Control Design for Grid-Connected Inverters via Lyapunov Approach

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Abstract

This paper develops a Lyapunov approach to design for grid-connected inverter. The control objective is to create the output current to track a reference current proportional with the fundamental harmonic of the grid voltage. The grid voltage is described as the outputs of an autonomous linear oscillatory system, and its harmonics can be estimated via an observer. A state-space description for the whole system is obtained from the state of the inverter circuit and that of the oscillatory system for the grid voltage. Based on the state-space description, a Lyapunov approach is applied to design a state-feedback controller with minimal tracking error. The design problem is cast into an optimization problem, which can be effectively solved with linear matrix inequality (LMI) toolbox in Matlab. The effectiveness of the Lyapunov approach is validated via SimPower simulation.

1. Introduction

As the demand for power is increasing significantly, renewable energy sources have recently received a lot of attention as an alternative way of generating directly electricity. Using renewable energy systems can eliminate harmful emissions from polluting the environment while also offering inexhaustible resources of primary energy. There are many sources of renewable energy, such as solar energy, wind turbines, water turbines, and geothermal energy. Most of the renewable energy technologies produce direct current (DC) power and hence inverters are required to convert the DC to the alternating current (AC) power.

Stand-alone (island) and grid-connected are two kinds of inverters. These two types have several similarities, but are different in terms of control function. A stand-alone inverter is used in off-grid applications. The generated power from renewable energy is delivered to loads, or can be stored in batteries. That systems require complexity and high maintenance, such that rechargeable batteries. That also increases the size and cost for the system. Grid-connected inverters overcome this limitation. For grid-connected inverters, they must follow the voltage and the frequency characteristics of the utility generated power presented on the distribution line. We consider grid connected inverters in this paper, as its main advantage is that no battery is required for storing the energy from renewable sources, which reduces the size and cost of the system. Moreover, it is easier to create a portable inverter due to the compact size of the system.

Investigations of different configurations and control methods for grid-connected inverters are being focused in recent years. A comprehensive review of single-phase grid-connected inverters [1] has covered some of the standards that inverters for grid applications must be fulfilled. It also provided a classification of the inverters regarding the stage (single, dual stage, and multi-string inverter), transformers and types of Interconnections, and types of grid interfaces (line-commutated current-source inverter and self-commutated voltage-source inverter).

Reducing the harmonics is a main problem which needs much effort in power inverters these days. The IEEE 929 standard stated that the Total Harmonic Distortion (TDH) of voltage and current should be lower than 5% in normal operation. Harmonics are not desirable because they cause overheating, decreased volt-ampere capacity, increased losses, distorted voltage and current waveform, etc. Several researches have been proposed to reduce the voltage THD of inverters. For example, the repetitive control theory has been successfully applied to PWM inverters [2]-[9], active filters [10]-[12], dead-beat control [13], [14], to reduce THD. Harmonic droop control technique [15] is also presented. Repetitive control has an excellent...
ability in eliminating periodic disturbances, however, in practical; this technique is limited in slow dynamics, poor tracking accuracy, and poor performance to non-periodic disturbances. Dead-beat and sliding-mode controls have excellent dynamic performance in control of output voltage, but these techniques suffer from complexity, sensitivity, and steady-state errors. In order to eliminate the current distortion, some current control methods are proposed, such that proportional resonant controller and multi-resonant controllers in [16], active power filters in [12], [17].

A promising control technique in grid-connected inverter is output current tracking. The inverter’s current polarity must be controlled to match the voltage polarity of the grid. Various synchronization methods are summarized in [18], [19] and [20]. The current hysteretic comparison control method, timing control of current instantaneous comparison method and the triangle wave comparison control method of timing tracking current are proposed in [21]. In [22], algorithms of current decoupling are derived for performing the reactive power control of grid-connected inverter. Through zero-crossing detecting circuit in [23] and [24], the inverter is controlled so as to generate the output current in phase with the grid voltage. A current control employing internal model principle in [25] is proposed to suppress the harmonic currents injected into the grid. Although most existing controllers give satisfactory results, the theory behind the dynamics and performances is not clearly described.

