right)\left(1-\nu^2\right)a^2\bar{\rho}}{\bar{E}H_0^{\,2}}\frac{\partial^2 w}{\partial t^2}+\frac{12\left(1-\nu^2\right)}{\bar{E}H_0^{\,3}}K_f w\right.$$
$$\left.+\frac{12\left(1-\nu^2\right)}{\bar{E}H_0^3}K\frac{\partial w}{\partial t}\right]=0 \qquad (2)$$

Solution

For damped harmonic vibrations, the solution is given by

$$w(R,t)=\bar{W}(R)e^{-\gamma t}\cos pt \qquad (3)$$

Substituting (3) in (2) and solving we get,

$$\left(1-\alpha R^2\right)^4 \frac{\partial^4 \overline{W}}{\partial R^4}+\left[-12\alpha\left(1-\alpha R^2\right)^3 R+\alpha\left(1-\alpha R^2\right)^4 R^{-1}\right]\frac{\partial^3 \overline{W}}{\partial R^3}$$
$$+\left[-6\alpha\left(1-\alpha R^2\right)^3+24\alpha^2\left(1-\alpha R^2\right)^2 R^2-\left(1-\alpha R^2\right)^4 R^{-2}-\right.$$
$$\left.-12\alpha\left(1-\alpha R^2\right)^3-6\nu\alpha\left(1-\alpha R^2\right)^3\right]\frac{\partial^2 \overline{W}}{\partial R^2}+\left[-6\nu\alpha\left(1-\alpha R^2\right)^3 R^{-1}\right.$$
$$\left.+24\nu\alpha^2\left(1-\alpha R^2\right)^2 R+6\alpha\left(1-\alpha R^2\right)^3 R^{-1}+\left(1-\alpha R^2\right)^4 R^{-3}\right]$$
$$\frac{\partial \overline{W}}{\partial R}+\left[E_F\, C^*\left(1-\alpha R^2\right)-D_k^2\left(I^*\right)^2-\Omega^2 I^*\left(1-\alpha R^2\right)^2\right]\overline{W}=0 \quad (4)$$

where , $E_F=\dfrac{12K_F\left(1-v^2\right)}{\overline{E}}$ $,C^*=\dfrac{1}{H_0^3}$

$$D_k=\frac{3\left(1-v^2\right)K^2}{\overline{\rho}\,\overline{E}} \qquad ,I^*=\frac{1}{H_0^2}$$

$$\Omega^2=\frac{12\left(1-v^2\right)a^2\,\overline{\rho}\ p^2}{\overline{E}},$$

where p= circular frequency,

Ω = Frequency parameter ,

D_k = damping parameter ,

E_F = Elastic foundation parameter

A series solution for $\overline{w}$ is assumed in the form,

$$\overline{w}(R)=\sum_{\lambda=0}^{\infty} a_\lambda\, R^{C+\lambda}\ ,\ a_0\neq 0, \qquad (5)$$

where c is exponent of singularity

Substituting (5) in equation (4) one obtains

$$\sum_{\lambda=0}^{\infty} a_\lambda F_1(\lambda)R^{(C+\lambda-4)}+\sum_{\lambda=0}^{\infty} a_\lambda F_2(\lambda)R^{(C+\lambda-2)}+\sum_{\lambda=0}^{\infty} a_\lambda F_3(\lambda)R^{(C+\lambda)}+$$
$$\sum_{\lambda=0}^{\infty} a_\lambda F_4(\lambda)R^{(C+\lambda+2)}+\sum_{\lambda=0}^{\infty} a_\lambda F_5(\lambda)R^{(C+\lambda+4)}=0 \qquad (6)$$

For the series expression (5) to be the solution the coefficient of different powers of R in the equation (6) must be identically equal to zero. Thus equating to zero the coefficient of lowest power of R, one gets the identical equation , $a_0\, F_1(0)=0$ Since $a_0\neq 0$ i.e. F_1 (0) = 0

$$F_1(0)=N_1(1)\,b_0(3)+N_1(2)\,b_0(2)+N_1(3)\,b_0(1)+N_1(4)\,b_0(0)=0$$
$$\left[1c(c-1)(c-2)(c-3)+2c(c-1)(c-2)-c(c-1)+c\right]=0$$

c = 0, 0, 2, 2

the following indicial roots are obtained c= 0, 0, 2, 2 further, equating to zero the coefficient of the next subsequent power of R, one finds that a_1=0 and a_2 is indeterminate for c=0 hence a_2 can be written as an arbitrary constants along with a_0. Similarly equating to zero the coefficients of next higher power of R the constant a_3 is obtained in terms of a_0, and a_2 and a_λ (λ = 4, 5, 6, --) can be written in terms of a_0 and a_2.

