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Damage Detection On Composite Beam: FEA&Experiment

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Abstract **-It is required that structures must safely work during its service life. But, damages initiate a breakdown period on the structures. Cracks are among the most encountered damage types in the structures. Structures are weakened by cracks. When the crack size increases in course of time, the structure becomes weaker than its previous condition. Finally, the structure may breakdown due to a minute crack. Therefore, crack detection plays an important role for structural health monitoring applications.**

Delamination is the separation of layers which are bonded together in composite laminate. In the case of bending loads delamination usually leads to significant loss of bending stiffness and strength. Therefore it is important to detect the presents of delamination at an early stage. The delamination causes reduction of stiffness and thereby the modal frequencies also change.

In order to find the damage location and size present in the composite materials, we have to determine changes in modal frequencies of the damaged structure as a function of the damage parameters such as delamination and crack location, size, interface at which it is present. In this work modal analysis is used to determine the crack and delamination in the composite structure. This is done by creating a four layer multi directional laminate of E-Glass /Epoxy in the lamination code of 90/0/0/90 in ANSYS. A parametric study is conducted by varying location, width and length of the crack and delamination.The change in frequency of undamaged and damaged shows that the natural frequency is affected by delamination and crack.

The results are validated with experimental modal analysis.From the results, a reverse design process is implemented for finding the damage location at a particular natural frequency.

I. INTRODUCTION

A composite is a structural material that consists of two or more combined constituents that are combined at a macroscopic level and are not soluble in each other One constituent is called the reinforcing phase and the one in which it is embedded is called the matrix. The reinforcing phase material may be in the form of fibers, particles, or flakes. The matrix phase materials are generally continuous. Examples of composite systems include concrete reinforced with steel and epoxy reinforced with graphite fibers, carbon fiber, glass fibers etc. Composites as structural material are being used in aerospace, military and civilian applications because of their tailor made properties. Preventing failure of composite material systems has been an important issue in engineering design generally, aircraft structures made of fiber reinforces composite materials are designed such that the fibers carry the bulk of the applied load. Interlaminar failure such as delamination refers to debonding of adjacent lamina. The possibility that interlaminarfailure occur in structural components is considered as design limit, and establishes restrictions on the usage of full potential of composites. Similar to isotropic materials, composite materials are subjected to various types of damage, mostly cracks and delamination. The delamination in a composite structure may reduce the structural stiffness and strength, redistribute the load in a way that the structural failure is delayed, or may lead to structural collapse. The presence of a crack could not only cause a local variation in the stiffness but it could affect the mechanical behaviour of the entire structure to a considerable extent. Cracks may be caused by fatigue under service conditions as a result of the limited fatigue strength. Small cracks are known to propagate due to fluctuating stress conditions. If these propagating cracks remain undetected and reach their critical size, then a sudden structural failure may occur. Hence it is possible to use natural frequency measurements to detect cracks. Specifically, crack damage can cause a stiffness reduction, with an inherent reduction in natural frequencies, an increase in modal damping, and a change in the mode shapes. From these changes the crack position and magnitude can be identified.

II. THEORY OF FREE VIBRATION OF CANTILEVER BEAMS

For a cantilever beam subjected to free vibration, and the system is considered as continuous system in which the beam mass is considered as distributed along with the stiffness of the shaft, the equation of motion can be written as:-

$$\frac{d^2}{dx^2}\{EI(x)\frac{d^2Y(x)}{dx^2}\}=\omega_n{}^2 m(x)Y(x) \qquad (1)$$

Where, E is themodulus of rigidity of beam material,

I is the moment of inertia of the beam cross-section,

Y(x) is displacement in y direction at distance x from

Fixed end,ω_n is the circular natural frequency, m is the mass per unit length,

m = $\rho A(x)$ ρ is the material density xis the distance measured from the fixed end.

