Control Design for Grid-Connected Invertersvia Lyapunov Approach

Vu Tran
Department of Electrical and Computer Engineering, University of Massachusetts, Lowell, USA

Mufeed MahD
Department of Electrical and Computer Engineering, University of Massachusetts, Lowell, USA

Abstract

This paper develops a Lyapunov approach to design for grid connected inverter. The control objective is to create the output current to track a reference currentin proportional with the fundamental harmonic of the grid voltage.
The grid voltage is described as the outputs of an autonomous linear oscillatory system, andits harmonics can be estimated via an observer. A state space description for the whole system is obtained from the state of the inverter circuit and that of the oscillatory system for the grid voltage. Based on the state space description, a Lyapunov approach is applied to design a state-feedback controller with minimal tracking error. The design problem is cast into an optimization problem, which can be effectively solved with linear matrix inequality (LMI) toolbox in Matlab. The effectiveness of the Lyapunov approach is validated via SimPower simulation.

1. Introduction

As the demand for power is increasing significantly, renewable energy sources have recently received a lot of attention as an alternative way of generating directly electricity. Using renewable energy systems can eliminate harmful emissions from polluting the environment while also offering inexhaustible resources of primary energy. There are many sources of renewable energy, such as solar energy, wind turbines, water turbines, and geothermal energy. Most of the renewable energy technologies produce direct current (DC) power and hence inverters are required to convert the DC to the alternating current (AC) power.

Stand-alone (island) and grid-connected are two kinds of inverters. These two types have several similarities, but are different in terms of control function. A stand-alone inverter is used in off grid applications. The generated power from renewable energy is delivered to loads, or can be stored in batteries. That systems requires complexity and high maintenance, such that rechargeable batteries. That also increases the size and cost for the system. Grid-connected inverters overcome this limitation. For grid-connected inverters, they must follow the voltage and the frequency characteristics of the utility generated power presented on the distribution line. We consider grid connected inverters in this paper, as its main advantage is that no battery is required for storing the energy from renewable sources, which reduces the size and cost of the system. Moreover, it is easier to create a portable inverter due to the compact size of the system.

Investigations of different configurations and control methods for grid-connected inverters are being focused in recent years. A comprehensive review of single-phase grid-connected inverters [1] has covered some of the standards that inverters for grid applications must be fulfilled. It also provided a classification of the inverters regarding the stage (single, dual stage, and multi-string inverter), transformers and types of Interconnections, and types of grid interfaces (line-commutated current-source inverter and self-commutated voltage-source inverter).

Reducing the harmonics is a main problem which needs much effort in power inverters these days. The IEEE 929 standard stated that the Total Harmonic Distortion (TDH) of voltage and current should be lower than 5% in normal operation. Harmonics are not desirable because they cause overheating, decreased volt-ampere capacity, increased losses, distorted voltage and current waveform, etc. Several researches have been proposed to reduce the voltage THD of inverters. For example, the repetitive control theory has been successfully applied to PWM inverters [2]-[9], active filters [10]-[12], dead-beat control [13], [14], to reduce THD. Harmonic droop control technique [15] is also presented. Repetitive control has an excellent ability in eliminating periodic disturbances, however, in practical; this technique is limited in slow dynamics, poor tracking accuracy, and poor performance to non-periodic disturbances. Dead-beat and sliding-mode controls have excellent dynamic performance in control of output voltage, but these techniques suffer from complexity, sensitivity, and steady-state errors In order to eliminate the current distortion, some current control methods are proposed, such that proportional resonant controller and multi-resonant controllers in [16], active power filters in [12], [17].

A promising control techniques in grid-connected inverter is output current tracking. The inverter's current polarity must be controlled to match the voltage polarity of the grid. Various synchronization methods are summarized in [18], [19] and [20]. The current hysteretic comparison control method, timing control of current instantaneous comparison method and the triangle wave comparison control method of timing tracking current are proposed in [21]. In [22], algorithms of current decoupling are derived for performing the reactive power control of grid-connected inverter. Through zero-crossing detecting circuit in [23] and [24], the inverter is controlled so as to generate the output current in phase with the grid voltage. A current control employing internal model principle in [25] is proposed to suppress the harmonic currents injected into the grid. Although most existing controllers give satisfactory results, the theory behind the dynamics and performances is not clearly described.