Hence assuming ,

$$a_\lambda=A_\lambda a_0+\ B_\lambda\, a_2=(\lambda=0,1,2,3,--) \qquad (7)$$

The following solution, corresponding to c=0 is obtained,

$$\overline{W}=a_0\left[1+\sum_{\lambda=4}^{\infty}A_\lambda\, R^\lambda\right]+a_2\left[R^2+\sum_{\lambda=4}^{\infty}B_\lambda R^\lambda\right] \qquad (8)$$

It is evident that no new solution will arise corresponding to other values of c, i.e. for c=2, it is already contained in the solution (8) with arbitrary constants a_0 and a_2.

Convergence of the Solution

Lamb's technique is applied to test the convergence of the solution (8). Rewriting recurrence relation

$$\frac{a_{\lambda+8}}{a_\lambda}+\frac{a_{\lambda+6}}{a_\lambda}\frac{F_2(\lambda+6)}{F_1(\lambda+8)}+\frac{a_{\lambda+4}}{a_\lambda}\cdot\frac{F_3(\lambda+4)}{F_1(\lambda+8)}+\frac{a_{\lambda+2}}{a_\lambda}\cdot\frac{F_4(\lambda+2)}{F_1(\lambda+8)}$$
$$+\frac{F_5(\lambda)}{F_1(\lambda+8)}=0$$

$$\mu_8+\mu_6\frac{F_2(\lambda+6)}{F_1(\lambda+8)}+\mu_4\cdot\frac{F_3(\lambda+4)}{F_1(\lambda+8)}+\mu_2\cdot\frac{F_4(\lambda+2)}{F_1(\lambda+8)}$$
$$+\frac{F_5(\lambda)}{F_1(\lambda+8)}=0$$

where $\mu=\text{limit}_{\lambda\to\infty}\dfrac{a_{\lambda+1}}{a_\lambda}$ where $\lambda\to\infty$

$\mu\to\infty$,

Hence the infinite series is uniformly convergent when $|\mu|<1$. Hence the solution is convergent.

Boundary Conditions and Frequency Equations

The frequency equations for clamped and simply supported circular plates have been obtained by employing the appropriate boundary conditions.

Clamped Plate: For a circular plate clamped at edges r=a, the deflection w and slope of the plate element at edges should be zero.

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$\overline{W}$ (r, t)$|_{r=a}$ =

$$\left.\frac{\partial \overline{W}(r,t)}{\partial r}\right|_{r=a} = 0 \qquad \text{or} \qquad \overline{W}\,|_{R=1} =$$

$$\left.\frac{\partial \overline{W}}{\partial R}\right|_{R=1} = 0$$

Using above equation and applying the boundary conditions one obtains the frequency equation for (clamped plate) as

$$\begin{vmatrix} V_1(\Omega) & V_2(\Omega) \\ V_3(\Omega) & V_4(\Omega) \end{vmatrix} = 0 \qquad (9)$$

where $V_1(\Omega) = 1 + \sum_{\lambda=4}^{\infty} A_\lambda$ $\qquad V_2(\Omega) = 1 + \sum_{\lambda=4}^{\infty} B_\lambda$

$$V_3(\Omega) = \sum_{\lambda=4}^{\infty} \lambda\, A_\lambda \qquad V_4(\Omega) = 2 + \sum_{\lambda=4}^{\infty} \lambda\, B_\lambda$$

Simply Supported Plate: For a circular plate simply supported at the edge r=a, the deflection W and the moments M_r at the edge should be zero.

$$W(r,t)\big|_{r=a} = M_r(r,t)_{r=a} = 0 \qquad \text{or,}$$

$$\overline{W}_{R=1} = \left[\frac{\partial^2 \overline{W}}{\partial R^2} + \frac{\nu}{R}\frac{\partial \overline{W}}{\partial R}\right]_{R=1} = 0$$

