Convective Instability Of Strongly Magnetized Ferrofluids

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Abstract

The application of differential equations towards stability analysis of ferrofluids is analysed. Strongly magnetized ferrofluids are considered. Weekly non- linear analysis is carried out. The non-dimensional thermal Rayleigh number R_a and magnetic Rayleigh number R_m are analysed with allowable range of parameters. Also using finite amplitude technique.

Key words: weakly nonlinear equation, Stability analysis, Strongly magnetized ferrofluids and Heat transfer also finite amplitude technique.

1. Review of literature

There are many fascinating materials which have been attracting scientists and researchers for their extraordinary physical properties and technical usage. Ferrofluid is one of such smart materials not available in nature freely, but is to be synthesized by different processes. Ferrofluid is a liquid which becomes strongly magnetized in the presence of magnetic field. There are at least three components required to prepare ferrofluid i.e. magnetic particles of colloidal size, carrier liquid and stabilizer (surfactant).They are stable suspensions of colloidal single domain ferromagnetic particles of the order of 10nm in suitable non-magnetic carrier liquid (Bibik and Lavrov 1965;Rosensweig et al. 1965; Rosensweig 1985; Berkovsky et al. 1993, Odenbach 2009).

If the size of permanently magnetized nano-particles will be less than 1- 2 nm, the magnetic properties will disappear and colloidal motion increases withincreasing the size of the particle. The colloidal particles, typically made from magnetite (Fe3O4), are coated with surfactants to avoid their agglomeration under Vander Waals attraction forces and dipole-dipole interaction among them. The presence of surfactant helps to maintain proper spacing between the particles to provide colloidal stability. A rich set of flow patterns and instabilities in the presence of DC, AC and rotating magnetic fields is exhibited by ferrofluids which are opaque to visible light (Cowley and Rosensweig 1967, Rosensweig 1997). Ferrofluids were first discovered at National Aeronautics and Space Administration (NASA) Research Centre in mid 1960‟s. The scientists at NASA found that they could make to flow this amazing ferrofluid by varying the external magnetic field. After the discovery of ferrofluid, not only original publications in journals and conferences have been released, but some textbooks like “Ferrohydrodymics” by Rosensweig (1985), “Magnetic Fluids: Engineering Applications”by Berkovsky *et al.* (1993), “Magnetic Fluids and Applications Handbook” by Berkovsky and Bastovoy (1996), “Magnetic Fluids” by Blums *et al.* (1997), “Magnetoviscous Effects in Ferrofluids” by Odenbach (2002) etc. also have been published in this area to supplement the basis for its engineering applications.

2 . Introduction

Heat transfer through ferrofluids subjected to strong magnetic fields, has notable application in technology of generator and motors. The ferrofluids have a distinct advantage as it can be effectively used as a coolant, used in heat transfer in armature of generators and motors, which rotate with constant angular velocity.

Convective instability of magnetized ferrofluids is strongly affected by the magneto and thermophoretic transfer of magnetic grains. The magnetic susceptibility of ferrofluids lies between paramagnetic and ferromagnetic material, thermal convection in the case of strongly magnetized ferrofluids in a topic of current technical importance.

Heat transfer though ferrofluids, subjected to very high magnetic fields, finds remarkable applications in transformer technology. In an attempt to replace solid core by liquid core, the Ferrofluids have an added advantage as it can also be effectively used as a heat transformer.

The method of formation of Ferrofluids was evolved in the early of mid -1960s.Due to the availability of colloidal magnetic fluids (ferrofluids) many uses of these fascinating liquids have been identified, which are concerned with the remove positioning and control of the magnetic fluid using magnetic force fields.