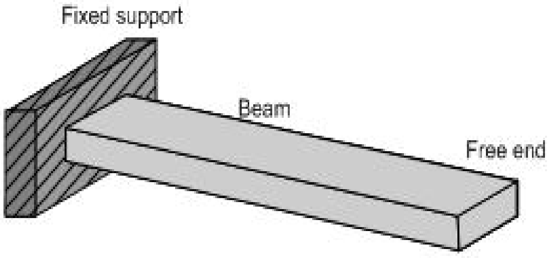


Figure 1 : Cantilever beam

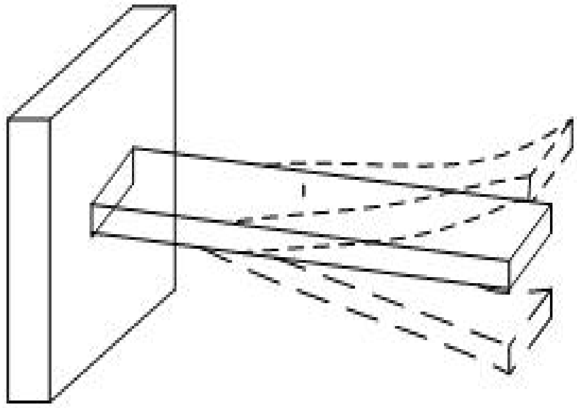


Figure 2 : Free vibration of a cantilever beam

Fig.2 shows of a cantilever beam with rectangular cross section, which can be subjected to bending vibration by giving a small initial displacement at the free end; and Optimum material utilization for stiffness is achieved by forcing the bending load carrying components to the surface of the structure with an extremely light weight core material.

1) ClassicalLamination Theory(CLT)

Classical lamination theory (CLT) as presented herein is applicable to orthotropic continuous fiber laminated composites only. The approach used in formulating CLT is similar to that used in developing load-stress relationships in elementary strength of materials courses. An initial displacement field consistent with applied loads is assumed. Through the strain-displacement fields and an appropriate constitutive relationship, a state of stress is defined. By satisfying the conditions of static equilibrium, a load-strain relation is defined, and subsequently a state of stress is defined for each lamina.

Basic Assumptions for Classical Lamination Theory (CLT)

1. Each layer of the laminate is quasi-homogeneous and orthotropic.
2. The laminate is thin compared to the lateral dimensions and is loaded in its plane.
3. State of stress is plane stress.
4. All displacements are small compared to the laminate thickness.
5. Displacements are continuous throughout the laminate.
6. Straight lines normal to the middle surface remain straight and normal to that surface after deformation. In-plane displacements vary linearly through the thickness.
7. Transverse shear strains are negligible.
8. Transverse normal strain is negligible compared to the in-plane strains x and y.
9. Strain-displacement and stress-strain relations are linear.

2) Classical Lamination Theory from ClassicalPlate Theory:

The classical lamination theory is almost identical to the **classical plate theory**; the only difference is in the material properties (stress-strain relations). The classical plate theory usually assumes that the material is isotropic, while a fiber reinforced composite laminate with multiple layers (plies) may have more complicated stress-strain relations. The four cornerstones of the lamination theory are the kinematic, constitutive, force resultant, and equilibrium equations. The outcome of each of these segments is summarized as follows: Theoretical evaluations of the effective properties of the facings was done using composite laminate theory (CLT). CLT consists of a collection of mechanics of materials type of stress and deformations hypotheses. By use of this theory, one can consistently proceed directly from the basic building block, the lamina, to the end result, a structural laminate. The assumptions in composite laminate theory are as follows:

- Each lamina is orthotropic
- Each lamina is homogeneous
- The laminate is thin and is loaded only in its plane
- Each lamina is elastic
- No slip occurs between the lamina interfaces
- Displacements are continuous and small throughout the laminate

3) CRACK THEORY:

Physical Parameters Affecting DynamicCharacteristics of Cracked Structures:

Usually the physical dimensions, boundary conditions, the material properties of the structure play important role for the determination of its dynamic response. Their vibration cause changes in dynamic characteristics of structures. The following aspects of the crack greatly influence the dynamic response of the structure.
(i)The position of crack
(ii)The depth of crack
(iii)The orientation of crack
(iv)The number of cracks

4) Classification of Cracks:

Based on their geometries, cracks can be broadly classified as follows:
1) Cracks perpendicular to the beam axis are known as “transverse cracks”. These are the most common and most serious as they reduce the cross-section and thereby weaken the beam. They introduce a local flexibility in the stiffness of the beam due to strain energy concentration in the vicinity of the crack tip.
2) Cracks parallel to the beam axis are known as “longitudinal cracks”. They are not that common but they posed anger when the tensile load is applied is at right angles to the crack direction i.e. perpendicular to beam axis or the perpendicular to crack.
3) “Slant cracks” (cracks at an angle to the beam axis) are also encountered, but are not very common. These influence the torsion behavior of the beam. Their effect on lateralvibrations is less than that of transverse cracks of comparableseverity.
4) Cracks that open when the affected part of the material is subjected to tensile stresses and close when the stress is reversed are known as “breathing cracks”. The stiffness of the component is most influenced when under tension. The breathing of the crack results in non-linearity’s in the vibration behavior of the beam. Cracks breathe when cracks is small, running speeds are low and radial forces are large.
5) Cracks that always remain open are known as “gaping cracks”. They are more correctly called “notches”.
6) Cracks that open on the surface are called “surface cracks”. They can normally be detected by techniques such as dye-penetrates or visual inspection.
7) Cracks that do not show on the surface are called “subsurface cracks”. Special techniques such as ultrasonic, magnetic particle, radiography or shaft voltage drop are needed to detect them. Surface cracks have a greater effect than subsurface cracks on the vibration behavior of shafts.