For a more clearly understanding of the whole system's dynamics, a systematic state space approach is developed in this paper. The design problem will be proposed using advanced nonlinear control system theory and linear matrix inequality (LMI) optimization technique, as in [26]-[30]. The problem of tracking current error with minimal tracking error will be casted into Lyapunov framework, which ensures stability and the total harmonics distortion (THD) requirements.

The paper is organized as follows. Section II describes the open loop description for the inverter circuit; following is the state-space description for the grid voltage and an observer, and control objective. Section III reviews the main tool to be used in this paper -Lyapunov approach to evaluate the tracking error. Section IV casts the problem of tracking error into Lyapunov framework and converts the design problem into the LMI optimization. Section V uses SimPower in MATLAB to simulate and verify the results. Section VI concludes the paper.

2. State space description and control objective

2.1. Open-loop description for the circuit

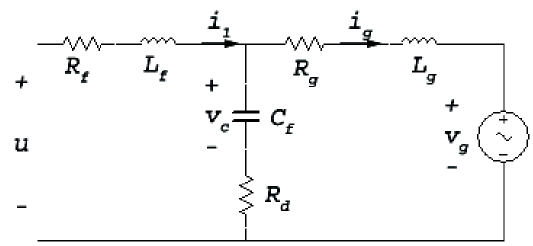

Figure 1. Equivalent circuit for a grid-connected inverter

Fig. 1 describes the equivalent circuit of a grid-connected inverter, where v_g is the grid voltage. The input voltage u is actually the output of the transistor bridge driven by a pulse-width-modulated (PWM) signal.

Let the duty cycle of the PWM signal be d and the DC voltage supply be V_S. Under the assumption of ideal switching, $u = (2d - 1)V_S$. Thus u can be considered as the input.

Let the state of the circuit as $x_c = \begin{bmatrix} v_c \\ i_1 \\ i_g \end{bmatrix}$

Define

$$A = \begin{bmatrix} 0 & \frac{1}{C_f} & -\frac{1}{C_f} \\ -\frac{1}{L_f} & -\frac{R_f + R_d}{L_f} & \frac{R_d}{L_f} \\ \frac{1}{L_g} & \frac{R_d}{L_g} & -\frac{R_d + R_g}{L_g} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{L_f} \\ 0 \end{bmatrix} \quad E = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{L_g} \end{bmatrix}$$

The circuit can be described as

$$\dot{x_c} = Ax_c + Bu + Ev_g \tag{1}$$

where u is the control input and v_g can be considered as an external disturbance.

2.2. State-space description for the grid voltage and an observer

The grid voltage is periodic with frequency 50Hz or 60Hz. The frequency may subject to some perturbation but can be measured. Let the fundamental frequency be β_0 (rad/second). According to [31], the grid voltage $v_g(t)$ can be written as:

$$v_g(t) = \sum_{k=1}^{\infty} b_k \sin(k\beta_0 t + \Phi_k) \tag{2}$$

The magnitude b_k and the phase ϕ_k for each harmonic can be evaluated with a bank of resonantfilters [16], [32], [33], or a composite observer [34], [35]. The resonant filters are described with transfer functions, while the composite observers are described via state-space equations. They are all based on the internal model principle in [36], [37]. Here we adopt the main ideas in [35] to describe v_g via state space equations and then construct an observer to estimate the state. The advantage of using the state space description is that the dynamics of the whole system can be simply described by stacking up the state equation for v_g and that for the circuit, i.e., (1). The resulting state equation for the whole system makes it very convenient to study the interaction between the grid voltage and the dynamics of LCL filter. Furthermore, it facilitates analysis of system performance via advanced tools developed in recent years, such as the Lyapunov approach and the linear-matrix-inequality (LMI) based optimization.

Let

$$S_0 = \begin{bmatrix} 0 & -\beta_0 \\ \beta_0 & 0 \end{bmatrix}$$

Define

$$S_g = \begin{bmatrix} S_0 & 0 & \cdots & 0 & 0 \\ 0 & 2S_0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & (N-1)S_0 & 0 \\ 0 & 0 & \cdots & 0 & NS_0 \end{bmatrix}$$

where all 0's in the above matrix are 2 by 2 blocks. Also define

Γ_g=[1 0 1 0 ⋯ 1 0]

Then v_g is the output of the following autonomous linear oscillatory system:

$$\dot{w}_g = S_g w_g, \quad v_g = \Gamma_g w_g \tag{3}$$

where$w_g \in \mathbf{R}^{2N}$. A feature of the matrix S_g is that $S_g + S_g^T = 0$. Because of this, we have $w_g(t)^T w_g(t) = w_g(0)^T w_g(0) = \|w_g(0)\|^2$ for all t. This kind of state-space descriptions for periodic signals has been widely used in the output regulation literature for tracking periodic references or rejection of periodic disturbances [26]-[28], [36], where the linear system (3) wasreferred to as the exogenous system, or simply, exosystem.