Ferrofluids are highly applied in lubrication, printing, and vacuum technology Schlichting [1] has investigated the boundary layer theory. Newringer and Rosensweig [3] have investigated the Magnetic fluids. Convective instability of uniform vertical magnetic field has been considered by Finlayson [4] Verma and Singh [5] studied the Magnetic fluid flow through porous annulus. The novel zero-leakage rotary-shaft seals are used in computer disk drives (Bailey [6]). This monograph reviews several applications of heat transfer through ferrofluids and one such phenomenon is enhanced convective cooling having a temperature-dependent magnetic moment due to magnetization of the fluid. This magnetization, in general, is a function of the magnetic field, temperature and density of the fluid. Any variation of these quantities can induce a change of body force distribution in the fluid. This leads to convection is ferrofluids. Is the presence of magnetic field gradient. This mechanism is known as Ferro convection, which is similar to Benard convection (Chadrasekarhar [2]). Sunil and Mahajan studied the non-linear stability analysis of magnetized ferrofluids, with internal angular momentum, heated from bellow saturating a porous medium of high permeability via generalized energy method using Brinkman model. Attia [11] studied the effect of the porosity of the medium on velocity components and temperature for the steady flow and heat transfer. Blennerhassett et al. [7] have studied the heat transfer through strongly magnetized ferrofluids. Here analysed the linear and weakly nonlinear thermo convective stability of a ferrofluids, confined between rigid horizontal plates at different temperatures and subjected to a strong uniform external magneto static field in the vertical direction. Sekar and Vaidyanathan [8] investigated the convective instability of a magnetized ferrofluids in a rotating porous medium. Differential equations are effectively used in the stability analysis of ferrofluids. Shliomis [9] have investigated the ferrofluids as thermal ratchets. Sunil and AmitMahajan [10] A nonlinear stability analysis in a double-diffusive magnetized ferrofluids layer saturating a porous medium

3 . Mathematical Formulation

An infinitely spread thin layer of ferrofluids contained between two rigid boundaries heated from below is studied. The fluid is assumed to satisfy the Oberbeck-Boussinesq approximation. The magnetisation **M** of the ferrofluids is assumed to be parallel to the local magnetic field **H**. The present analysis deals with the special case of very strong magnetic fields

The governing equations are:

$$\nabla . q = 0 \qquad (1)$$

$$\frac{\partial q}{\partial t} + (q.\nabla)q = -\frac{\nabla p}{\rho_0} + \frac{\rho g}{\rho_0} + \nabla.(BH) + \nu \nabla^2 q \qquad (2)$$

where p is the pressure which include the magnetic contribution centripetal acceleration, $q=(u,v,w)$ the velocity, $g=(o,o,-g)$ the acceleration due to gravity, **B** the magnetic induction, **H** the magnetic field , ρ the density, ρ_0 the density at $T=T_0$ and υ is the kinematic viscosity.

Energy equation is

$$\frac{\partial T}{\partial t}+\left(q.\nabla\right)T=\kappa\nabla^2 T \qquad (3)$$

Usual magnetic equation of state and other related equation are considered where T is the temperature and κ is the thermal conductivity where M is the magnetization, M_0 is the constant mean value of the magnetisation

For strong magnetic applied field in the Ferrofluids,

$M=(\overline{M}+K\beta d\theta)\hat{k}$ where K is Pyromagnetic coefficient (4)

When the fluid is assumed to be non-conductive

$$\nabla\times H=0 \qquad (5)$$

$H=\overline{H}+K\,\beta\,d\,\nabla\Phi$ (For magnetic field in the Ferro fluids) (6)

where Φ is the magnetic potential.

Following stability analysis procedure. The non dimensional variable are $R_T=\dfrac{g\alpha\beta d^4}{kv}$

is the conventional thermal Rayleigh number, and $R_M=\dfrac{\mu_0 K^2\beta^2 d^4}{kv\rho_0}$ is the magnetic Rayleigh number and $p_r=\dfrac{v}{k}$ is the Prandtl number.

