Critical Evaluation of Behavior of an Mdof System Subjected to Seismic Excitation Using Fluid Viscous Damping in Specific Context to its Response Reduction

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Abstract— **Fluid viscous dampers (FVDs) have been widely used as a means of achieving structural control, through energy dissipation. While linear FVDs have been in vogue, nonlinear FVDs too show considerable promise due to their superior energy dissipation characteristics and significant reduction in the damper force compared to a linear fluid viscous damper for the same peak displacement. This paper presents an analytical study to evaluate the effect of supplemental damping in the form of incorporating both, linear and nonlinear FVDs on SDOF and MDOF systems when they are subjected to seismic excitations. Covered in the paper are the basics of the properties and characteristics of both linear and nonlinear FVDs, besides the development of design charts giving the plots of time periods of SDOF systems versus deformation (displacements), relative velocities, total acceleration and the damper force. These design charts in turn form the basis in the form of readily available charts for preliminary decisions on parameters of supplemental dampers to be used in design required for a particular system to meet the desired response stipulations. The detailed mathematical formulations and a numerical study to evaluate the response of an example MDOF system to a seismic excitation is discussed using two methods - response spectrum method being used to evaluate the response in terms of reduction in storey drifts and the second method used is mode superposition method to evaluate the response in terms of reduction in storey displacements through time history analysis.**

Keywords— Energy Dissipation; Fluid Viscous Dampers(FVDs); Response Control; Natural Damping Coefficient; Supplemental Damping; Linear FVD; nonlinear FVD; Damper Force; Response Spectra; Design Charts; Storey Drifts; Storey Displacements

I. INTRODUCTION

In conventional seismic design, acceptable performance of a structure during earthquake shaking is based on the lateral force resisting system, being able to absorb and dissipate the earthquake energy in a stable manner for a large number of cycles. Energy dissipation occurs in specially detailed regions of concentrated damage to the gravity frame, namely plastic hinges which are often irreparable. The occurrence of inelastic deformations results in softening of the structural system which itself reduces the absolute input energy.

Another approach to improving earthquake response performance and damage control is that of supplemental energy dissipation systems. In these systems, mechanical devices are incorporated into the frame of the structure and dissipate energy throughout the height of the structure. The means by which energy is dissipated is either yielding of mild steel, sliding friction plates, motion of a piston or a plate within a viscous fluid, orificing of fluid, or viscoelastic action in polymeric materials.

In-structure damping, or energy dissipation, encompasses any component to reduce the movement of structures under lateral loads such as wind and earthquakes. Usual structural engineering processes attempt to achieve more capacity than demand by increasing the capacity of the structure. Passive control takes the opposite approach and attempts to reduce the demand on the structure. This strategy attempts to reduce the demand on a structure, rather than more usual approach of adding capacity. The focus of vibration control of structures in this paper is through incorporation of Fluid Viscous Dampers (as type of Passive Energy Dissipation Devices (EDDs)) on 'framed structure' applications, although the basic working principles are the same for bridges and other structures. [1][2]

II. FLUID VISCOUS DAMPERS

'Fluid Viscous Dampers (FVDs)', are a class of 'Passive Energy Dissipation System'. They are commonly used as passive energy dissipation devices for seismic protection of structures. They can dissipate large amount of energy over a wide range of load frequencies. The damping force generated by the damper is due to the pressure differential across the piston head and fluid compressibility. Such dampers consist of a hollow cylinder filled with fluid, the fluid typically being silicon based. As the damper piston rod and piston head are stroked, fluid is forced to flow through orifices either around or through the piston head. The resulting differential in pressure across the piston head very high pressure on the upstream side and very low pressure on the downstream side can produce very large forces that resist the relative motion of the damper. These dampers will generally not increase the strength and stiffness of a structure unless the excitation frequency is high. They operate on principles of fluid orificing and sloshing. Damping force in these devices is proportional to 'velocity', i.e. they are 'rate dependent' devices. A purely viscous device is a special case of viscoelastic device with zero stiffness and frequency independent properties i.e. at any excitation frequency will not add stiffness.

Therefore, when an FVD is to be applied to the energy dissipation design of a structure, the natural frequency of the structure will not be affected, so, it is more convenient and simpler in design. The reduction in deformations can be in the tune of 30% to 70%, which are comparable to those achieved by using other passive energy dissipation devices such as the metallic, friction and the viscoelastic dampers. The main advantages of incorporating FVDs in a superstructure apart from those mentioned above include activation at low displacements, requirement of minimal restoring force, their properties are largely frequency and temperature independent besides they have a proven record in military applications. A disadvantage, however of using FVDs is from reliability point of view that there can arise a possibility of fluid seal leakage. [1][2][3]

III. ORGANIZATION OF THE PAPER

The present paper based on the objectives is organized in the following manner:

1) A brief introduction and general information on the concept of energy dissipation through supplemental damping in structures subjected to seismic excitation forms the initial part.
2) Evaluation of damper properties and characteristics of FVDs (both, linear and nonlinear) is covered.
3) Developing formulation of Supplemental Damping Ratio (ξ_{sd}) for use of nonlinear fluid viscous dampers under seismic excitation, besides, also covered briefly is development of 'Design Charts' in specific context to El Centro ground acceleration for preliminary selection of linear and nonlinear damper parameters for seismic control of SDOF structures.
4) Evaluation of the response taking example MDOF structures using linear and nonlinear fluid viscous dampers, duly incorporating the inputs from the so obtained design charts; and
5) Finally, inferences are drawn for the preliminary selection of proposed FVD based on storey displacement/storey drift stipulations, as found suitable for the MDOF system selected, concludes this paper.

IV. METHODOLOGY ADOPTED FOR THE CURRENT INVESTIGATION

Based on the objectives set for the current investigation, the basic methodology adopted for the investigation, included, utilization of the established equations of motion as pertaining to the response of a single degree of freedom (SDOF) system subjected to a seismic excitation (in this case, the El Centro earthquake of 1940 has been taken as the input excitation) as a bare frame with its inherent damping and thereafter analyzing effect on the response of the same very system upon incorporating a supplemental damping (through a fluid viscous damper, with both linear and nonlinear properties) into it. Design charts (site specific) for SDOF systems (i.e. plots of displacement versus time period, relative velocity versus time period, total acceleration versus time period and damper force versus time period) developed have been incorporated in obtaining the inputs ('S_a/g' values for the dynamic analysis of multi degree of freedom (MDOF) systems). The supplemental damping effect was thereafter analyzed for example MDOF System (i.e. 4 storey frame with assumed parameters) using these design charts. The overall analysis has been based on Newmark's Average Acceleration Method using MATLAB and MS Excel.

V. EVALUATING DAMPER PROPERTIES AND CHARACTERISTICS FOR A FLUID VISCOUS DAMPER AND DEVELOPING DESIGN CHARTS FOR PRELIMINARY SELECTION OF LINEAR AND NONLINEAR DAMPER PARAMETERS [3] [4] [5]

The mathematical modeling for the two types of FVDs i.e. linear and nonlinear, is done in terms of parameters α and c_α (i.e. the nonlinearity parameter and the damping coefficient, respectively) constituting the equation of the damping force, f_d:

$$f_d = c_\alpha |\dot{u}|^\alpha \operatorname{sgn}(\dot{u}), \text{ where}$$

$\dot{u}$, is the relative velocity between the two ends of the damper; α is the exponent between 0 and 1 (α =1 for linear viscous dampers while α=0 exhibits the characteristics of a friction damper).

For a given earthquake ground acceleration (in this case, El Centro earthquake of 1940), we develop the design charts that directly give us the structural deformation, relative velocity, total acceleration and the damper force for a specific *Time Period* value of an SDOF system over a range of 3 seconds; these design charts are useful in thus selecting an FVD that limits the structural deformation to a design value.

A. Nonlinear Fluid Viscous Damper

The force (f_D)-velocity ($\dot{u}$) relation for nonlinear fluid viscous dampers (FVDs) can be analytically expressed as a fractional velocity power law:

$$f_D = c_\alpha \operatorname{sgn}(\dot{u})|\dot{u}|^\alpha \quad (1)$$

where, c_α is the experimentally determined damping coefficient with units of force per velocity raised to the α power; α is a real positive exponent with typical values in the range of 0.35 – 1 for seismic applications; and sgn (.) is the signum function. Equation (1) becomes $f_D = c_1\dot{u}$ for $_\alpha = 1$, which represents a linear FVD and $f_D = c_0 \operatorname{sgn}(\dot{u})$ for α=0, which represents a pure friction damper. Thus, α characterizes the nonlinearity of FVDs.