For a more clearly understanding of the whole system’s dynamics, a systematic state space approach is developed in this paper. The design problem will be proposed using advanced nonlinear control system theory and linear matrix inequality (LMI) optimization technique, as in [26]-[30]. The problem of tracking current error with minimal tracking error will be casted into Lyapunov framework, which ensures stability and the total harmonics distortion (THD) requirements.

The paper is organized as follows. Section II describes the open loop description for the inverter circuit; following is the state-space description for the grid voltage and an observer, and control objective. Section III reviews the main tool to be used in this paper -Lyapunov approach to evaluate the tracking error. Section IV casts the problem of tracking error into Lyapunov framework and converts the design problem into the LMI optimization. Section V uses SimPower in MATLAB to simulate and verify the results. Section VI concludes the paper.

2. State space description and control objective

2.1. Open-loop description for the circuit

![Figure 1. Equivalent circuit for a grid-connected inverter](image_url)

Let the state of the circuit as $x_c = \begin{bmatrix} v_c \\ i_1 \\ i_g \end{bmatrix}$

Define

$$A = \begin{bmatrix} 0 & \frac{1}{C_f} & -\frac{1}{C_f} \\ -\frac{1}{L_f} & \frac{-R_f + R_d}{L_f} & \frac{R_d}{L_f} \\ \frac{1}{L_g} & \frac{-R_g}{L_g} & -\frac{R_g}{L_g} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{L_f} \\ 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 \\ \frac{1}{L_g} \\ 0 \end{bmatrix}$$

The circuit can be described as

$$\dot{x}_c = Ax_c + Bu + Ev_g$$

(1)

where $u$ is the control input and $v_g$ can be considered as an external disturbance.

2.2. State-space description for the grid voltage and an observer

The grid voltage is periodic with frequency 50Hz or 60Hz. The frequency may subject to some perturbation but can be measured. Let the fundamental frequency be $f_0$ (rad/second). According to [31], the grid voltage $v_g(t)$ can be written as:

$$v_g(t) = \sum_{k=1}^{\infty} b_k \sin(kf_0t + \phi_k)$$

(2)

The magnitude $b_k$ and the phase $\phi_k$ for each
harmonic can be evaluated with a bank of resonant filters [16], [32], [33], or a composite observer [34], [35]. The resonant filters are described with transfer functions, while the composite observers are described via state-space equations. They are all based on the internal model principle in [36], [37]. Here we adopt the main ideas in [35] to describe \( v_g \) via state space equations and then construct an observer to estimate the state. The advantage of using the state space description is that the dynamics of the whole system can be simply described by stacking up the state equation for \( v_g \) and that for the circuit, i.e., (1). The resulting state equation for the whole system makes it very convenient to study the interaction between the grid voltage and the dynamics of LCL filter. Furthermore, it facilitates analysis of system performance via advanced tools developed in recent years, such as the Lyapunov approach and the linear-matrix-inequality (LMI) based optimization.

Let
\[
S_0 = \begin{bmatrix} 0 & -\beta_0 \\ \beta_0 & 0 \end{bmatrix}
\]
Define
\[
S_g = \begin{bmatrix} S_0 & 0 & \cdots & 0 \\ 0 & 2S_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (N-1)S_0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}
\]
where all 0's in the above matrix are 2 by 2 blocks. Also define
\[
\Gamma_g = \begin{bmatrix} 1 & 0 & 1 \cdots & 1 & 0 \end{bmatrix}
\]
Then \( v_g \) is the output of the following autonomous linear oscillatory system:
\[
\dot{w}_g = S_gw_g, \quad v_g = \Gamma_g w_g \quad (3)
\]
where \( w_g \in \mathbb{R}^{2N} \). A feature of the matrix \( S_g \) is that \( S_0 + 2S_0 = 0 \). Because of this, we have \( w_g(t) = w_g(0) \) for all \( t \). This kind of state-space descriptions for periodic signals has been widely used in the output regulation literature for tracking periodic references or rejection of periodic disturbances [26]-[28], [36], where the linear system (3) was referred to as the exogenous system, or simply, exosystem.