5) Stress Intensity Factor (K)

It is defined as a measure of the stress field intensity near the tip of an ideal crack in a linear elastic solid when the crack surfaces are displaced in the opening mode (Mode I). (SIFs) are used to define the magnitude of the singular stress and displacement fields (local stresses and displacements near the crack tip).The SIF depends on the loading, the crack size, the crack shape, and the geometric boundaries of the specimen. The recommended units for K are MPa√m. it is customary to write the general formula in the form $K=Y\sigma\sqrt{\pi a}$ where σ is the applied stress, a is crack depth, Y is dimensionless shape factor.

III. OBJECTIVE

- To conduct a modal analysis of a composite laminate
    - Intact model
    - Control model
- To predict the position and area of delamination and crack present in the composite laminate.
- To validate the numerical analysis, conduct an experimental modal analysis of composite laminate for a selected model.

Methodology and scope of the present work:

To study the variation in natural frequency by varying delamination length, width and location of the crack for different ply orientation in a four layered composite beam using Ansys 15.

The modes of frequencies are extracted by experimental modal analysis.

The aim is to work out a composite beam finite element with a single delamination of different area and crack at different positions from the fixed end. It has been assumed that the delamination and crack changes only the stiffness of the element whereas the mass of the element is unchanged. For theoretical modeling of delaminated and cracked composite beam dimensions, delamination and crack locations, delamination length and width, material properties is specified. By using the present model the following effects due to the delamination and crack of the cantilever composite beam have been analyzed.

(1) The influence of delamination and crack position and area of delamination on the natural frequencies.
(2) The one of the presented results were compared with experimental modal analysis

IV. FINITE ELEMENT ANALYSIS

Numerical modal analysis based on the finite element (FE) modeling is performed for studying the dynamic response of a structure. The natural frequencies and mode shapes are important modal parameters in designing a structure under dynamic loading conditions. The numerical analysis is carried out by using the commercial finite element program ANSYS. It is mainly used to verify the effectiveness of delamination detection method used in this study. In the present study, since the specimen is made of fiberglass/epoxy composite material, the FE modelling of the composite beam is simulated with the layered element. The composite plate consists of four orthotropic layers, and it is considered as orthotropic material. The FE analysis software ANSYS was used in the modal analysis to obtain mode shapes.

A Cantilever beam of the dimensions Length 250mm Width of 50 mm of composite material was considered for the numerical analysis. The beam was modelled with twenty nodded shell elements so as to introduce the delamination and crack.

Modal analysis on ANSYS:

Modal Analysis is a tool used to determine vibration characteristics or natural frequencies of a mechanical structure. It can also be used for dynamic analysis, harmonic response, and transient dynamic analysis. Modal analysis in ANSYS® is linear analysis. In this research natural frequencies and mode shapes are concentrated upon.

SOLID186 Element Description:

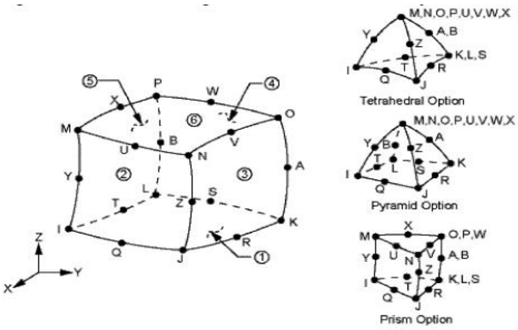
SOLID186 is a higher order 3-D 20-node solid element that exhibits quadratic displacement behavior. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The element supports plasticity, hyper elasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyper elastic materials.

SOLID186 is available in two forms:

- Homogenous Structural Solid
- Layered Structural Solid

SOLID186 Homogenous Structural Solid Element Description:

SOLID186 Homogenous Structural Solid is well suited to modeling irregular meshes (such as those produced by various CAD/CAM systems). The element may have any spatial orientation.