The energy dissipated by the damper during a cycle of harmonic motion $u = u_0 \sin \omega t$ is:

$$E_D = \oint f_D\, du = \int_0^{2\pi/\omega} f_D \dot{u}\, dt = \int_0^{2\pi/\omega} c_\alpha |\dot{u}|^{1+\alpha}\, dt \quad (2)$$

Integrating Equation 2 results in:

$$E_D = \pi \beta_\alpha c_\alpha \omega^\alpha u_0^{\alpha+1} \quad (3)$$

Where, the constant β_α is:

$$\beta_\alpha = (2^{2+\alpha}\Gamma^2(1+\alpha/2)) / (\pi\Gamma(2+\alpha)) \quad (4)$$

and Γ is the gamma function. For a linear FVD, (α = 1), $\beta_\alpha = 1$ and Equation 3 becomes:

$$E_D = \pi c_1 \omega u_0^2 \quad (5)$$

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In the limit case of pure friction dampers, ($\alpha = 0$), $\beta_\alpha = 4/\pi$ and Equation (3) reduces to

$E_D = 4 c_0 u_0$.

Nonlinear and linear FVDs dissipate an equal amount of energy per cycle of harmonic motion if the two results (Equations (3) and (5)) for E_D are the same; this equality leads to

$$c_\alpha = ((\omega u_0)^{1-\alpha} / \beta_\alpha) c_1 \quad (6)$$

B. *Equivalent Linear Viscous Damping* [3]

We characterize the energy dissipation capacity of energy-equivalent nonlinear FVDs by supplemental damping ratio ξ_{sd} and their nonlinearity by α. For a linear single-degree-of freedom (SDOF) system with mass m, stiffness k, and a nonlinear FVD defined by Equation (1), the supplemental damping ratio ξ_{sd} due to the FVD is defined based on the concept of equivalent linear viscous damping as follows:

$$\xi_{sd} = ED/4\pi ES_0 = ED/2\pi k u_0^2 \quad (7)$$

where ES0 is the elastic energy stored at the maximum displacement, u0. Substituting Equation (3) evaluated at $\omega=\omega_n$ into Equation (7) gives ξ_{sd} as a function of the displacement amplitude, u_0:

$$\xi_{sd} = ((\beta_\alpha c_\alpha)/2ku_0)(\omega_n u_0)\alpha = ((\beta_\alpha c_\alpha)/2m\omega_n))(\omega_n u_0)^{\alpha-1} \quad (8)$$

Equation (8) reduces to the amplitude-independent damping ratio $\xi_{sd} = (c_1/2m\omega_n)$ for a linear FVD (α=1) and to $\xi_{sd} = 2c_0/\pi k u_0$ for a friction damper (α=0).

C. *Equation of Motion and System Parameters* [1][4][5]

(1) *Equation of motion*

The equation governing the motion of the SDOF system with mass *m*, elastic stiffness *k*, linear viscous damping coefficient *c*, and a nonlinear FVD subjected to ground acceleration $\ddot{u}_g(t)$ is

$$m\ddot{u} + c\dot{u} + ku + c_\alpha \,\mathrm{sgn}(\dot{u}) |\dot{u}|\alpha = -m\ddot{u}_g(t) \quad (9)$$

Given c_α and $\alpha \neq 1$ values, Equation (9) is nonlinear, therefore, the response u of the system depends nonlinearly on the excitation intensity. Thus, parameterizing this equation and studying the effect of supplemental damping on system response become complicated because of the nonlinear term involving two parameters c_α and α, wherein cα is not a dimensionless parameter. Therefore, we replace c_α by Equation (9) for energy-equivalent FVDs and divide the resulting equation by m to obtain

$$\ddot{u} + 2\xi \omega_n \dot{u} + \omega_n^2 u + (2 \xi_{sd} \omega_n) / \beta\alpha (\omega_n u_0)1\text{-}\alpha \,\mathrm{sgn}(\dot{u})|\dot{u}|\alpha = -\ddot{u}_g(t) \quad (10)$$

Where, $\omega_n = \sqrt{(k/m)}$ and $\xi = c/2m\omega_n$ are the natural vibration frequency and the damping ratio of the system, respectively; and ξ_{sd} is the supplemental damping ratio due to the nonlinear FVD. Equation (10) governs the motion of SDOF systems with energy-equivalent nonlinear FVDs, which are characterized by the same ξ_{sd} value but different α values. In particular, when α =1 and α =0 in Equation (10), we obtain the governing equations for linear-viscous and friction-supplemental damping, respectively.

Although Equation (10) is nonlinear and involves the unknown displacement amplitude u_0 (for $\alpha \neq 1$) in the supplemental damping term, it offers the following advantages over Equation (9): (i) the response u of the system varies linearly with the excitation intensity, i.e. scaling the $\ddot{u}_g(t)$ by doubling the peak ground acceleration $\ddot{u}_g0$ will double u(t); (ii) effects of nonlinear FVDs on the system response can be investigated in terms of two independent, dimensionless parameters, ξ_{sd} and α; and (iii) the accuracy of the corresponding linear viscous system in estimating the response of the system with nonlinear FVDs can be evaluated.

(2) *System Parameters*

As indicated by Equation (9), the response of energy-equivalent SDOF systems with nonlinear FVDs is controlled by four parameters: (i) damper nonlinearity parameter α, which controls the shape of the damper force hysteresis loop; (ii) supplemental damping ratio ξ_{sd}, which represents the energy dissipation capacity of the FVD independent of the α value; (iii) natural vibration period of the system $T_n = 2\pi / \omega_n$; and (iv) damping ratio, ξ, which represents the inherent (natural) energy dissipation capacity of the system.

D. *Earthquake Response*

The earthquake response history selected for the current evaluation is that of the El Centro 1940 ground motion as has been taken in this paper. Accordingly all the system responses (deformation, velocity and the total acceleration responses) are based in particular to the El Centro seismic excitation.

(1) *Influence of Damper Nonlinearity*

Although the mean response spectra for deformation, relative velocity, and total acceleration presented in Figs. 1 (a)–(c), 2 (a)-(c) and 3 (a)-(c) respectively, are affected very little by damper nonlinearity, the influence increases at longer periods and for smaller values of α, implying more nonlinearity.

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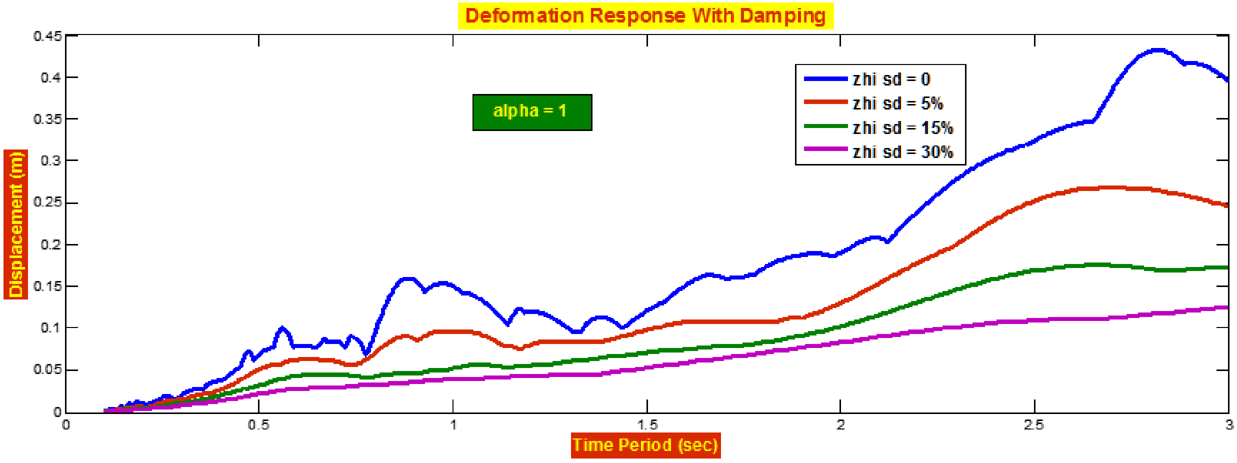
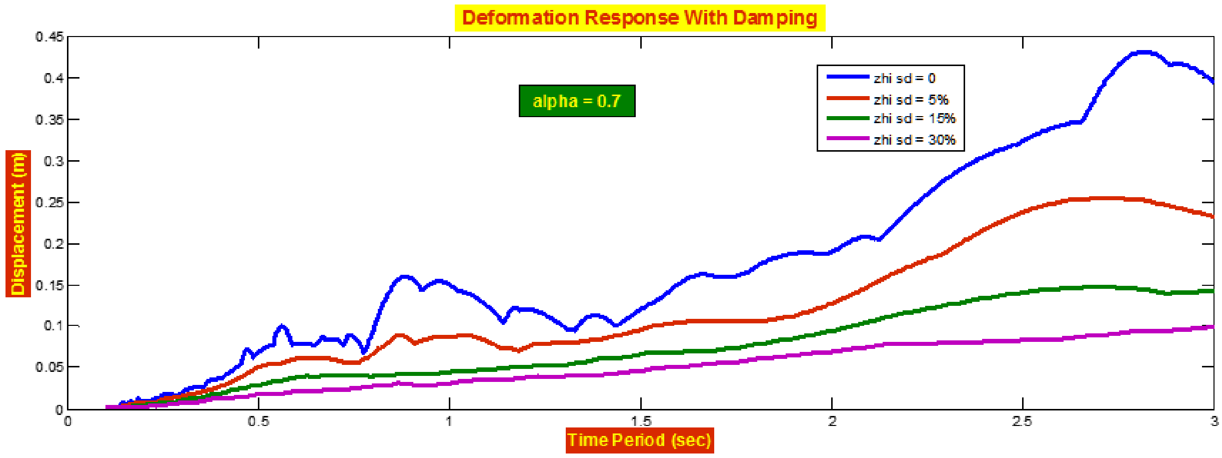
Damper nonlinearity has essentially no influence on system response in the velocity-sensitive spectral region and small influence in the acceleration- and displacement-sensitive regions. Thus, the system response is only weakly affected by damper nonlinearity. This observation has the useful implication for design applications that, for a given ξ_{sd}, the response of systems with nonlinear FVDs can be estimated to a sufficient degree of accuracy by analyzing the corresponding linear viscous system (α=1).