The state \( w_g \in \mathbb{R}^{2N} \) can be decomposed as
\[
w_g = \begin{bmatrix} w_{g1}^T \\ w_{g2}^T \\ \vdots \\ w_{gN}^T \end{bmatrix}
\]
where \( w_{gi} \in \mathbb{R}^2 \), \( k = 1, ..., N \). By the structure of \( \Gamma_g \), we have \( v_{gk}(t) = \sum_{i=1}^{N} [1 \ 0] w_{gi}(t) \). The \( k \)th harmonic is exactly \( [1 \ 0] w_{gk}(t) = \beta_k \sin(k \omega t + \phi_k) \). Let \( C_k \) be a \( 1 \times 2N \) row vector whose first element is one and the rest are all zero. Then the first harmonic, denoted \( v_{gi} \), is \( v_{gi}(t) = C_iw_{gi}(t) \). It is easy to verify that the system (3), in particular, the pair \((\Gamma_g, S_g)\), is observable. Thus an observer can be constructed to estimate the state \( w_g \), and hence \((\beta_k, \phi_k)\) for all \( k \). Let the state of the observer be \( \hat{w}_g \). We have
\[
\dot{\hat{w}}_g = S_g\hat{w}_g - L(\Gamma_g w_g - v_g) \quad (4)
\]
where \( L \) is the observer gain which can be designed via various approaches.

If the frequency \( \beta_0 \) for the observer is exactly the same as the frequency of the grid voltage, then the observer error \( w_g(t) - \hat{w}_g(t) \) will go to 0 asymptotically and we can use the estimated state \( \hat{w}_g \) for various purposes. If the grid frequency is subject to perturbation, this frequency can be measured on line and used for the observer. Due to robustness, the same gain \( L \) should be stabilizing for a certain range of \( \beta_0 \) and \( S_g(\beta_0) \). The discrete-time version of the observer is usually used in practice. More details can be found in [35]. With the estimated state \( \hat{w}_g \), the first harmonic of \( v_g \) is estimated as \( C_iw_i \).

2.3. The control objective
Ideally, we would like to feed the grid a sinusoidal current \( i_g \), which is in phase with \( v_g \). The magnitude of \( i_g \) can be varied depending on the need of the grid and the local energy storage devices. This objective can be stated as a reference tracking problem where the reference for the grid current is given as
\[
i_{g,ref}(t) = r v_g(t) \quad (4)
\]
where \( r \) is a positive number that can be changed. It would be more convenient to introduce another exosystem:
\[
w_i = S_0 w_i, \quad i_{g,ref} = [1 \ 0]w_i \quad (5)
\]
where \( w_i \in \mathbb{R}^2 \). The condition (4) can be satisfied if \( w_i(0) = rw_g(0) \). Since \( S_0 w_i = w_i(t) \), we have \( w_i(t) = rw_i(t) \). There is some redundancy introducing (5). The purpose is to make it easier to handle \( r \).

The control objective is to minimize the magnitude of the tracking error at steady state.
\[
e(t) = i_g(t) - i_{g,ref}(t) \quad (6)
\]
Note that \( i_g \) can be considered as an output to the inverter system (1): \( i_g = [0 \ 0 \ 1]x \). The two exosystems (3) and (5) can be combined to obtain a \( 2(N+1) \)-order system.

Define
\[
w = [w_g \ \ w_i]
\]
and
\[
S = \begin{bmatrix} S_g & 0 \\ 0 & S_0 \end{bmatrix} \quad I_1 = [I_g \ 0 \ 0]
\]
\[
I_2 = [0 \ 0 \ 1 \ 0]
\]
Then
\[
w = Sw, \quad v_g = I_1 w, \quad i_{g,ref} = I_2 w \quad (7)
\]
This system describes all the dynamics of the grid voltage and the reference current. Combined with the
state-space description of the LCL filter, the dynamics of the whole system can be described. It is clear that the exosystem (7) evolves all by itself and is driven by its initial condition \(w(0)\). Recall that \(w\) consists of the states for all the harmonics of \(v_1\) and for \(i_{\text{ref}}\), in particular,

\[
\begin{bmatrix}
W_{g1} \\
W_{g2} \\
\vdots \\
W_{gN}
\end{bmatrix}
\]

Since \(S + S^T = 0\), we have

\[
w(t)^T w(t) = w(0)^T w(0) = \|w(0)\|^2
\]

\[
w_{gk}(t)^T w_{gk}(t) = w_{gk}(0)^T w_{gk}(0) = b_k^2
\]

\[
w_r(t)^T w_r(t) = w_r(0)^T w_r(0) = r^2 b_1^2
\]

for all \(t\). Thus \(\|w_{gk}(0)\|^2\) represents the power of the \(k\)th harmonics of \(v_g\) and \(\|w(0)\|^2\) the total power of the harmonics of \(v_g\) plus the power of the reference current.