The non-dimensional governing equations are

$$\nabla.q=0 \qquad (7)$$

$$pr^{-1}\left[\frac{\partial}{\partial t}+(q.\nabla)\right]q+R_M\theta\nabla\Phi_Z$$
$$=\nabla^2 q+(R_a+R_M)\theta\hat{k}-R_M\Phi_Z\hat{k}-\nabla p \qquad (8)$$

$$\left(\frac{\partial}{\partial t}+\left(q.\nabla\right)\theta\right)=\nabla^2\theta+w \qquad (9)$$

$$\nabla^2\Phi-\theta_z=0 \qquad (10)$$

The linear stability analysis is carried out

f= f(z)exp($iax+\sigma t$) (11)

Using normal mode technique, 0n eliminating pressure p, the non dimensional governing equations are Ψ is stream function, Θ is the temperature and Φ is the magnetic potential are given by

$$-pr^{-1}\sigma(D^2-a^2)\Psi+(D^2-a^2)^2\Psi$$
$$-iaR_T\left(\frac{1+R_T}{S}\right)\Theta+ia\left(\frac{R_T{}^2}{S}\right)D\Phi=0 \qquad (12)$$

$$-\sigma\Theta-ia\Psi+(D^2-a^2)\Theta=0 \qquad (13)$$

$$-D\Theta+(D^2-a^2)\Phi=0 \qquad (14)$$

The boundary condition for rigid, conducting boundaries on Ψ and θ are given by

$\Psi=D\Psi=\Theta=0$, for z =±1/2 (15)

$$(D\pm a)\Phi_{z=\pm\frac{1}{2}}=0 \qquad (16)$$

Simplifying the Rayleigh number can be easily obtained as

$$R_T=\frac{-hS+h\left[h^2S^2+4Sh^4\right]^{1/2}}{2a^2} \qquad (17)$$

For weakly non linear system, finite amplitude technique adopted by Maluku’s and Veronis(1958) has been used retaining only first order terms.When layer of fluid is heated from below and cooled from above, a cellular regime of steady state convection is set up at values of the Rayleigh number exceeding the critical value. A method is presented here to determine the form and amplitude convection. The nonlinear equation describing the field of motion and temperature are expanded in a square of homogeneous linear equations depends upon the solution of the linear stability problem. We find that there are infinite number steady state finite amplitude solutions.

Dynamical variables *f(x,z,t)* are expanded in the following manner.

$$f(x,z,t) = \varepsilon^{1/2}\left[f_{11}(z,\tau)E + cc\right] + \varepsilon\left\{f_{02}(z,\tau) + \left[f_{22}(z,\tau)E^2 + cc\right]\right\} + \varepsilon^{3/2}\left[f_{13}(z,\tau)E + cc\right] \quad (18)$$

Nusselt number is obtained as

$$Nu - 1 = \left\{\frac{(R_T - R_{Tc})}{K_R}\right\}\left(D\Theta_{02}\right)_{z=\frac{1}{2}} \quad (19)$$

Nu-1 measures the ratio of the heat transfer by convection to that by conduction whereas the Nusselt number measure the ratio of the total heat transfer across a horizontal plane to the heat transfer by conduction alone.

4 . Conclusion

The system heated from below favours convective heat transport. The heat transport decreases when the system tends to loose magnetic character. For ferrofluids system heated from above, conductive heat transport is favoured when it tends to loose its magnetic character.

Thus differential equation has greater role to play right from forming governing equations has obtaining expression for Nu-1, differential equations are highly applicable in heat transfer problems

5. References

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List of symbols

H - Magnetic field
M - Magnetization of the ferrofluids
B - Magnetic induction
P - Pressure
q= (u,v,w) the velocity
g= *(o,o,-g)* the acceleration due to gravity
ρ_0 Density at T=T_0 ^{0}
υ Kinematic viscosity·
T Temperature
κ Thermal conductivity
M_0 Constant mean value of the magnetisation
K Pyromagnetic coefficient
Φ Magnetic potential.
R_T Conventional thermal Rayleigh number
R_M Magnetic Rayleigh number
p_r Prandtl number.
T Temperature
t- Time
Ψ Stream function
Θ Amplitude of perturbed temperature gradient
α Thermal expansion coefficient
k Unit vector in z direction
z Amplitude of vertical component of vorticity
d- Thickens of the layer
D- Differential operator
β - Temperature gradient
T_0 Temperature of the lower boundary
a- Over all horizontal wave number
H_0 –Imposed uniform vertical magnetic field
Nu- nessult number