The state $w_g \in \mathbf{R}^{2N}$ can be decomposed as

$$w_g = \begin{bmatrix} w_{g1}^T & w_{g2}^T & \ldots & w_{gN}^T \end{bmatrix}^T$$

where$w_{gk} \in \mathrm{R}^2$, $k = 1, \ldots, N$. By the structure of Γ_g, we have$v_g(t) = \sum_{k=1}^{N} [1 \quad 0] w_{gk}(t)$. The$k^{th}$ harmonic is exactly $[1 \quad 0] w_{gk}(t) = b_k \sin(k\beta_0 t + \Phi_k)$. Let C_1 be a 1 by $2N$ row vector whose first element is one and the rest are all zero. Then the first harmonic, denoted v_{g1}, is $v_{1g}(t) = C_1 w_g(t)$

It is easy to verify that the system (3), in particular, the pair (Γ_g,S_g), is observable. Thus an observer can be constructed to estimate the state w_g, and hence (b_k,ϕ_k) for all k. Let the state of the observer be w_z. We have

$$\dot{w}_z = S_g w_z - L(\Gamma_g w_z - v_g)$$

where L is the observer gain which can be designed via various approaches.

If the frequency β_0 for the observer is exactly the same as the frequency of the grid voltage, then the observer error $w_z(t) - w_g(t)$ will go to 0 asymptotically and we can use the estimated state w_z for various purposes. If the grid frequency is subject to perturbation, this frequency can be measured on line and used for the observer. Due to robustness, the same gain L should be stabilizing for a certain range of β_0 and $S_g(\beta_0)$. The discrete-time version of the observer is usually used in practice. More details can be found in [35]. With the estimated state w_z, the first harmonic of v_g is estimated as $C_1 w_z$.

2.3. The control objective

Ideally, we would like to feed the grid a sinusoidal current i_g, which is in phase with v_g. The magnitude of i_g can be varied depending on the need of the grid and the local energy storage devices. This objective can be stated as a reference tracking problem where the reference for the grid current is given as

$$i_{g,ref}(t) = r v_{g1}(t) \tag{4}$$

where$r$ is a positive number that can be changed. It would be more convenient to introduce another exosystem:

$$\dot{w}_r = S_0 w_r, \quad i_{g,ref} = [1 \quad 0] w_r \tag{5}$$

where$w_r \in \mathbf{R}^2$. The condition (4) can be satisfied if $w_r(0) = r w_{g1}(0)$. Since$\dot{w}_{g1} = S_0 w_{g1}$, we have $w_r(t) = r w_{g1}(t)$. There is some redundancy introducing (5). The purpose is to make it easier to handle r.

The control objective is to minimize the magnitude of the tracking error at steady state.

$$e(t) = i_g(t) - i_{g,ref}(t) \tag{6}$$

Note that i_g can be considered as an output to the inverter system (1): i_g=[0 0 1]x_c. The two exosystems (3) and (5) can be combined to obtain a $2(N + 1)$ -order system.

Define

$$w = \begin{bmatrix} w_g \\ w_r \end{bmatrix}$$

and

$$S = \begin{bmatrix} S_g & 0 \\ 0 & S_0 \end{bmatrix} \qquad \Gamma_1 = [\Gamma_g \quad 0 \quad 0]$$

$$\Gamma_2 = [0 \quad \cdots \quad 0 \quad 1 \quad 0]$$

Then

$$\dot{w} = Sw, \quad v_g = \Gamma_1 w, \quad i_{g,ref} = \Gamma_2 w \tag{7}$$

This system describes all the dynamics of the grid voltage and the reference current. Combined with the state-space description of the LCL filter, the dynamics of the whole system can be described. It is clear that the exosystem (7) evolves all by itself and is driven by its initial condition $w(0)$. Recall that $w$consists of the states for all the harmonics of v_g and for $i_{g,ref}$, in particular,

$$w = \begin{bmatrix} w_{g1} \\ w_{g2} \\ \vdots \\ w_{gN} \\ w_r \end{bmatrix}$$