(a)



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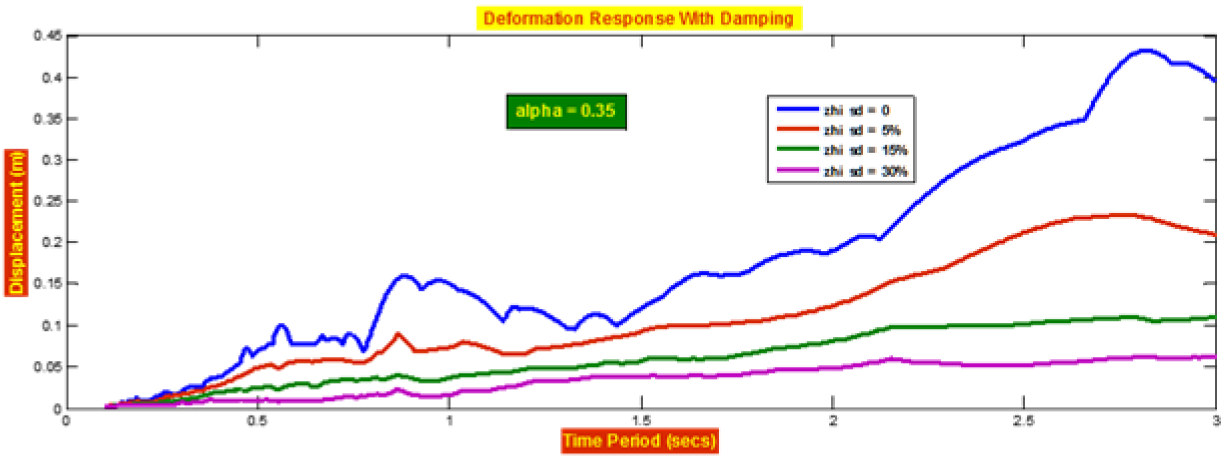

(c)

Fig. 1 Mean response spectra for deformation for the example SDOF system with ξ = 2% and supplemental damping ξ_{sd} = 0, 5%, 15% and 30% due to nonlinear FVDs with different α values

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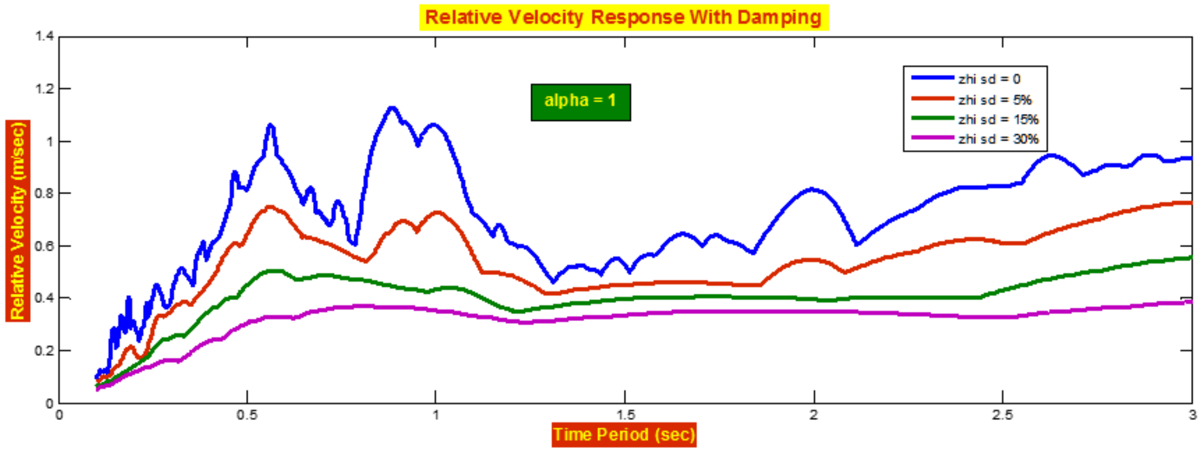
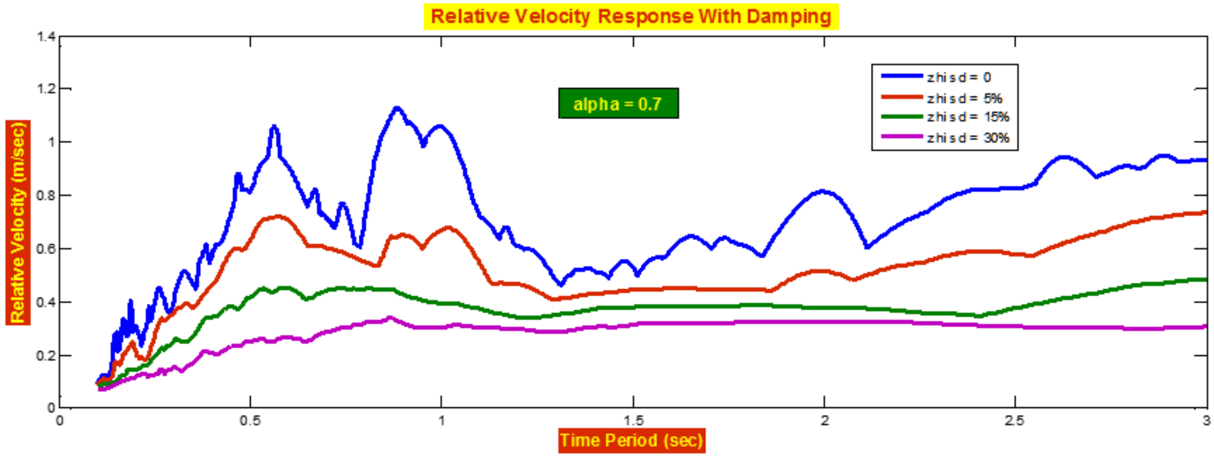


(a)



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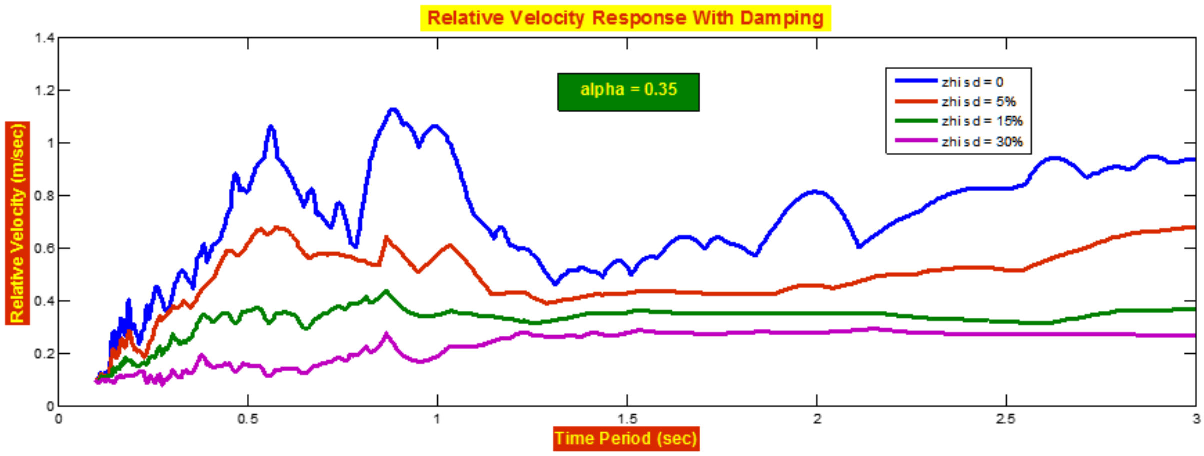

(c)

Fig. 2 Mean response spectra for relative velocity for the example SDOF system with ξ = 2% and supplemental damping ξ_{sd} = 0, 5%, 15% and 30% due to nonlinear FVDs with different α values