Applying these boundary conditions on the equation , one gets the frequency equation for simply supported plate as,

$$\begin{vmatrix} V_1(\Omega) & V_2(\Omega) \\ V_5(\Omega) & V_6(\Omega) \end{vmatrix} = 0 \qquad (10)$$

where $\qquad V_5(\Omega) = \sum_{\lambda=4}^{\infty} \lambda(\lambda+\nu-1)A_\lambda$

$$V_6(\Omega) = 2(1+\nu) + \sum_{\lambda=4}^{\infty} \lambda(\lambda+\nu-1)B_\lambda$$

Deflection Functions and Moments

Again enforcing the boundary condition W=0 at X=1 and adopting the same value of a_0 and a_2 the non dimensional parameter is obtained in the form

$$\overline{M} = \frac{M_x}{D_0} = -\left(1-\alpha R^2\right)^3\left[\sum_{\lambda=3}^{\infty} \lambda(\lambda+\nu-1)A_\lambda R^{\lambda-2}\right] +$$

$$\left(1-\alpha R^2\right)^3\left[\frac{1+\sum_{\lambda=3}^{\infty} A_\lambda}{1+\sum_{\lambda=3}^{\infty} B_\lambda}\left\{2(1+\nu) + \sum_{\lambda=3}^{\infty} \lambda(\lambda+\nu-1)B_\lambda R^{\lambda-2}\right\}\right] \qquad (11)$$

Where $\quad D_0 = \dfrac{\overline{E}H_0^3}{12(1-\nu^2)}$

The values of Ω for both edge conditions have been taken from equation (9) and (10).

Result and Discussion : Numerical results for an isotropic circular plate of parabolically varying thickness resting on elastic foundation have been computed by using computer technology. In all the cases considered the Poisson's ratio has been assumed to remain constant and it has been taken to be 0.3. Terms of series up to an accuracy of 10^{-8} in their absolute values have been retained. Frequency parameter corresponding to first two modes of vibration of a clamped and simply supported isotropic circular plate has been computed for different values of taper constant, damping parameter and foundation effect have been computed. All the results are tabulated in tables and graphically shown in figures (1.1) to (1.8). The results up to accuracy of 10^{-4} have been given in the tables.

Verification of work is obtained by allowing damping parameter and elastic foundation parameter to be zero, the problem reduce to well known problem of a homogenous circular plate of parabolically varying thickness. The results so good agreements with the already published work of Gupta .

Figure (1.1) and (1.2) shows the effect of variation of a taper constant on the frequency parameter for a circular plate of parabolically varying thickness resting on elastic foundation, (i.e., for D_K = .01, E_F = .01, h =.1 and D_K = .02, E_F = .02, h =.1) with simply supported (S-S) and clamped edge edge conditions. From figure it is observed that the first mode remain near about constant and the second mode will be decreases in frequency parameter with the increasing of taper constant on the both mode of vibration for simply supported and clamped edge plates.

Figure (1.3) and (1.4) shows the effect of variation of damping constant on the frequency parameter for a circular plate of parabolically varying thickness resting on elastic foundation (i.e., for α = .01, E_F = .01, h =.1 and α = .02, E_F = .02, h =.1) with simply supported and clamped edge conditions. From figure it is observed that there is a decreasing in the frequency parameter with the increasing of damping parameter on the both mode of vibration but this decreasing is some greater for the first mode than the second mode for the simply supported and clamped edge plates.

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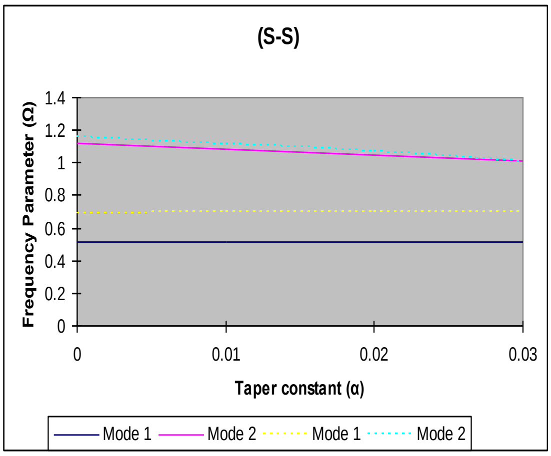
Figure (1.5) and (1.6) shows the effect of variation of foundation parameter for a circular plate of parabolically varying thickness resting on elastic foundation (i.e., for $\alpha = .01$ $D_K = .01$, h =.1 and $\alpha = .02$, $D_K = .02$, h =.1) with simply supported and clamped edge conditions. From figure it is observed that there is a increasing in the frequency parameter with the increasing of foundation effect but this increasing is some greater for the first mode than the second mode on the both mode of vibration for the simply supported and clamped edge plates.