Fig; 3:SOLID186 Homogenous StructuralSolid Element

Model geometry:

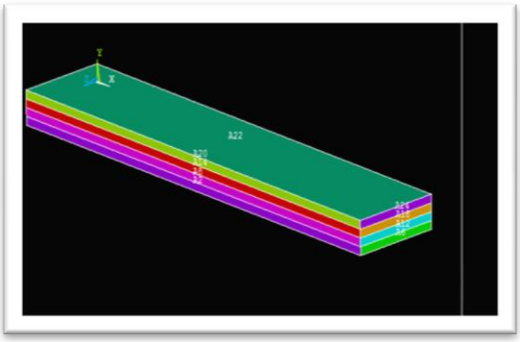


Fig. 4: Model geometry

Length of specimen 250mm, width 50mm, Height 5mm and Number of layers 4

Mesh generation

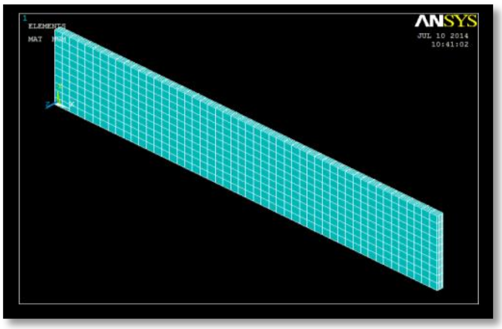


Fig. 5: Meshed model.

Number of divisions on the length is 50 and number of divisions on the width is 10

Apply loads and obtain the result:

There are several mode-extraction methods that can be selected in ANSYS®. These include: Block Lanczos, Supernode, PCG Lanczos, reduced, unsymmetrical, damped, and QR damped. Damping in the structure can be accomplished by the damped and QR damped methods. Block Lanczos is used to obtain the solution and boundary condition used is as cantilever beam.

Visualize/review the results:

Results from a modal analysis are written to a structural result file called, Job name. RST.Results could include: Natural frequencies, expanded mode shapes. The results can be reviewed in the general postprocessor.

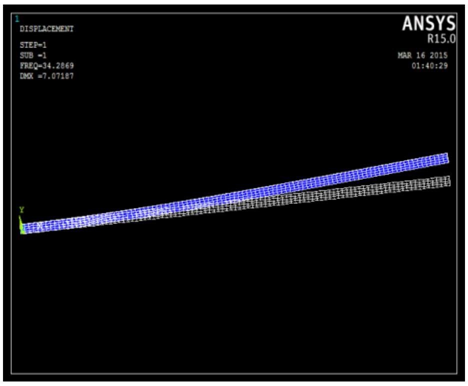


Fig 6: mode shape of mode 1

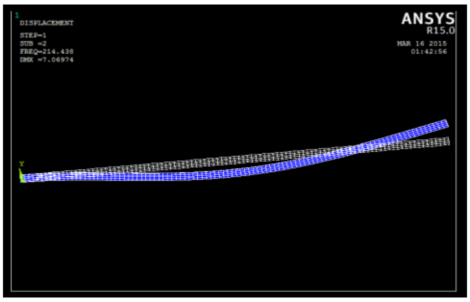


Fig 7: mode shape of mode 2

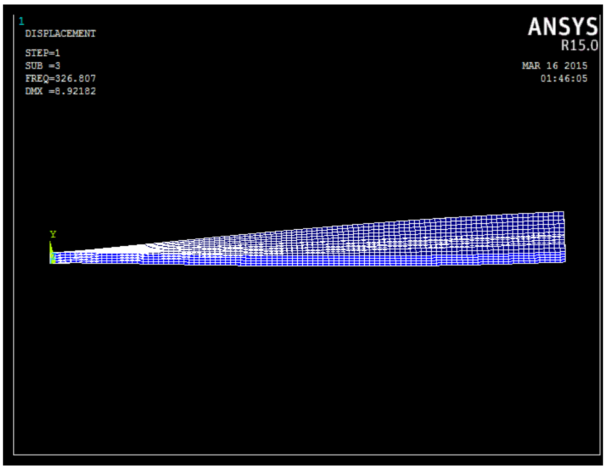


Fig 8:mode shape of mode 3

Requirement specifications:

This step is done in preprocessing in ANSYS. In this work the composite beam element model used was solid 186 and it has specification at the preprocessing stage. The parameters indicated below in the table are entered in to the analysis. The Layer orientation selected for this study are 90/0/0/90.Each layer ismodeled as unidirectional.