The condition \(w_r(t) = r w_g(t)\) implies that \(i_{\text{ref}}\)'s change proportional to the first harmonic of \(v_g\). In terms of the combined state \(w\), this can be written as

\[
w_1 w_{2N+2} = w_2 w_{2N+1}
\]

This condition will be used as a constraint in an optimization problem to be formulated.

3. Evaluation of the tracking error using Lyapunov approach

3.1. Evaluation of tracking error for general exosystem

From (7), both the grid voltage and the reference current are driven by an autonomous linear system. According to [38], the state variable of an autonomous system contains all the information that determines its future behavior, it can be effectively used to correct the dynamic behavior of the whole system. In the case of the inverter circuit, the state \(w\) can be used to minimize the tracking error of the grid current (6).

The state of the whole system is a combination of the circuit state \(x\) and the exosystem state \(w\). They can be either measured or estimated via an observer, thus state feedback is feasible. If the tracking error can be effectively evaluated via a certain performance measure, the next step would be minimizing this performance measure via a certain optimization algorithm.

The Lyapunov approach developed in [26] can be applied for this kind of problems. It deals with more general systems (nonlinear, time-varying) with periodic excitations, which could be disturbance or reference. The objective is to evaluate the magnitude of certain output at steady state, which could be the tracking error.

To apply the Lyapunov approach, we first need the state space description for the whole system, which can be easily obtained by combining the state-space equation (1) for the circuit and the state-space equation (7) for the \(v_g\) and \(i_{\text{ref}}\). Since \(i_{\text{ref}} = [0 \; 0 \; 1]\), if we let \(C = [0 \; 0 \; 1 - \Gamma_2]\), then

\[
\begin{bmatrix}
\dot{x}_c \\
\dot{w}
\end{bmatrix} = \begin{bmatrix} A & E \Gamma_1 \end{bmatrix} \begin{bmatrix} x_c \\
w \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} u
\]

\[
e = C \begin{bmatrix} x_c \\
w \end{bmatrix}
\]

To reduce the tracking error, we apply a simple state feedback

\[
u = K_i x_c + K_2 w = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_c \\
w \end{bmatrix}
\]

Substitute the feedback law in (13) into (12), we have the closed-loop system

\[
\begin{bmatrix} \dot{x}_c \\
\dot{w}
\end{bmatrix} = \begin{bmatrix} A + BK_1 & E \Gamma_1 + BK_2 \end{bmatrix} \begin{bmatrix} x_c \\
w \end{bmatrix} \begin{bmatrix} S \end{bmatrix} \begin{bmatrix} x_c \\
w \end{bmatrix}
\]

\[
e = C \begin{bmatrix} x_c \\
w \end{bmatrix}
\]

As long as \(A + BK_1\) is stable, the solution for the above system will be bounded and for any initial condition, the solution will approach a steady state oscillation. Since \(A + BK_1\) is stable, the effect of the initial condition of \(x_c(0)\) will vanish asymptotically. Thus the steady state oscillation, in particular, the tracking error, depends only on the initial condition \(w(0)\). Hence, a gain from the norm of the initial condition \(\|w(0)\| = \sqrt{w(0)^T w(0)}\) to the magnitude of \(e\) at steady state can be defined. The main result of [26] was applied to estimate this gain via a quadratic Lyapunov function \(V(x_c, w) = \begin{bmatrix} x_c^T & w^T \end{bmatrix} P \begin{bmatrix} x_c \\
w \end{bmatrix}\).