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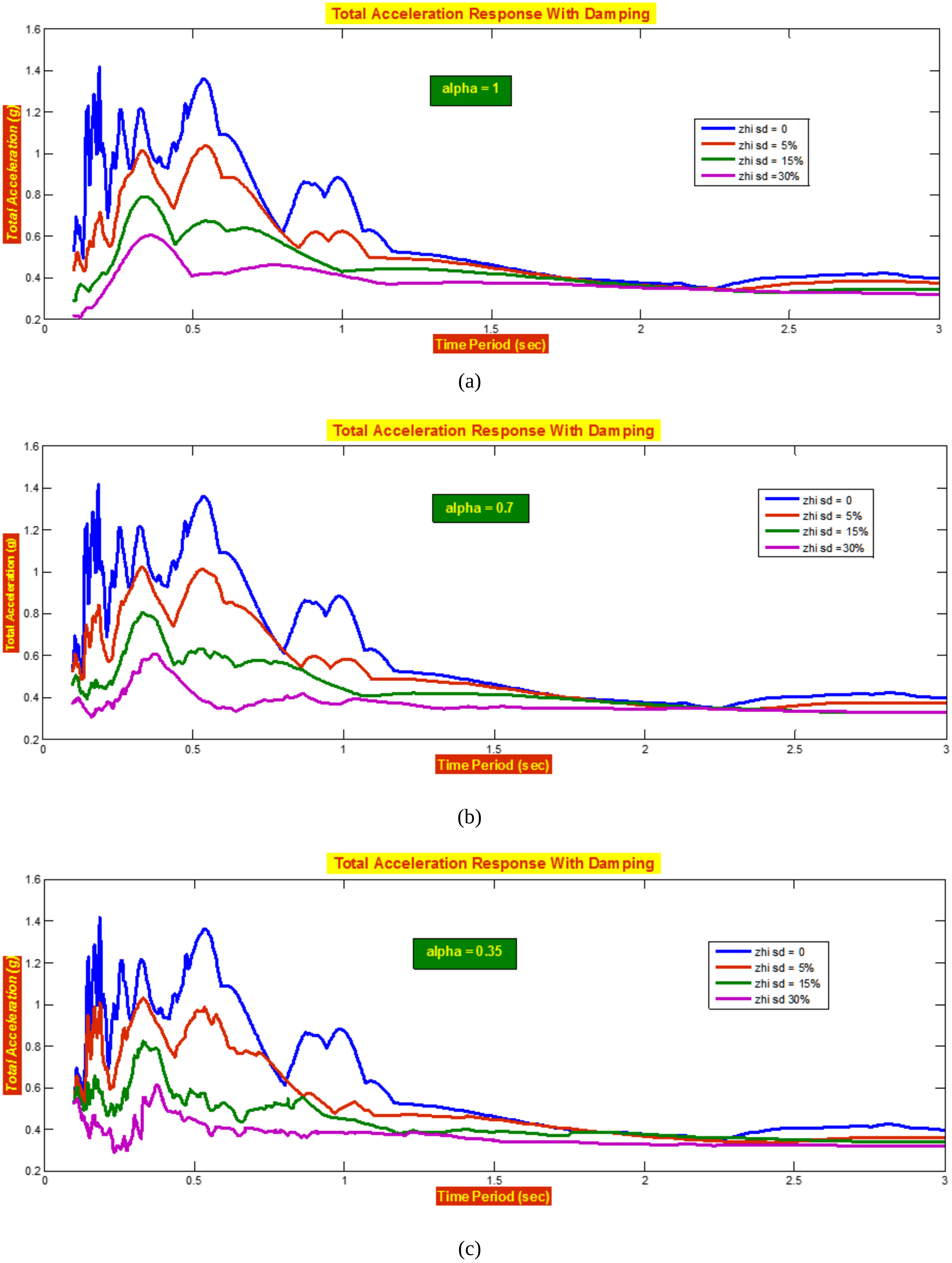

Fig. 3 Mean response spectra for total acceleration for the example SDOF system with $\xi = 2\%$ and supplemental damping ξ_{sd} = 0, 5%, 15% and 30% due to nonlinear FVDs with different α values

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(2) Influence of Supplemental Damping

As expected, supplemental damping reduces structural response, with greater reduction achieved by the addition of more damping (Figs. 1, 2 and 3); the reduction achieved with a given amount of damping is different in the three spectral regions. As Tn→0, supplemental damping does not affect response because the structure moves rigidly with the ground. And as $T_n \to \infty$, supplemental damping again does not affect the response because the structural mass stays still while the ground underneath moves.

The response reduction is significant over the range of periods considered. As observed from the plots, as little as 5% supplemental damping reduces the deformation response in a range of over 20% averaged over the three acceleration-, velocity-and displacement-sensitive spectral regions, respectively. The corresponding reductions are close to about 40% - 50% range for moderate supplemental damping (ξ_{sd} =15%) and higher for large supplemental damping (ξ_{sd} =30%). Consistent with earlier observations, the reduction in responses is essentially unaffected by damper nonlinearity in the velocity-sensitive region and only weakly dependent in the acceleration- and displacement-sensitive regions. It is thus indicated that supplemental damping reduces all responses.

E. Damper Force

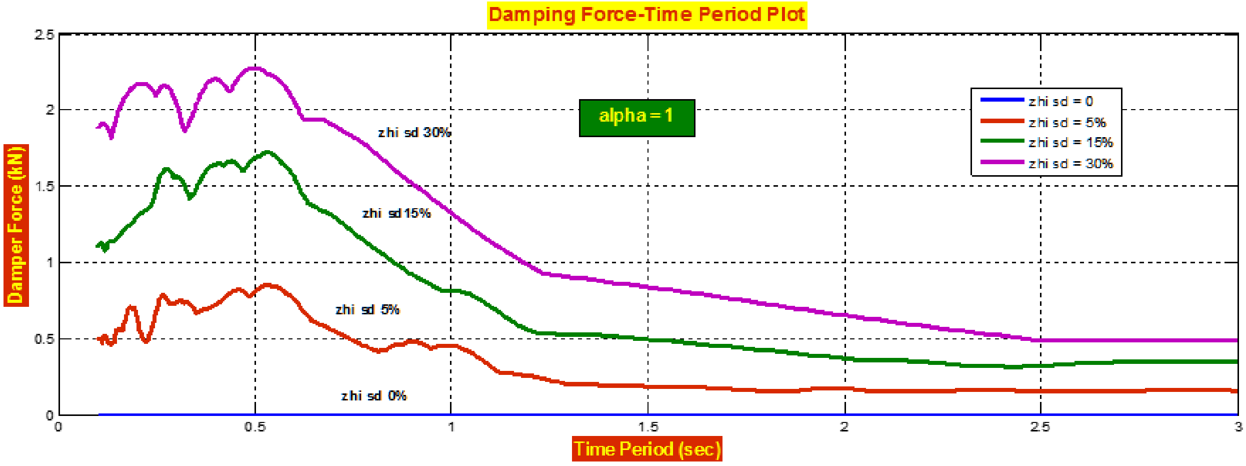
The response spectrum for damper force shown in Figs. 4 (a), (b) and (c) permits two salient observations:

(i) the damper force is larger for larger dampers, as indicated by their ξ_{sd} values; and

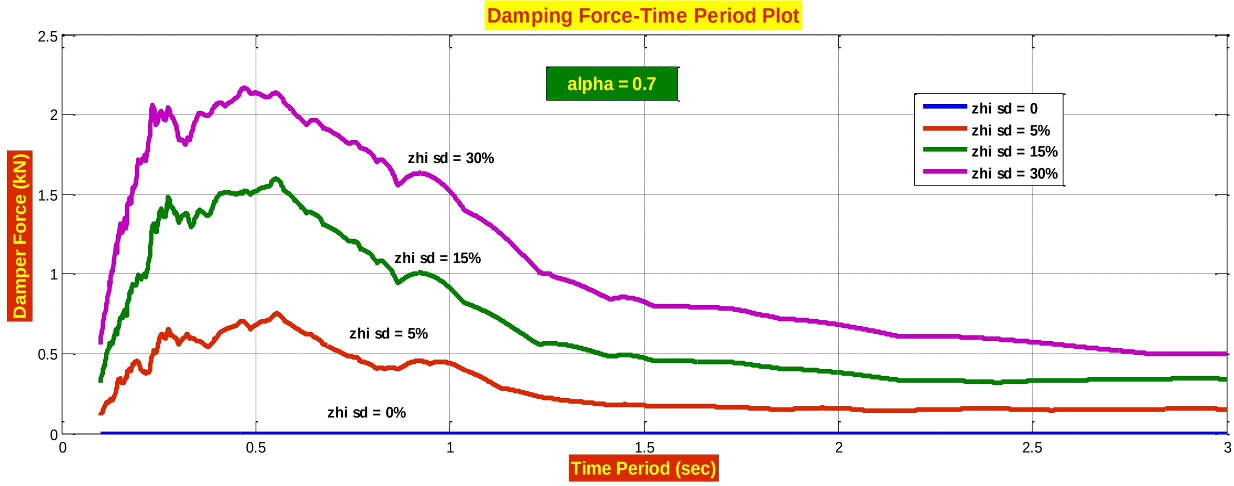
(ii) for a selected ξ_{sd} for supplemental damping, the damper force is smaller for nonlinear FVDs, as can be observed for time periods lesser than 0.8 seconds; the more nonlinear the damper (i.e. smaller the α value), the smaller is the damper force (comparison between Figs. 4 (a), (b) and (c)).