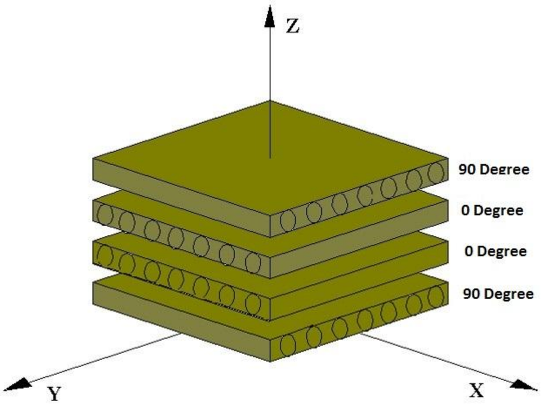


Fig. 9: Quasi –isotropic composite laminate

Table 1:Material properties

Length of beam	250mm
Width	50mm
Number of layers	4
Density	1901kg/m3
E11	135.5GPa
E22	9GPa
E33	9GPa
Poisson’s ratio	0.24
Shear modulus	5.2GPa
Shear modulus ,G_{13}	3.08GPa

Modal analysis results of undamaged composite beam for the stacking sequence of 90/0/0/90 is presented in table 2

Table 2: Mode frequencies of undamaged model

Modes	1	2	3	4	5
Mode frequencies	41.7	260.8	397.6	889.3	994.1

Table 3: Comparison on modal frequencies

Ansys results		
	Without crack	*With crack*
Mode 1	41.71Hz	40.9Hz
Mode 2	260.8 Hz	247.7Hz
Mode 3	397.6 Hz	341.8Hz
	Without delamination	*With delamination*
Mode 1	41.7 Hz	40.162Hz
Mode 2	260.8 Hz	221.68Hz
Mode 3	397.6 Hz	318.12Hz

Ansys modal analysis:

Modal analysis was carried out in delamination and cracked induced model, delamination area and position are varied in between the layers. The length of delamination selected as 50mm and delamination width as 100mm .First position of delamination length starting from the 25mm.

V. EXPERIMENTAL WORK

Introduction:

The experiment work was carried in machine Dynamics lab at College of engineering Trivandrum. The experiment modal analysis was conducted using control and intact GFRP composite cantilever beam. The aim of this experimental work is to investigate the modal parameters (Frequency, mode shape). The test beam is prepared by manual layup method.

The block diagram of the experiment set up contains
The specimen
Accelerometer
Impact hammer
Data acquisition system.

Specimen preparation:

The composite specimens were prepared from the unidirectional Glass fiber. There were four layers each having 1.375mm thickness. The total thickness was 5.5mm length of the specimen was 200mm width was 40mm.The laminate code was 90/0/0/90.The two sets of specimen were prepared.

- First set having no delamination.
- The second set was with delamination induced model. The delamination induced at the mid interface and at the middle portion by inserting Teflon tape at the location for through width of the specimen.