Since $S + S^T = 0$, we have

$$w(t)^T w(t) = w(0)^T w(0) = \|w(0)\|^2 \qquad (8)$$

$$w_{gk}(t)^T w_{gk}(t) = w_{gk}(0)^T w_{gk}(0) = b_k^2 \qquad (9)$$

$$w_r(t)^T w_r(t) = w_r(0)^T w_r(0) = r^2 b_1^2 \qquad (10)$$

for all t. Thus $\|w_{gk}(0)\|^2$ represents the power of the k^{th} harmonics of v_g and $\|w(0)\|^2$ the total power of the harmonics of v_g plus the power of the reference current.

The condition $w_r(t)= rw_{g1}(t)$ implies that $i_{g,ref}$is proportional to the first harmonic of v_g. In terms of the combined state w, this can be written as

$$w_1 w_{2N+2} = w_2 w_{2N+1} \qquad (11)$$

This condition will be used as a constraint in an optimization problem to be formulated.

3. Evaluation of the tracking error using Lyapunov approach

3.1. Evaluation of tracking error for general exosystem

From (7), both the grid voltage and the reference current are driven by an autonomous linear system. According to [38], the state variable of an autonomous system contains all the information that determines its future behavior, it can be effectively used to correct the dynamic behavior of the whole system. In the case of the inverter circuit, the state w can be used to minimize the tracking error of the grid current (6).

The state of the whole system is a combination of the circuit state x_c and the exosystem state w. They can be either measured or estimated via an observer, thus state feedback is feasible. If the tracking error can be effectively evaluated via a certain performance measure, the next step would be minimizing this performance measure via a certain optimization algorithm.

The Lyapunov approach developed in [26] can be applied for this kind of problems. It deals with more general systems (nonlinear, time-varying) with periodic excitations, which could be disturbance or reference. The objective is to evaluate the magnitude of certain output at steady state, which could be the tracking error.

To apply the Lyapunov approach, we first need the state space description for the whole system, which can be easily obtained by combining the state-space equation (1) for the circuit and the state-space equation (7) for the v_g and $i_{g,ref}$. Since i_g=[0 0 1]x_c, if we let C =[0 0 1 − Γ_2], then

$$\begin{bmatrix} \dot{x}_c \\ \dot{w} \end{bmatrix} = \begin{bmatrix} A & E\Gamma_1 \\ 0 & S \end{bmatrix} \begin{bmatrix} x_c \\ w \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \qquad (12)$$

$$e = C \begin{bmatrix} x_c \\ w \end{bmatrix}$$

To reduce the tracking error, we apply a simple state feedback

$$u = K_1 x_c + K_2 w = [K_1 \quad K_2] \begin{bmatrix} x_c \\ w \end{bmatrix} \qquad (13)$$

Substitute the feedback law in (13) into (12), we have the closed-loop system

$$\begin{bmatrix} \dot{x}_c \\ \dot{w} \end{bmatrix} = \begin{bmatrix} A + BK_1 & E\Gamma_1 + BK_2 \\ 0 & S \end{bmatrix} \begin{bmatrix} x_c \\ w \end{bmatrix} \qquad (14)$$

$$e = C \begin{bmatrix} x_c \\ w \end{bmatrix} \qquad (15)$$

As long as $A + BK_1$ is stable, the solution for the above system will be bounded and for any initial condition, the solution will approach a steady state oscillation. Since $A + BK_1$ is stable, the effect of the initial condition of $x_c(0)$ will vanish asymptotically. Thus the steady state oscillation, in particular, the tracking error, depends only on the initial condition $w(0)$. Hence, a gain from the norm of the initial condition $\|w(0)\| = \sqrt{w(0)^T w(0)}$to the magnitude of e at steady state can be defined. The main result of [26] was applied to estimate this gain via a quadratic Lyapunov function$V(x_c, w) = [x_c^T w^T] P \begin{bmatrix} x_c \\ w \end{bmatrix}$.