From Figs. 4 (a), (b) and (c)), it is also observed that for longer time periods, increase in nonlinearity increases the damping force in comparison to the linear dampers. Thus, it could be inferred that such nonlinear dampers would find usefulness in cable suspension bridges which have long time periods. Nonlinear FVDs are advantageous because they achieve essentially the same reduction in response (Figs. 1, 2 and 3) but with a significantly reduced damper force (Fig. 4).

The above observations are valid for the range of system period considered, except for very short-period systems ($T_n < 0.1$ sec).



(a)



(b)

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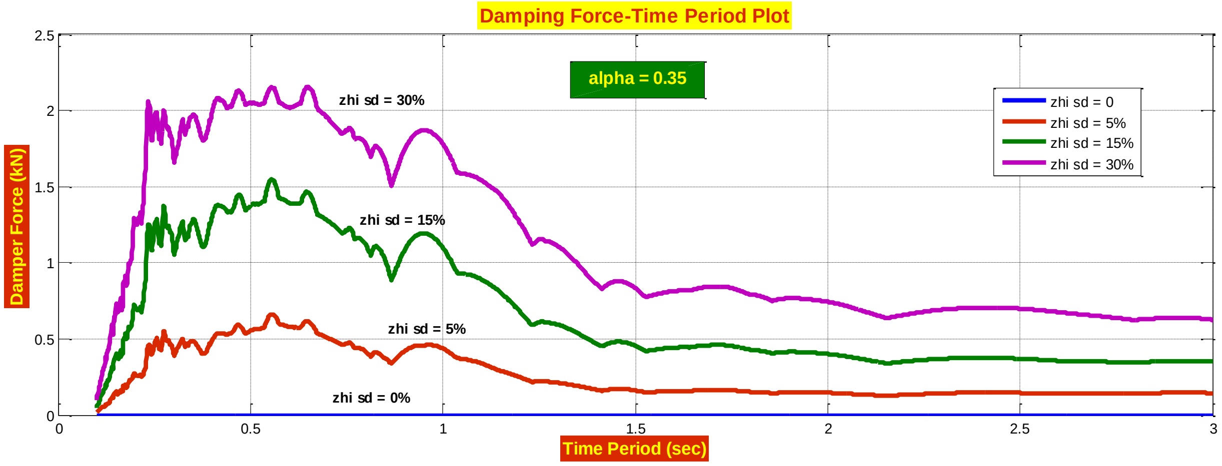

(c)

Fig. 4 Damper force spectra for FVDs with supplemental damping ξ_{sd} = 5%, 15% and 30% and α = 1, 0.7 and 0.35

VI. RESPONSE EVALUATION TAKING AN EXAMPLE MDOF SYSTEM

In this part of the paper, we evaluate the responses of an example MDOF system – a four storey frame of given parameters (dimensions and stiffness). In this example, the first method adopted to analyze the responses (in terms of 'storey drifts'), was, the *Response Spectrum Method* which has been used considering initially, a bare frame with its inherent natural damping ratio of 2%, thereafter, incorporating supplemental damping of 5%, 15% and 30%. The *design charts* (Fig.3) used for the S_a/g values are the ones developed for an SDOF system pertaining to the El Centro ground acceleration, and are therefore site specific. The response spectrum method adopted is as per the provisions of the IS 1893 (Part 1): 2016.

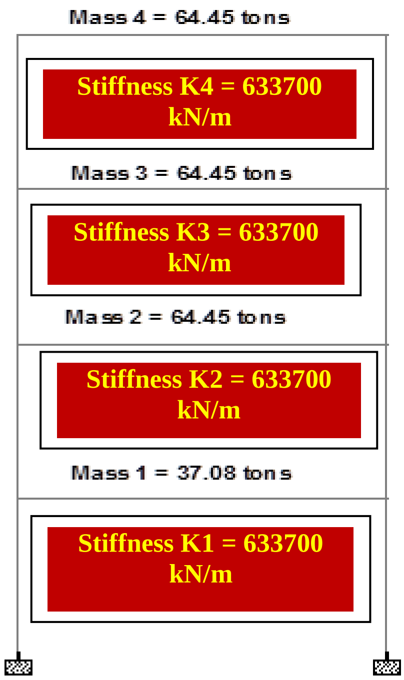

Fig. 5 An example four MDOF system

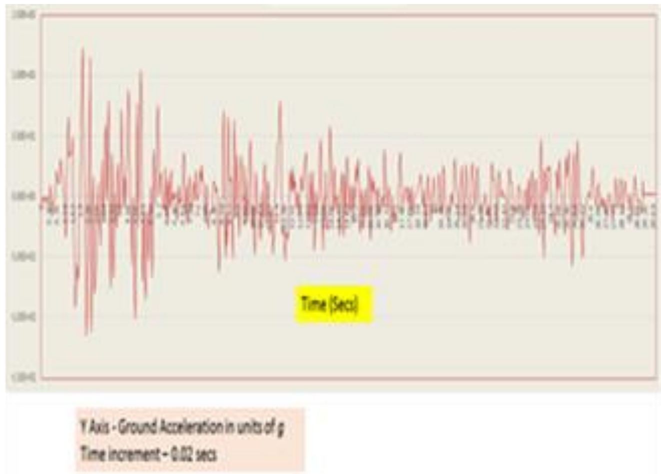

Fig. 6 El Centro Earthquake Ground Acceleration (1940)

The second method considered for analyzing the response in terms of 'displacements' of each storey (physical coordinates) the same four storey framed structure subjected to the same El Centro ground acceleration has been done using Time History Analysis. Here too, initially, only the bare frame with its natural damping ratio is considered for evaluation of displacement response, thereafter the same frame is evaluated for response after incorporating two types of supplemental damping at a supplemental damping ratio of 30% but with linear and nonlinear FVDs.

1) *Response Evaluation using Response Spectrum Method for a Four DOF System*

The four storey frame considered is as shown in the Fig. 5. The parameters of mass and stiffness are as illustrated. The frame is subjected to the El Centro ground acceleration (Fig. 6). The procedure adopted for evaluation of response is to first calculate the natural frequencies for the various modes as also to find out the mode shapes. The storey shears are then determined from dynamic analysis to finally arrive the storey drifts.

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The S_a/g values for response spectrum method are the ones adopted from the design charts developed for El Centro ground acceleration. The results of the responses for storey drifts for the four storey frame using response spectrum method with 5%, 15% and 30% supplemental damping considering bare frame (i.e. without supplemental damping, ξsd = 0 and natural damping, ξ =2%) and with different supplemental damping ratios (ξ_{sd} = 5%, 15% and 30%) using the values of S_a/g from the developed design charts are tabulated as under:

TABLE 1: COMPARATIVE RESULTS

Storey	ξsd =0 (Bare frame) Storey Drift Dynamic (cm)	ξ_{sd} =5% Storey Drift Dynamic (cm)	% Reduction in Storey Drift	ξ_{sd} =15% Storey Drift Dynamic (cm)	% Reduction in Storey Drift	ξ_{sd} =30% Storey Drift Dynamic (cm)	% Reduction in Storey Drift
4	0.00874	0.00694	20.5	0.00463	47.0	0.00295	66.2
3	0.01668	0.01320	20.9	0.00873	47.7	0.00547	67.2
2	0.02275	0.01800	20.9	0.01190	47.7	0.00746	67.2
1	0.02464	0.01949	20.9	0.01288	47.7	0.00807	67.2
	S_a/g=1.279		S_a/g=1.012		S_a/g=0.6688		S_a/g=0.419

To enable a validation for the results of storey drifts obtained by response spectrum method, the same four storey frame is analyzed using mode superposition method for obtaining the responses in terms of storey displacements using time history analysis. The effect of introducing a supplemental damping of 30% (both, linear and nonlinear) on individual storey displacements is covered in the next sub-section wherein the same example four DOF system has been considered.

2) Response Evaluation using Mode Superposition Method for a Four DOF System (Bare Frame) [6] [7]

We evaluate the response of the four storey plane frame model in Figure 5 (i. e. a Four Degree of Freedom System) subjected to El Centro ground acceleration (Fig. 6), using mode superposition method (incorporating time history (Tedesco et. al. 1999)). The equation of motion for a multi-degree-of-freedom system in matrix form can be expressed as:

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = -\ddot{x}_g(t)\,[m]\{I\} \qquad (11)$$

Where,

$[m]$ = mass matrix

$[k]$ = stiffness matrix

$[c]$ = damping matrix

$\{I\}$ = unit vector

$\ddot{x}_g(t)$ = ground acceleration

The solution of equation of motion for any specified forces is difficult to obtain, mainly due to coupling of the variables {x} in the physical coordinates. In mode superposition analysis or a modal analysis, a set of normal coordinates is defined, such that, when expressed in those coordinates, the equations of motion become uncoupled.

In mode superposition analysis or a modal analysis, a set of normal coordinates is defined, such that, when expressed in those coordinates, the equations of motion become uncoupled The physical coordinates {x} may be related with normal or principal coordinates {q} from the transformation expression as,

{x} = [Ø]{q}, where [Ø] is the modal matrix.