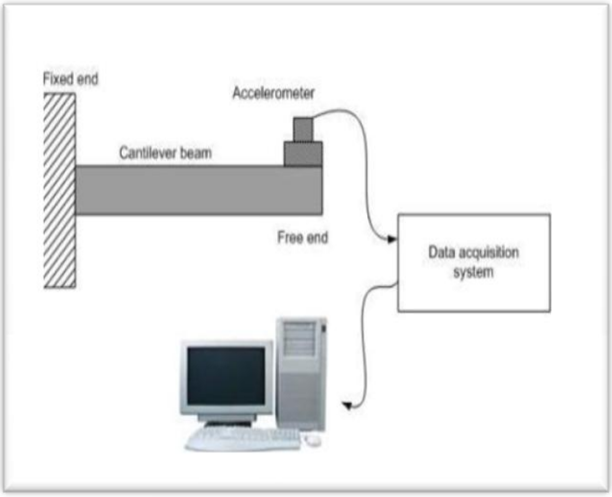


Fig. 10: Block diagram of experiment set up

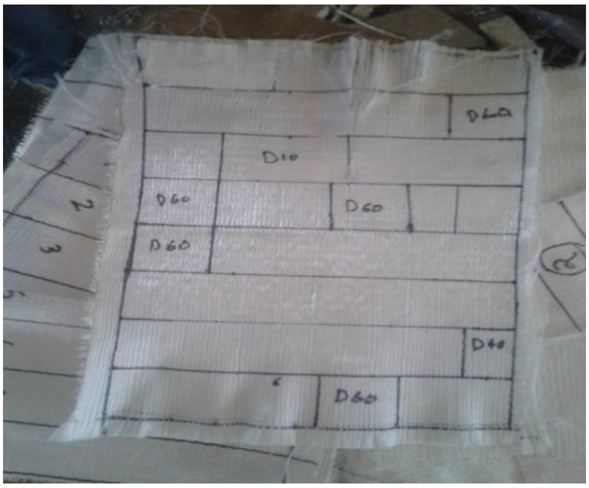


Fig 11: glass fibre sheets marked with areas for delamination

Fig 12: Plying up with suitable adhesives

Fig 13: Finalcomposite laminate

Fig. 14: Specimen without delamination

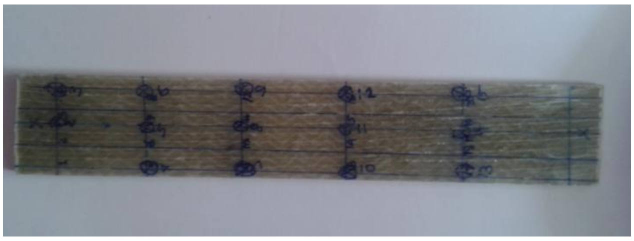


Fig. 15: Specimen with delamination

Using an impact hammer is the simplest and fastest way of exciting a structure into vibration. It is very suitable to use in operational conditions. Moreover, it does not influence the structure by attaching the exciter, which is an advantage itself. The hammer consists of a head, force transducer, tip and handle. The equipment of a hammer is usually completed with a set of tips of different stiffness and with a set of heads of different masses. A force transducer detects the magnitude of the force felt by the impactor which is assumed to be equal and opposite to that experienced by the structure.Each PCB® Modally Tuned®, ICP® instrumented impact hammer features a rugged, force sensor that is integrated into the hammer’s striking surface.

“Modal Tuning” is a feature that ensures the structural characteristics of the hammer do not affect measurement results. This is accomplished by eliminating hammer resonances in the frequency range of interest from corrupting the test data, resulting in more accurate and consistent measurements.

The force sensor serves to provide a measurement of the amplitude and frequency content of the energy stimulus that is imparted to a test object. Accelerometers are used in conjunction with the hammer to provide a measurement of the object’s structural response due to the hammer blow. A variety of tips supplied with each hammer permit the energy content of the force impulse to be tailored to suit the requirements of the item under test.

Fig. 16: Impact hammer

Data acquisition card:

The data acquisition card used in the experiment is National Instuments, the photograph of The NI USB-4431 which is displayed in figure 6.16. It is a five-channel dynamic signal acquisition module for making high-accuracy measurements from IEPE sensors. The USB-4431 delivers 100 dB of dynamic range and incorporates software-selectable IEPE (2.1 mA constant current) signal conditioning for accelerometers and microphones. The module consists of four analog input channels for reading from IEPE sensors with a single analog output. The four analog input channels simultaneously acquire at rates from 2 to 102.4 kS/s. In addition, each channel includes built-in antialiasing filters that automatically adjust to your sampling rate. The USB-4431 is ideal for a wide variety of portable test applications such as frequency response audio tests or suspension shaker tests.

. The data acquisition system includes a data acquisition box (DAQ) and a host computer which displays the data in real-time and provides a graphical-user interface (MEScope). Combined with MEScope and the PC, we can analyze and process acquired signals and control simple processes anytime, anywhere.DAQ provides analog input (AI), analog output (AO), digital input and output (DIO), audio, power supplies, and digital multimeter (DMM) functions in a compact USB device.

Fig. 17: Data acquisition card

Fast Fourier transform (FFT):

Frequency Analysis Based on the Fast Fourier Transform (FFT) Algorithm is the tool of choice for measurement and diagnostic of vibration. The FFT Analyzer is recently developed pc based virtual instrument. It uses impulse execution & either frequency domain analysis or time – domain Analysis to entrant the model Parameter from the response measurement in real time. Following impulse are execution of the specimen , the measured analog response signal maybe digitalized & analyzed using the domain techniques or transformed for analysis in the frequency domain using FFT Analyzer. The peaks in the frequency response spectrum are the location of natural frequency.

The model parameter can be entranced from a set of frequency response function (FRF) measurements between one or more reference positions & measurement position required in model. The response frequency and damping value can be found from any of the FRF measurements. On the structure the execution of the model parameter from FRF can be done using a variety of mathematical curve fitting algorithm. The FRF can be obtained using multichannel FFT measurements. The determination of frequency with the help of PULSE software requires the determination of mode frequencies.