Here we summarize the main result of the Lyapunov approach when applied to the linear system (14). Denote

\[
A_L(K_1, K_2) = \begin{bmatrix} A + BK_1 & E \Gamma_1 + BK_2 \end{bmatrix}\]

**Theorem:** For \(\gamma > 0\), if there exist a positive definite matrix \(P = P^T \in \mathbb{R}^{(2N+5) \times (2N+5)}\), and a number \(\eta > 0\) such that

\[
C^T \begin{bmatrix} A_L(K_1, K_2) \end{bmatrix} P + PA_L(K_1, K_2) \leq -\eta \begin{bmatrix} P - \gamma I_{2N+2} \\
0
\end{bmatrix}
\]

where \(I_{2N+2}\) is an identity matrix of dimension \(2N + 2\), then for any initial condition \(x_c(0)\) and \(w(0)\), \(x_c(t)\) and \(e(t)\) will converge to a bounded set. Moreover, the tracking error \(e\) at steady state is bounded by \(\|e(t)\| \leq \gamma \|w(0)\|\).

The number \(\gamma\) satisfying Theorem is called a bound on the steady state gain from \(\|w(0)\|\) to the tracking error \(e\). For given \(K_1, K_2\), this steady state gain can be evaluated by minimizing \(\gamma\) satisfying the LMI constraints (16) and (17), by using the LMI toolbox in Matlab.
Here we note that the norm of the initial condition, \( \|w(0)\|^2 \), is closely related to the magnitude of \( v_g \) and \( i_{g,ref} \). Furthermore
\[
\|w(0)\|^2 = \sum_{k=1}^{N} \|w_{g,k}\|^2 + r^2 \|w_0\|^2 = \sum_{k=1}^{N} b_k^2 + r^2 b_1^2
\]
Thus \( \|w(0)\|^2 \) represents the total power of all the harmonics of \( v_g \) plus the power of the conference \( i_{g,ref} \).

### 3.2. Improved evaluation using structural information

In practice, the THD of the grid voltage is below a certain level and the magnitude of the reference current is within a given range. In this section, such structural information will be effectively utilized to improve the evaluation of the tracking error. Specifically, the structural information will be exactly expressed in terms of quadratic inequalities and incorporated in the Lyapunov approach to obtain less restrictive constraints, thus reducing the minimal value of \( \gamma \) for the optimization problem.

1) THD bound condition: Consider the grid voltage expressed in (2). The THD value is
\[
THD_{v_g} = \frac{\left(\sum_{k=2}^{\infty} b_k^2\right)^{1/2}}{b_1}
\]
Suppose that a known bound on the THD is \( \epsilon_0 \). Then we have
\[
\sum_{k=2}^{\infty} b_k^2 \leq \epsilon_0^2 b_1^2
\]
Recall from (9) that \( b_k^2 = w_{g,k}(t)^T w_{g,k}(t) \) for all \( t \), the above inequality can be expressed as
\[
\sum_{k=2}^{\infty} w_{g,k}^T w_{g,k} \leq \epsilon_0^2 w_{g,1}^T w_{g,1}
\]
In terms of the combined state \( \begin{bmatrix} x_c^T & w^T \end{bmatrix} \), this inequality can be expressed as
\[
\begin{bmatrix} x_c^T & w^T \end{bmatrix} W_{THD} \begin{bmatrix} x_c \n w \end{bmatrix} \geq 0 \tag{19}
\]
where
\[
W_{THD} = \begin{bmatrix}
0_3 & 0 & 0 & 0 \\
0 & \epsilon_0^2 I_2 & 0 & 0 \\
0 & 0 & -I_{2(N-1)} & 0 \\
0 & 0 & 0 & 0_2
\end{bmatrix}
\]
and \( 0_3 \) denotes a \( 3x3 \) zero block and other \( 0 \)'s have compatible dimensions.

2) Magnitude of reference current condition: The reference current is proportional to the first harmonic \( v_g \), and is set as \( i_{g,ref}(t) = r v_g(t) \). Suppose that \( r \) is bounded by \( r_{max} \). Then we have \( w_r \leq r_{max} w_{g,1} w_{g,1} \).

In terms of \( \begin{bmatrix} x_c^T \n w^T \end{bmatrix} \), this constraint can be written as
\[
\begin{bmatrix} x_c^T \n w^T \end{bmatrix} W_{rm} \begin{bmatrix} x_c \n w \end{bmatrix} \geq 0 \tag{20}
\]
where \( W_{rm} \) is given by
\[
W_{rm} = \begin{bmatrix}
0_3 & 0 & 0 & 0 \\
0 & r_{max}^2 I_2 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -I_2 & 0_2
\end{bmatrix}
\]

3) Phase of reference current condition: The reference current is in phase with the first harmonic \( v_g \). This implies that the state \( w_r \) is proportional to the state \( w_{g,1} \). Let \( w_r = \frac{w_{r,1}}{w_{r,2}} w_{g,1} = \frac{w_{g,1}}{w_{g,2}} \).