Time derivatives of {x} are,

$$\{\dot{x}\} = [\emptyset]\{\dot{q}\}$$

$$\{\ddot{x}\} = [\emptyset]\{\ddot{q}\}$$

Substituting the time derivatives in the equation of motion, and pre-multiplying by $[\emptyset]^T$ results in,

$$[\emptyset]^T[m][\emptyset]\{\ddot{q}\} + [\emptyset]^T[c][\emptyset]\{\dot{q}\} + [\emptyset]^T[k][\emptyset]\{q\} = (-\ddot{x}_g(t)\,[\emptyset]^T\,[m]\{I\})$$

More clearly represented as follows:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = P_{eff}(t)$$

Where,

$[M] = [\emptyset]^T[m][\emptyset]$, the diagonalized modal mass matrix,

$[C] = [\emptyset]^T[c][\emptyset]$, the diagonalized modal damping matrix,

$[K] = [\emptyset]^T[k][\emptyset]$, the diagonalized modal stiffness matrix, and

$P_{eff}(t) = (-\ddot{x}_g(t)\,[\emptyset]^T\,[m]\{I\})$, the effective modal force vector.

The mass and stiffness matrices for the plane frame with infills are:

K =	1267400	-633700	0	0	
	-633700	1267400	-633700	0	in kN/m
	0	-633700	1267400	-633700	
	0	0	-633700	633700	
M =	64.4500	0	0	0	
	0	64.4500	0	0	in Tons
	0	0	64.4500	0	
	0	0	0	37.0800	

Natural frequencies and mode shape for the plane frame model:

$$[\omega] = \begin{bmatrix} 37.9751 & 0 & 0 & 0 \\ 0 & 108.1569 & 0 & 0 \\ 0 & 0 & 161.9460 & 0 \\ 0 & 0 & 0 & 191.6197 \end{bmatrix} \text{ in rad/second}$$

$$T = \begin{bmatrix} 0.1655 & 0 & 0 & 0 \\ 0 & 0.0581 & 0 & 0 \\ 0 & 0 & 0.0388 & 0 \\ 0 & 0 & 0 & 0.0328 \end{bmatrix} \text{ in seconds}$$

$$\emptyset_1 = \begin{Bmatrix} -0.0328 \\ -0.0608 \\ -0.0798 \\ -0.0872 \end{Bmatrix}, \quad \emptyset_2 = \begin{Bmatrix} 0.0795 \\ 0.0644 \\ -0.0273 \\ -0.0865 \end{Bmatrix},$$

$$\emptyset_3 = \begin{Bmatrix} 0.0808 \\ -0.0540 \\ -0.0448 \\ 0.0839 \end{Bmatrix}, \quad \emptyset_4 = \begin{Bmatrix} -0.0397 \\ 0.0690 \\ -0.0799 \\ 0.0696 \end{Bmatrix}$$

The values for [M], [K] and [C] are:

$$[M] = [\emptyset]^T[m][\emptyset] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[K] = [\emptyset]^T[k][\emptyset] = \begin{bmatrix} 1442 & 0 & 0 & 0 \\ 0 & 11698 & 0 & 0 \\ 0 & 0 & 26227 & 0 \\ 0 & 0 & 0 & 36719 \end{bmatrix}$$

$$[C] = \text{diag}(2M_r\xi_r\omega_r) = \begin{bmatrix} 3.7975 & 0 & 0 & 0 \\ 0 & 10.815 & 0 & 0 \\ 0 & 0 & 16.1947 & 0 \\ 0 & 0 & 0 & 19.1621 \end{bmatrix}$$

Calculation of Effective Force Vector

The excitation function is,

$$P_{eff}(t) = (-\ddot{x}_g(t)\,[\emptyset]^T[m]\{I\}) \text{ or } (-\ddot{x}_g(t)\,\Gamma_r$$

Where,

$$\Gamma_r = \frac{[\emptyset]rT\,[m]\{I\}}{[\emptyset]rT\,[m]\{\emptyset\}r} = \frac{[\emptyset]rT\,[m]\{I\}}{Mr}$$

Modal participation factors for the plane frame are $\Gamma_r = \begin{bmatrix} -14.40 \\ 4.30 \\ 1.95 \\ -0.68 \end{bmatrix}$

$$P_{eff}(t) = (-\ddot{x}_g(t)\,[\emptyset]^T[m]\{I\}) = \begin{bmatrix} -14.40 \\ 4.30 \\ 1.95 \\ -0.68 \end{bmatrix}(-\ddot{x}_g(t))$$

Displacement Response in Physical Coordinates

The uncoupled equations in the normal coordinates are,

$$\ddot{q}_1 + 3.7975\,\dot{q}_1 + 1442\,q_1 = 14.40\,\ddot{x}_g(t)$$

$$\ddot{q}_2 + 10.815\,\dot{q}_2 + 11698\,q_2 = -4.3\,\ddot{x}_g(t)$$

$$\ddot{q}_3 + 16.1947\,\dot{q}_3 + 26227\,q_3 = -1.95\,\ddot{x}_g(t)$$

$$\ddot{q}_4 + 19.1621\,\dot{q}_4 + 36719\,q_4 = 0.68\,\ddot{x}_g(t)$$

The displacement response q_r is now evaluated using the Average Acceleration Method. The displacement response in physical coordinates {x} is calculated from the transformation expression:

$$\{x(t)\} = \sum_{r=1}^{n}\{\emptyset\}_r\,q_r(t)$$

$$\{x(t)\} = \{\emptyset\}_1 q_1(t) + \{\emptyset\}_2 q_2(t) + \{\emptyset\}_3 q_3(t) + \{\emptyset\}_4 q_4(t)$$

$$= \begin{Bmatrix} -0.0328 \\ -0.0608 \\ -0.0798 \\ -0.0872 \end{Bmatrix} q_1(t) + \begin{Bmatrix} 0.0795 \\ 0.0644 \\ -0.0273 \\ -0.0865 \end{Bmatrix} q_2(t) + \begin{Bmatrix} 0.0808 \\ -0.0540 \\ -0.0448 \\ 0.0839 \end{Bmatrix} q_3(t) + \begin{Bmatrix} -0.0397 \\ 0.0690 \\ -0.0799 \\ 0.0696 \end{Bmatrix} q_4(t)$$

$$= (-0.0328)\,q_1(t) + (0.0795)\,q_2(t) + (0.0808)\,q_3(t) + (-0.0397)\,q_4(t)$$
$$(-0.0608)\,q_1(t) + (0.0644)\,q_2(t) + (-0.0540)\,q_3(t) + (0.0690)\,q_4(t)$$
$$(-0.0798)\,q_1(t) + (-0.0273)\,q_2(t) + (-0.0448)\,q_3(t) + (-0.0799)\,q_4(t)$$
$$(-0.0872)\,q_1(t) + (-0.0865)\,q_2(t) + (0.0839)\,q_3(t) + (0.0696)\,q_4(t)$$

The numerical method used is the Average Acceleration Method to arrive at the values of q_1, q_2, q_3 and q_4 i.e. the normal coordinates. Thereafter the above equations are solved to get the values of the physical displacement responses (x_1, x_2, x_3 and x_4 in meters) of the four storeys as time history plots over the entire 30 seconds duration. These pertain to bare frame displacements that have only the natural damping ratio of 2% and there is no supplemental damping incorporated. The response of masses at various floor levels in the physical coordinates {x} are obtained as shown in Figs. 7 (a) to 7 (d).

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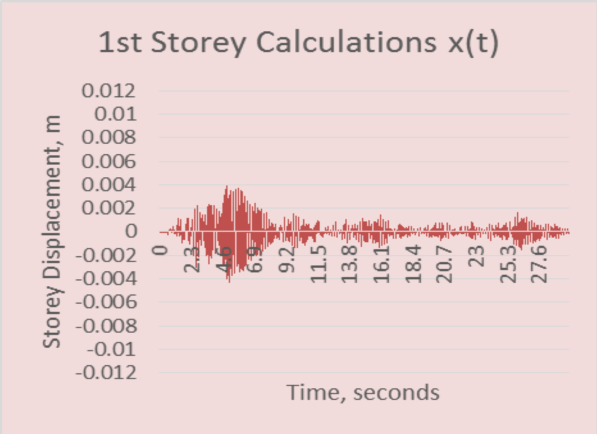
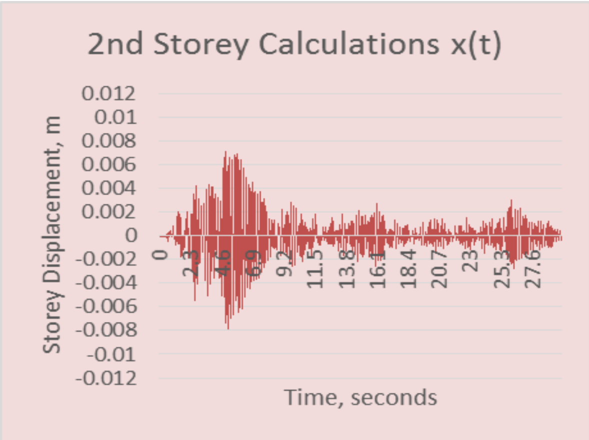