The Piezo-Electric accelerometer:

This accelerometer is based on the Piezo-electric effect. When a Piezo-electric crystal is subjected to a mechanical force or stresses along specific planes, a voltage is generated across the

Fig: 18: Piezo-electric accelerometer.

crystal. If the force on the crystal is due to an accelerometer, a measure of the voltage across the crystal becomes a measure of the acceleration.PCB's single axis and three axis (triaxial) accelerometer configuration parameters include sensitivity, temperature, frequency response, amplitude response, form factor and lead wire grounding. Accelerometers are critical for evaluating proper performance of equipment or structures. Such applications usually require proof of calibration of the entire measurement system from the sensor through to the final output. PCB calibrates all products traceable to NIST and includes a calibration certificate with each sensor shipped.

VI. EXPERIMENTAL PROCEDURE

Free vibration is conducted on the test specimens to obtain its including natural frequencies the beam is clamped on the table with the help of clamping device arrangement. The impact is applied by striking at the marked portion near the fixed end of the test specimen.

During free vibrations, the dynamic responses of the beam are measured through the accelerometer as shown in figure. For this test, the location of accelerometer different marked position in order to extract the signals of vibration. The layout of the sensors on the test specimen is depicted in Figure A data acquisition system i.e. Data card is used to store the record data and transfer measured data to the pc for post processing. Frequency response functions (FRFs) were obtained and analyzed using ME Scope software.

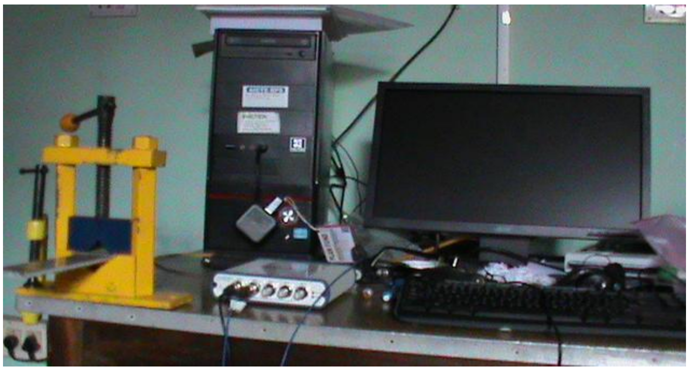


Fig. 19: Schematic diagram of experiment arrangement

Measurement procedure:

1. A composite beam of E Glass/Epoxy with, dimensions (L, w, d) was used as a cantilever beam.

2. The fixed end was made by fixing the beam with the help of Pipe wise fixed on thetable.

3. The connections of the accelerometer were properly made.

4. Accelerometer was placed at the different positions of the cantilever beam, to measure the vibration response.

5. The cantilever beam was struck with an impact hammer and beam starts vibrating.

6. All the data was recorded obtained from the vibrating beam with the help of accelerometer attached to it.

7. The experiments were repeated to check the repeatability of the experimentation.

8. The whole set of data was recorded and then the data was imported into the PC further processing and analysis was done using ME Scope software.

The signal obtained from the data acquisition system is used to extract the mode frequencies.

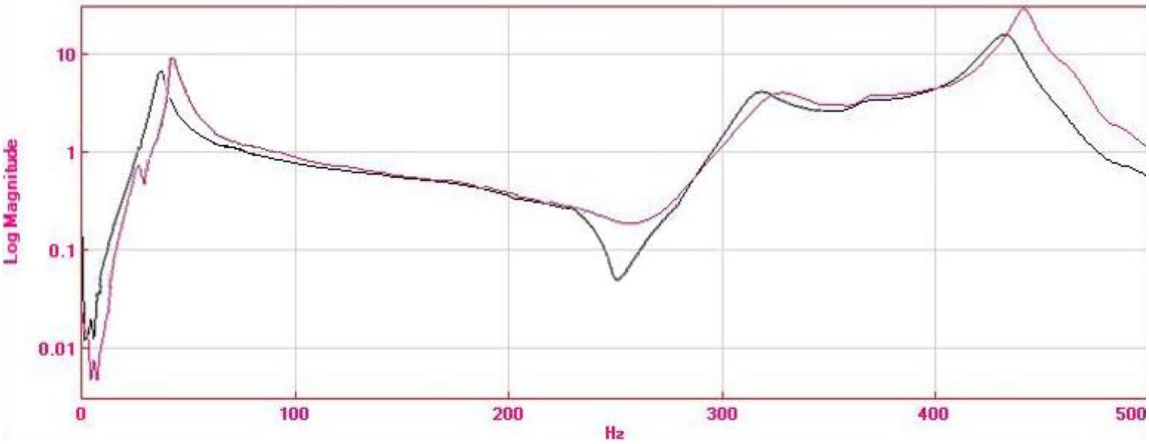
The Validation of Finite Element Model:

The first three mode frequencies obtained from the modal analysis in Ansys results were compared with the experimental modal analysis results and is presented in table

It is observed that the results showinggood agreement with each other. Thus the composite modelling technique in this work is acceptable.