Figure (1.7) and (1.8) shows the variation of deflection and moment parameter with respect to the different points on the plate surface from axis of symmetry.

FIGURE 1.1 (H=0.1, ν =0.3)

Graph _______= ($Dk=E_F=0.01$)

Graph --------- = ($Dk=E_F=0.02$)

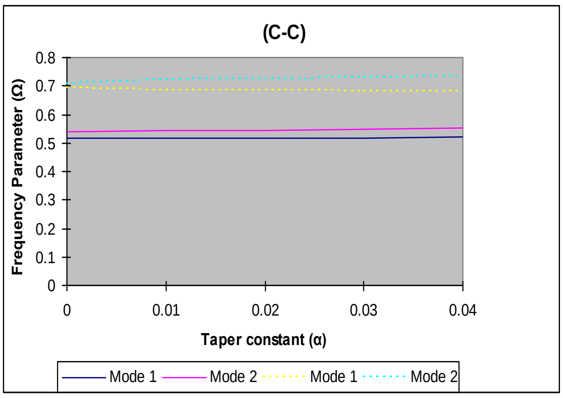


Variation of Ω for the vibration of a damped simply supported circular plate of parabolically varying thickness for different values of taper constant.

FIGURE 1.2 (H=0.1, ν =0.3)

Graph _______= ($Dk=E_F=0.01$)

Graph --------- = ($Dk=E_F=0.02$)

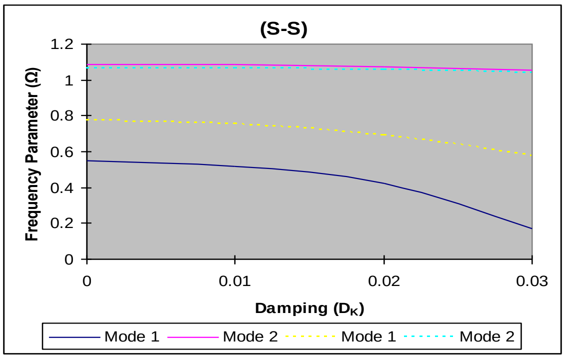


variation of Ω for the vibration of a damped clamped circular plate of parabolically varying thickness for different values of taper constant.

FIGURE 1.3 (H=0.1, ν =0.3)

Graph _______= ($Dk=E_F=0.01$)

Graph --------- = ($Dk=E_F=0.02$)

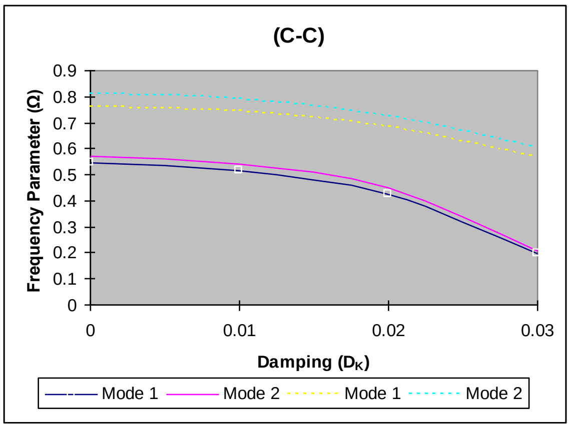


Variation of Ω for the vibration of a damped simply supported circular plate of parabolically varying thickness for different values of damping parameter.