Then \( w_{r,1} w_{g,1,2} = w_{r,2} w_{g,1,1} \). In terms of the whole state, this is equivalent to
\[
\begin{bmatrix} x_c^T \n w^T \end{bmatrix} W_{rp} \begin{bmatrix} x_c \n w \end{bmatrix} = 0 \tag{21}
\]
where \( W_{rp} \) is given by
\[
W_{rp} = \begin{bmatrix}
0_3 & 0 & 0 & 0 \\
0 & 0_2 & 0 & \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \\
0 & 0 & 0 & 0 \\
0 & 0 & -I_{2(N-1)} & 0 \\
0 & 1 & 0_2 & 0
\end{bmatrix}
\]

Now we can use the quadratic inequalities (19)-(21) to improve the evaluation of the tracking error.

Consider the closed loop system (14). Suppose that the THD of the grid voltage is less than \( \epsilon_0 \) and \( i_{g,ref}(t) = r v_g(t) \) with \( r \leq r_{max} \). For \( \gamma > 0 \), if there exist a positive definite matrix \( P = P^T \in \mathbb{K}^{(2N+5) \times (2N+5)} \), and numbers \( \eta, \alpha_1, \alpha_2 \geq 0, \alpha_3 \in \mathbb{R} \) such that
\[
C^T C \leq P \tag{22}
\]
\[
A_L(K_1, K_2)^T P + P A_L(K_1, K_2)^T \leq -\eta \begin{bmatrix} 0 & 0 \\ 0 & I_{2N+2} \end{bmatrix} \tag{23}
\]

where \( I_{2N+2} \) is a \( (2N+2) \) identity matrix, then for any initial condition \( x_c(0) \) and \( w(0) \), \( x_c(t) \) and \( e(t) \) will converge to a bounded set. Moreover, the tracking error \( e \) at steady state is bounded by
\[
|e(t)| \leq \gamma^2 (b_1^2 + \cdots + b_k^2 + r_{max}^2 b_1^2)^{1/2}
\]

For given feedback gain \( K_1, K_2 \), the magnitude of the tracking error \( e \) can be evaluated by solving an optimization problem with matrix inequality constraints. The constraint (23) is less restrictive than the corresponding condition (17) due to the additional parameters \( \alpha_1, \alpha_2, \alpha_3 \) in the terms \( -\alpha_1 W_{THD} - \alpha_2 W_{rm} - \alpha_3 W_{rp} \), which result from the structural information.
4. Design of state-feedback law via LMI-based optimization

The analysis problem in the previous section can be readily turned into a design problem by considering $K_1, K_2$ as additional optimization parameters. Putting everything together, we have the following optimization problem

$$\begin{align*}
\min_{P, K_1, K_2, \eta, a_1, a_2, \alpha} & \quad Y \\
\text{s.t.} & \quad C^T P C \leq P \\
& \quad b A_{g} (K_1, K_2)^T P + P A_{g} (K_1, K_2) \leq 0 \\
& \quad -\eta \left( P - \gamma \begin{bmatrix} 0 & 0 \\ 0 & I_{N+1} \end{bmatrix} \right) \\
& \quad -a_1 W_{THD} - a_2 W_{rm} - a_3 W_{rp} \\
& \quad P \succ 0, a_1, a_2, \eta \succ 0
\end{align*}$$

(24)

When $K_1$ and $K_2$ are considered as optimization parameters, the above optimization problem has bilinear terms in constraint b). We may use the path-following method as used in [39] to find the optimal or sub-optimal solution.

5. Computation and simulation results

The Simulink model was constructed for the inverter using SimPower in Matlab. The model is included a transistor bridge controlled by PWM signals. The parameters for the circuit in Fig. 1 are given as follows: $L_g = 150 \mu H$, $L_d = 450 \mu H$, $C_{ef} = 22 \mu F$, $R_1 = 0.02 \Omega$, $R_c = 0.02 \Omega$, $R_d = 1 \Omega$.