(a)



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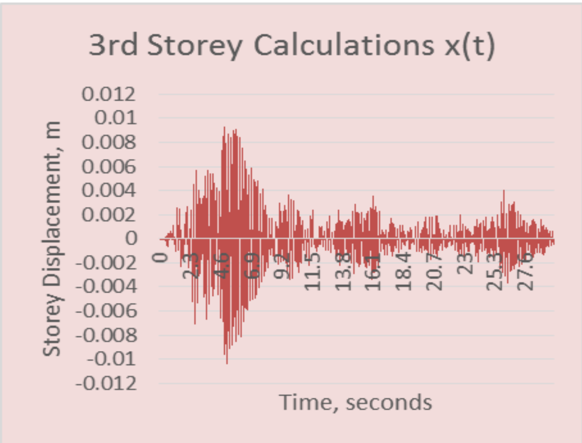

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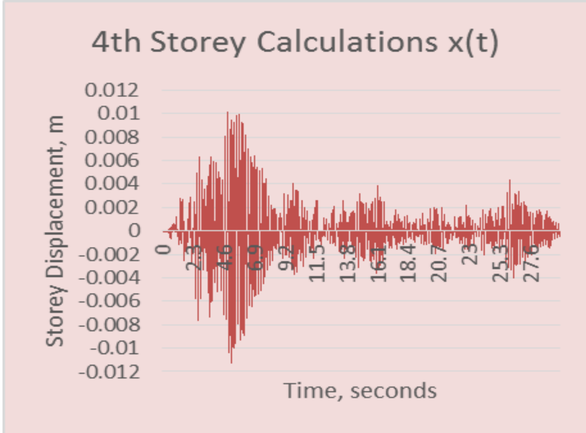


(d)

Fig. 7 Displacement response history in physical coordinates for the four storey frame: Bare Frame (without supplemental damping)

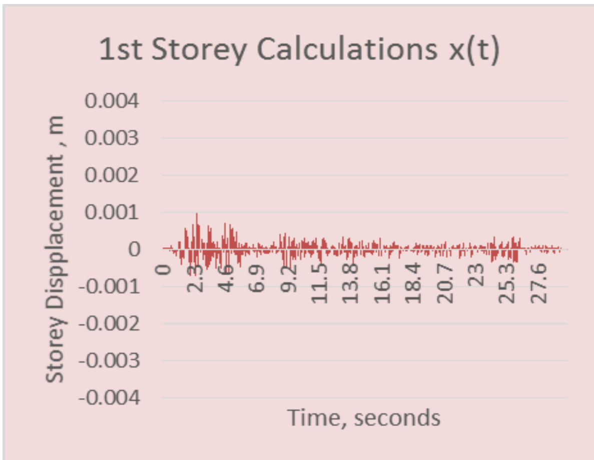
Displacement Response after incorporating Supplemental Damping

In order to reduce the response of the bare frame we incorporate supplemental damping in the form of linear and nonlinear FVDs in the frame and then analyze the effect on the structural response. The supplemental damping ratio of 30% is taken for both, linear and nonlinear damping. The alpha value (nonlinearity) taken is 0.5. This supplemental damping is taken to be present in each storey i.e. each storey has a supplemental damping ratio of 30%.

The effect of supplemental damping is analyzed in two steps, i.e. first we incorporate linear damping and obtain the individual storey displacement response history in physical coordinates and then, in step two, nonlinear damping is incorporated and similarly we obtain the individual storey displacement response history in physical coordinates. A comparison is then carried out to draw conclusions.

1) *Response using linear damping*

The displacement response history obtained for the same frame (all the four storeys) through the Average Acceleration Method using linear FVD is as reflected in the plots as under:



(a)

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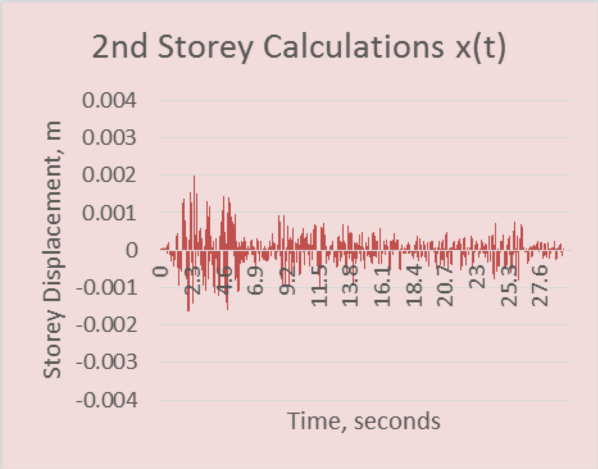

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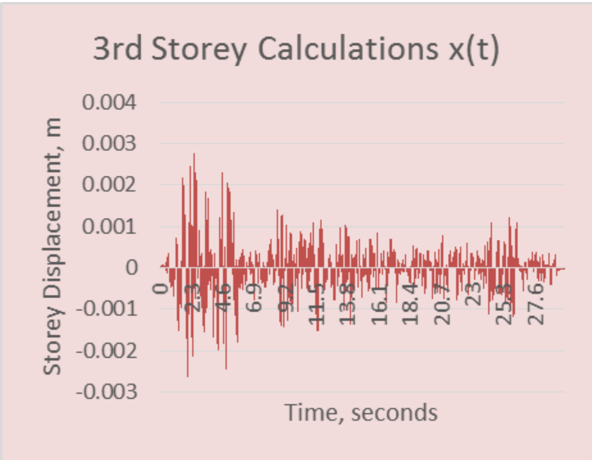

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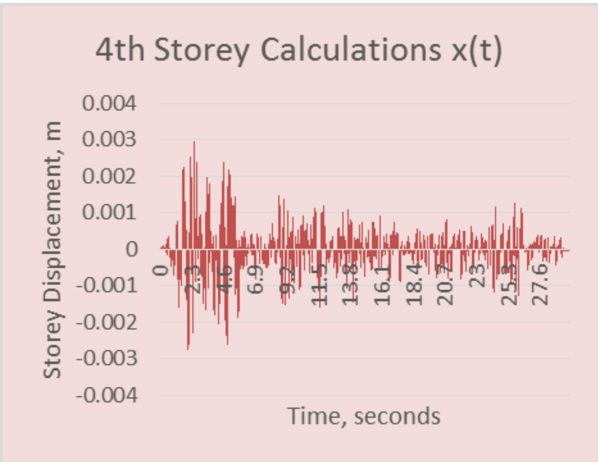
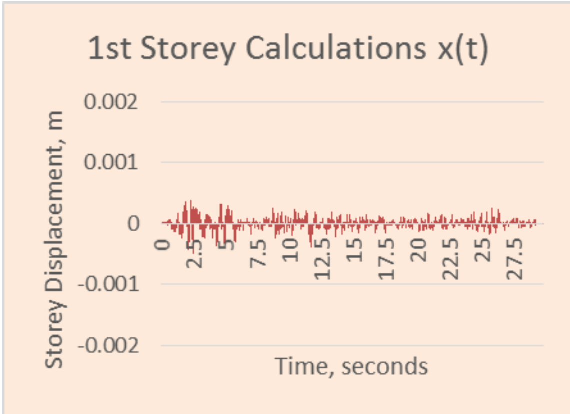


(d)

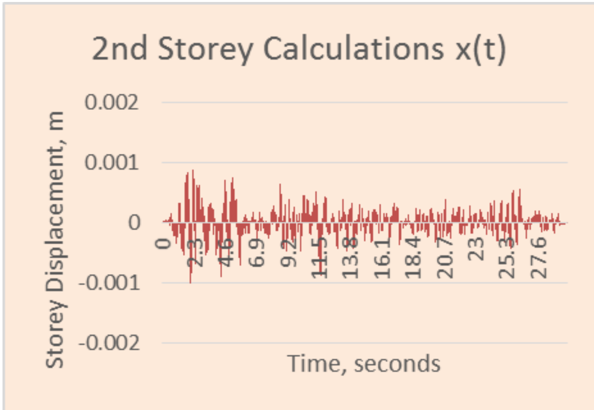
Fig. 8 Displacement Response History in physical coordinates for the Four Storey Frame: 30% Supplemental Damping (Linear)

2) Response using nonlinear damping

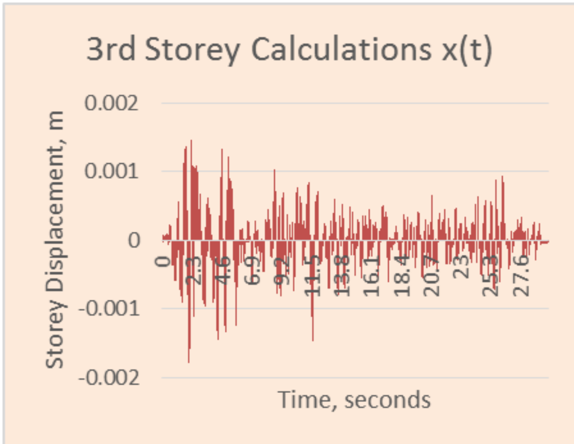
In the second step, the time history plots using nonlinear supplemental fluid viscous damping as mentioned above are obtained (Figs. 9 (a) to 9 (d)).