FIGURE 1.4 (H=0.1, ν =0.3)

Graph _______= ($\alpha=E_F=0.01$)

Graph --------- = ($\alpha=E_F=0.02$)

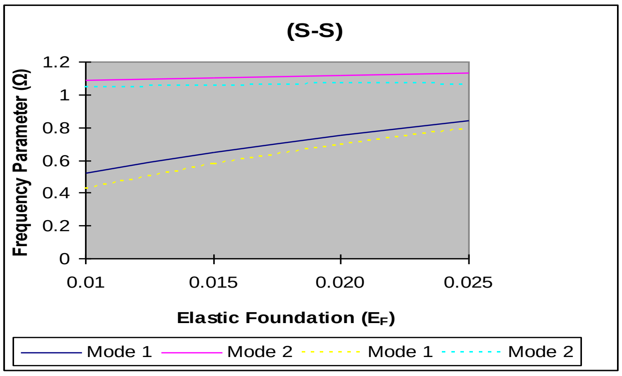


Variation of Ω for the vibration of a damped clamped circular plate of parabolically varying thickness for different values of damping parameter.

FIGURE 1.5 (H=0.1, ν =0.3)

Graph _______= ($\alpha=D_K=0.01$)

Graph --------- = ($\alpha=D_K=0.02$)

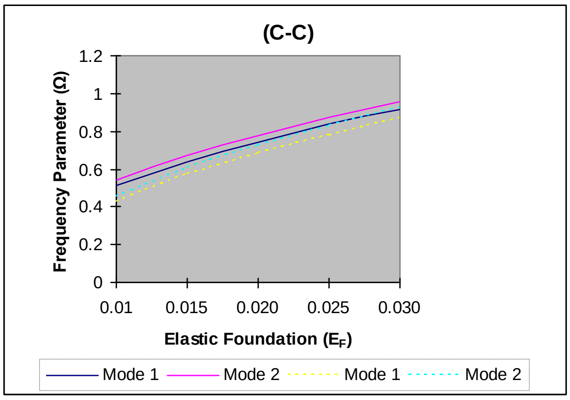


Variation of Ω for the vibration of a damped simply supported circular plate of parabolically varying thickness for different values of foundation parameter.

FIGURE 1.6 (H=0.1, ν =0.3)

Graph ________= (α =D $_K$ =0.01)

Graph --------- = (α =D $_K$ =0.02)



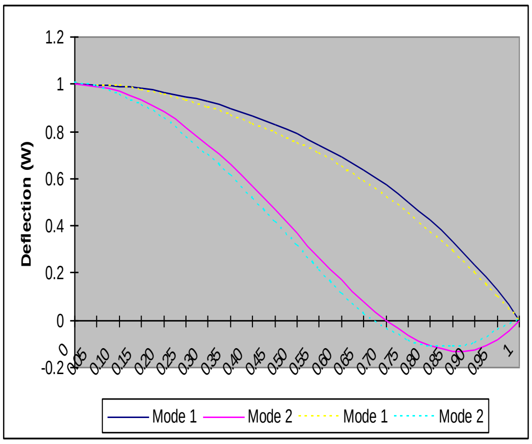
Variation of Ω for the vibration of a damped clamped circular plate of parabolically varying thickness for different values of foundation parameter.

FIGURE = 1.7

(H=0.1, ν =0.3) $\alpha = 0.0$, D_k =0.01, E_f =0.01

Graph_______=S-S

Graph--------=C-C



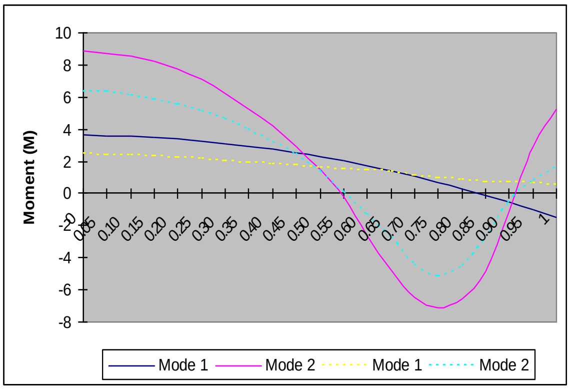
Transverse Deflection (W) for a circular plate of parabolically varying thickness.

FIGURE = 1.8

(H=0.1, ν =0.3) $\alpha = 0.0$, D_k =0.01, E_f =0.01

Graph______=S-S

Graph----------=C-C



Moment parameter (M) for a circular plate of parabolically varying thickness

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