The switching frequency is 20KHz and the DC voltage supply is 12V. The state feedback is processed by a low pass filter $\frac{1}{1+5f_{\text{switch}}}$ before sent to drive the transistor bridge.

The grid voltage $v_g$ was measured via a transformer and its harmonic components were analyzed in Matlab.

![Figure 2.Hamonic components of grid voltage](image)

Fig. 2 is the grid voltage harmonic components. Its fundamental frequency is $f_0 = 60Hz$. The total harmonic distortion (THD) is 2.434%. For simulation, only the first 5 harmonics are kept. The resulting $v_g$ used for simulation is

$$v_g(t) = 7.9554\sin(\beta t) - 0.4868 + 0.0084\sin(2\beta t) - 0.525 + 0.0299\sin(3\beta t) + 2.67 + 0.0032\sin(4\beta t) - 1.1385 + 0.1911\sin(5\beta t) + 0.3363$$

The reference current is $i_{g,\text{ref}} = 7.9554\sin(\beta t) - 0.4868$, with $r$ is the proportional factor. The output current $i_g$ will be created in proportion with $i_{g,\text{ref}}$.

By choosing different weighting for $K$, and solving the optimization problem, we obtained the feedback gain $K$.

$$K = \begin{bmatrix} -0.2005 & 7.1971 & -72.8533 & 0.2944 & -0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
0.2005 & 7.1971 & -72.8533 & 0.2944 & -0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
0.4868 & 0.0084 & 2.67 & 0.0032 & -1.1385 & 0.1911 & 0.3363 & -72.8533 & 0.2944 & -0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
0.0084 & 2.67 & 0.0032 & -1.1385 & 0.1911 & 0.3363 & -72.8533 & 0.2944 & -0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
0.0299 & 3.6377 & 1.1385 & 0.1911 & 0.3363 & -72.8533 & 0.2944 & -0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
3.6377 & 1.1385 & 0.1911 & 0.3363 & -72.8533 & 0.2944 & -0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
0.0032 & -1.1385 & 0.1911 & 0.3363 & -72.8533 & 0.2944 & -0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
-1.1385 & 0.1911 & 0.3363 & -72.8533 & 0.2944 & -0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
0.1911 & 0.3363 & -72.8533 & 0.2944 & -0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
0.3363 & -72.8533 & 0.2944 & -0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
-72.8533 & 0.2944 & -0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
0.2944 & -0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
-0.0012 & 0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
0.2942 & -0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
-0.0023 & 0.2940 & -0.0035 & 0.2936 & -0.0046 \\
0.2940 & -0.0035 & 0.2936 & -0.0046 \\
-0.0035 & 0.2936 & -0.0046 \\
0.2936 & -0.0046 \\
-0.0046 \\
\end{bmatrix}$$

![Figure 3a. Grid current $i_g$ and tracking error with $r=0.2$](image)

![Figure 3b. Grid current $i_g$ and tracking error with $r=0.3$](image)

Fig. 3a and 3b show the tracking performance by the PWM model with $r=0.2$ and 0.3 respectively. As expected, the output current $i_g$ tracks the reference $i_{g,\text{ref}}$, which is proportional with the first harmonic of grid voltage $v_g$. $i_g = i_{g, \text{ref}} = r v_g$. With $r=0.2$ and $r=0.3$, the magnitude of $i_g$ will be about 1.6 and 2.4, and the THD value for $i_g$ is 0.9369% and 6798% respectively.
magnitude of tracking error at stable state is about 0.08(A). Fig. 4a and 4b show the harmonic components of the current output \(i_g\).

![Figure 4a. Harmonic components of the current output \(i_g\) with \(r=0.2\)](image)

![Figure 4b. Harmonic components of the current output \(i_g\) with \(r=0.3\)](image)

6. Conclusions

This paper developed a Lyapunov approach to modelling and design the state-feedback control for grid-connected inverters. The method was applicable to inverters connected to the grid via a transformer. The model ensures the internal stability and makes efficient use of harmonic information of the grid voltage, and the magnitude/phase of the reference current. The effectiveness of this design was then validated by SimPower simulation. The results show the robustness of the design, the output of the inverter can be fed to the grid with low THD.

7. References