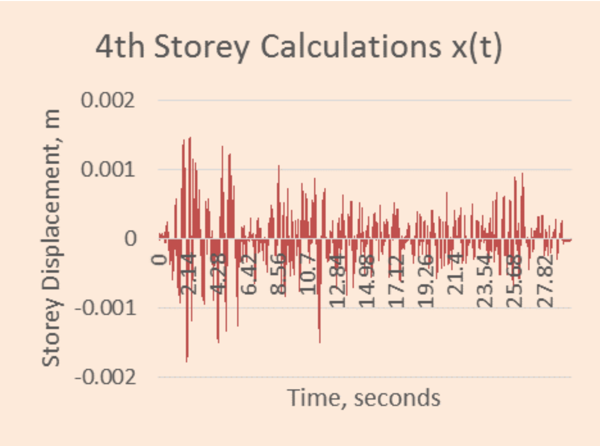
(a)



(b)



(c)



(d)

Fig. 9 Displacement Response History in physical coordinates for the Four Storey Frame: 30% Supplemental Damping (Nonlinear)

VI. CRITICAL EVALUATION

Based on the analysis of the example MDOF system, what stands out is that there is consistency in the expected results on how these systems respond to external excitation. Also there is considerable similarity in the change in the behavior once subjected to supplemental damping by means of incorporating an EDD. The results of important findings duly summarized are listed out below along with the concise discussions.

1) *Effect on Storey Drifts* (based on response spectrum method)

It is observed from Table 1 that as the supplemental damping in the given MDOF system increases, there is a pronounced reduction in the respective storey drifts. The reduction in the storey drift at the level of the four storey is about 20% for a 5% supplemental damping ratio compared to a bare frame with only its natural damping, while the reduction is about 47% and 67% for 15% and 30% supplemental damping ratios. Here, it could also be inferred that a reduction in respective storey drifts is implicative of reduction in storey displacements. The same is corroborated in the succeeding findings.

2) *Effect on Storey Displacements (based on mode superposition method)*

It is observed from Figs. 7, 8 and 9 that the maximum displacement in the bare frame (Figs. 7 (a) to (d)) for the first storey is 0.00432m (i.e. 4.32mm) while for the second, third and fourth storeys the physical displacements are 0.00714m (i.e. 7.14mm), 0.00931m (i.e. 9.31mm) and 0.010172m (i.e. 10.17mm), respectively. The displacements for the bare frame, as expected are increasing from the lowest storey i.e. the first storey to the topmost storey i.e. the fourth storey, progressively.

Also, from Figs. 8(a) to (d), once linear supplemental damping is introduced into the system, it can be clearly seen that there has been a significant reduction in the displacement response in each of the four storeys, i.e. the first storey displacement is observed to reduce from 0.00432m (i.e. 4.32mm) to 0.000985m (i.e. 1mm), for the second storey, the reduction in displacement is from 0.00714m (i.e. 7.14mm) to 0.00198m (i.e. 1.98mm), while for the third and fourth storeys it's from 0.00931m (i.e. 9.31mm) and 0.010172m (i.e. 10.17mm), to 0.00278m (i.e. 2.78mm) and 0.00297m (i.e. 2.97mm), respectively.

Again, it is evident from the plots in Figs. 9 (a) to (d), that there is further reduction in the storey displacements when there is nonlinearity in the damping, even at a constant supplemental damping of 30%. The fourth storey maximum displacement has now reduced from a maximum of 0.01017m (i.e. 10.17mm) to barely 0.001476m (i.e. 1.47mm). Table 2, summarizes the comparative results (including the response of the other storeys to nonlinear supplemental damping).

TABLE 2: COMPARATIVE RESULTS OF STOREY DISPLACEMENTS USING MODE SUPERPOSITION METHOD

	Maximum Displacement (*x* in meters)			
Supplemental Damping (ξ_{sd})	1st Storey	2nd Storey	3rd Storey	4th Storey
Bare Frame (only inherent damping, ξ =5%)	0.00432	0.00714	0.00931	0.01017
30% (Linear damping)	0.000985 (77.20% reduction)	0.00198 (72.20% reduction)	0.00278 (70.14% reduction)	0.00297 (67.20% reduction)
30% (Non-linear damping)	0.00037 (91.4% reduction)	0.00089 (87.53% reduction)	0.00146 (84.31% reduction)	0.00147 (85.54% reduction)

VII. CONCLUSIONS

The objectives that were set for the current investigation were successfully met and the various associated parameters were successfully evaluated. Preliminary selection of linear and nonlinear damper parameters too can be suitably implemented through the design charts that were developed for seismic control of SDOF systems (in terms of α i.e. the nonlinearity parameter and c_α in the form of the supplemental damping, ξ_{sd}). Following conclusions can be drawn from the foregoing discussions for supplemental damping in both, SDOF and MDOF systems.

1) Fluid viscous dampers have a high energy dissipation capacity. The dynamic characteristics of a nonlinear FVD can be described by its energy dissipation capacity, represented by supplemental damping ratio ξ_{sd}, and its nonlinearity by a parameter α, which defines the hysteresis loop shape. The much lesser is the value of velocity exponent, greater is the energy dissipation capacity of the damper.
2) Damper nonlinearity essentially has no influence on the peak responses—deformation u_0, relative velocity $\dot{u}_0$, and total acceleration $\ddot{u}_0$t of systems.
3) Nonlinear FVDs are advantageous because they achieve essentially the same reduction in system responses but with a reduced damper force (within specific time period range).
4) The design values of structural deformation and forces for a system (period T_n and inherent damping ξ) with nonlinear FVDs can be estimated directly from the design spectrum for the period T_n and total damping $\xi + \xi_{sd}$.

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Specific Conclusions from the Example MDOF systems

1) The response behavior of the example MDOF systems (first one i.e. 4 DOF system evaluated using the *response spectrum method* and the second i.e. 4 DOF system using *mode superposition method*) provide a complete overview and thus validate the various inferences drawn from the above evaluation, as to the behavior of these structures when subjected to seismic excitation. They go onto establishing the fact that the responses of a bare frame in terms of storey displacements or storey drifts have a significant reduction once supplemental damping is introduced (in our case, both types of fluid viscous dampers i.e. linear and nonlinear), as has been observed from the time histories for the four-storey frame.
2) The comparison between the storey displacements for the four storey frame using mode superposition method and storey drifts by response spectrum method helped in validating that there is consistency in the design charts developed for El Centro ground acceleration and the results are comparable.
3) Introduction of nonlinearity in a damper further contributes to reduction in the maximum storey displacements, as compared to the corresponding reduction due to linear damping. This has a direct impact on the structural control through energy dissipation, thus mitigating any adverse effect on the primary structural members.

REFERENCES

[1] Constantinou, M. C., Soong, T. T., and Dargush, G. F. (1998). "Passive Energy Dissipation Systems for Structural Design and Retrofit, MCEER, University at Buffalo, Buffalo, NY".
[2] Sinha, R., and Goyal, A. (2005). "Vibration Control of Structures and Equipments, Lecture Notes for Short-Term Course on Dynamics and Control of Machine Vibrations", Bombay, India.
[3] Chopra A. K., (1995). "Dynamics of structures theory and applications to earthquake engineering", Prentice- Hall, Englewood Cliffs, N.J.
[4] "Seismic Design of Structures with Viscous Dampers" by Jenn-Shin Hwang.
[5] "Earthquake Response of Elastic SDF Systems with Non-Linear Fluid Viscous Dampers" by Wen Hsiung Lin and Anil K. Chopra, December 2001.
[6] "Earthquake Resistant Design of Structures" by S. K. Duggal.
[7] Agrawal, P., and Shrikhande, M.(2012). "Earthquake Resistant Design of Structures", Prentice-Hall of India Learning Private limited, New Delhi.
[8] Journal of Engineering Mechanics, September 1997: "Structural Control : Past, Present and Future" by G W Housner, L A Bergman and A. G. Chassiakos, R. O. Claus, S. F. Masri, R E Skelton, T. T. Soong, B. F. Spencer and J. T. P. Yao.
[9] "Structural Dynamics – Theory and Applications" by Joseph W. Tedesco, William G. McDougal and C. Allen Ross.
[10] "Basics of Structural Dynamics and Aseismic Design" by S. R. Damodarasamy and S. Kavitha.
[11] "Structural Dynamics for Structural Engineers" by Gary C. Hart and Kevin Wong.
[12] "Getting Started with MATLAB 7" by Rudra Pratap.
[13] "Solving Vibration Problems using MATLAB" by Rao V. Dukkipati.
[14] "MATLAB and Its Applications in Engineering" by Raj Kumar Bansal, Ashok Kumar Goel and Manoj Kumar Sharma.
[15] IS-1893(Part-I):2016 - Criteria for Earthquake Resistance Design of Structures.
[16] IS-13920: 1993 - Ductile detailing of reinforced concrete structures subjected to Seismic forces – Code of practice.