Table 4: Comparison of Ansys andExperiment results

Ansys results			Experimental results	
	Without crack	*With crack*	*Without crack*	*With crack*
Mode 1	41.71Hz	40.9Hz	45 Hz	39.5 Hz
Mode 2	260.8 Hz	247.7Hz	265 Hz	260 Hz
Mode 3	397.6 Hz	341.8Hz	441 Hz	432 Hz
Ansys results			Experimental results	
	Without delamination	*With delamination*	*Without delamination*	*With delamination*
Mode 1	41.7 Hz	40.162Hz	45 Hz	40 Hz
Mode 2	260.8 Hz	221.68Hz	265 Hz	223.4 Hz
Mode 3	397.6 Hz	318.12Hz	441 Hz	418.5 Hz



- Intact beam,Cracked beam

Fig 20: Comparison of a cracked and undamaged beam.

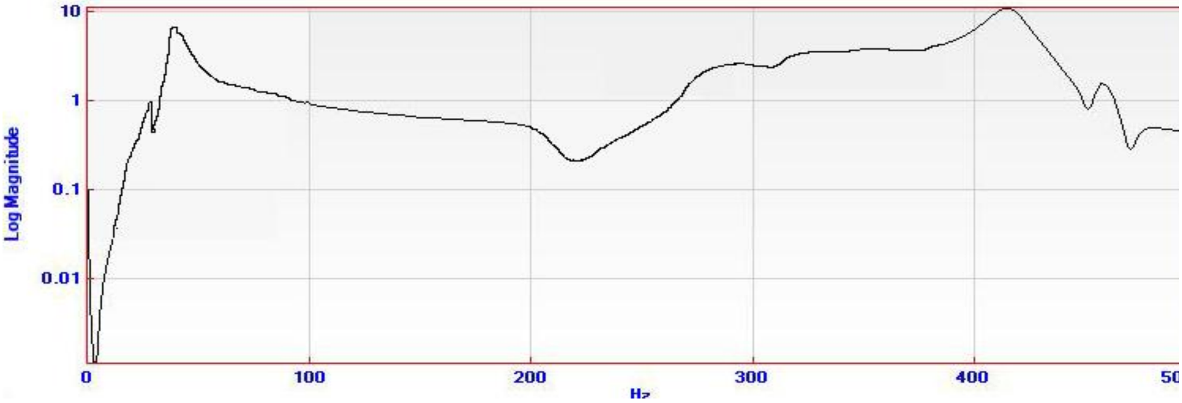


Fig 21: Frequency response of a delaminated beam

Fig 22: Fixing of accelerometer

Fig 23: Striking with impact hammer

VII. RESULTS AND DISCUSSIONS

- The variation in natural frequencies with the delamination length and crack position is carried out. The mode frequency variation of the interfaces are carried out in Ansys.
- The crack and delamination in the composite plate has an effect on the stiffness of the plate; this will affect the frequency of the composite plate. the stiffness of plate will decrease and this will cause a decreasing in the natural frequency of the composite structure.
- It is seen that for both the crack and delaminated structures the natural frequency get reduced and deviation is clearly visible from the graph shown above.
- Thus from the above result we could conclude that the presence of crack in composite structures will affect its life and it can identified by the change in the natural frequency. So by using suitable frequency measurement techniques we could detect the presence of crack in structures at the early stages and necessary remedial action could be implemented.

VIII. CONCLUSION

- The following conclusions can be made from the present study of the composite four layer beam from the finite element analysis and selected model experimental modal analysis**.**
- The finite element modal analysis were performed and studied the variation in natural frequencies. Variation study conducted by varying the crack position starting from the fixed end. It is observed that the variables have a significant influence on the mode frequencies.
- The natural frequency variation study conducted on the interfaces of the models. The reduction in natural frequencies were found on the delaminated model having lamination codeof 90/0/0/90.
- The crack and delamination in the composite plate has an effect on the stiffness of the plate; this will affect the frequency of the composite plate.
- So, with the presence of the crack and the delamination the stiffness of plate will decrease and this will cause a decreasing in the natural frequency of the corresponding mode (bending mode for crack and torsion mode for delamination)of the composite structure.
- An experimental validation study conducted on a selected models of intact and control model. There are fair agreements between experimental and finite element results.
- From the database of results we can identify the type of failure whether from a particular mode frequency.

IX. ACKNOWLEDGMENT

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