Here we summarize the main result of the Lyapunov approach when applied to the linear system (14). Denote

$$A_L(K_1, K_2) = \begin{bmatrix} A + BK_1 & E\Gamma_1 + BK_2 \\ 0 & S \end{bmatrix}$$

Theorem: for $\gamma > 0$, if there exist a positive definite matrix $P = P^T \in R^{(2N+5)\times(2N+5)}$, and a number $\eta > 0$ such that

$$C^T C \leq P \qquad (16)$$

$$A_L(K_1, K_2)^T P + P A_L(K_1, K_2) \leq -\eta \left(P - \gamma \begin{bmatrix} 0 & 0 \\ 0 & I_{2N+2} \end{bmatrix} \right) \qquad (17)$$

where I_{2N+2} is an identity matrix of dimension $2N + 2$, then for any initial condition $x_c(0)$ and $w(0)$, $x_c(t)$ and $e(t)$ will converge to a bounded set. Moreover, the tracking error e at steady state is bounded by $|e(t)| \leq \gamma^{\frac{1}{2}} \|w(0)\|$.

The number γ satisfying Theorem is called a bound on the steady state gain from $\|w(0)\|$ to the tracking error e. For given K_1, K_2, this steady state gain can be evaluated by minimizing γ satisfying the LMI constraints (16) and (17), by using the LMI toolbox in Matlab.

Here we note that the norm of the initial condition, $\|w(0)\|^2$, is closely related to the magnitude of v_g and $i_{g,ref}$. Furthermore

$$\|w(0)\|^2 = \sum_{k=1}^{N} \left\|w_{gk}\right\|^2 + r^2 w_r^2 = \sum_{k=1}^{N} b_k^2 + r^2 b_1^2$$

Thus $\|w(0)\|^2$ represents the total power of all the harmonics of v_g plus the power of the conference $i_{g,ref}$.

3.2. Improved evaluation using structural information

In practice, the THD of the grid voltage is below a certain level and the magnitude of the reference current is within a given range. In this section, such structural information will be effectively utilized to improve the evaluation of the tracking error. Specifically, the structural information will be exactly expressed in terms of quadratic inequalities and incorporated in the Lyapunov approach to obtain less restrictive constraints, thus reducing the minimal value of γ for the optimization problem.

1) THD bound condition: Consider the grid voltage expressed in (2). The THD value is

$$THD_{v_g} = \frac{(\sum_{k=2}^{\infty} b_k^2)^{\frac{1}{2}}}{b_1}$$

Suppose that a known bound on the THD is ε_0. Then we have

$$\sum_{k=2}^{\infty} b_k^2 \le \varepsilon_0^2 b_1^2$$

Recall from (9) that $b_k^2 = w_{gk}(t)^T w_{gk}(t)$ for all t, the above inequality can be expressed as

$$\sum_{k=2}^{\infty} w_{gk}^T w_{gk} \le \varepsilon_0^2 w_{g1}^T w_{g1} \tag{18}$$

In terms of the combined state $\begin{bmatrix} x_c \\ w \end{bmatrix}$, this inequality can be expressed as

$$[x_c^T \quad w^T] W_{THD} \begin{bmatrix} x_c \\ w \end{bmatrix} \ge 0 \tag{19}$$

where

$$W_{THD} = \begin{bmatrix} 0_3 & 0 & 0 & 0 \\ 0 & \varepsilon_0^2 I_2 & 0 & 0 \\ 0 & 0 & -I_{2(N-1)} & 0 \\ 0 & 0 & 0 & 0_2 \end{bmatrix}$$

and $0p$ denotes a $p \times p$ 0 block and other 0's have compatible dimensions.

2) Magnitude of reference current condition: The reference current is proportional tothe first harmonic $v_{g1}$and is set as $i_{2,ref}(t)=rv_{g1}(t)$. Suppose that r is bounded by r_{max}. Then we have$w_r^T w_r \le r_{max}^2 w_{g1}^T w_{g1}$.

In terms of$\begin{bmatrix} x_c \\ w \end{bmatrix}$, this constraint can be written as

$$[x_c^T \quad w^T] W_{rm} \begin{bmatrix} x_c \\ w \end{bmatrix} \ge 0 \tag{20}$$

whereW_{rm} is given by

$$W_{rm} = \begin{bmatrix} 0_3 & 0 & 0 & 0 \\ 0 & r_{max}^2 I_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_2 \end{bmatrix}$$

3) Phase of reference current condition: The reference current is in phase with the first harmonic v_{g1}. This implies that the state w_r is proportional to the state w_{g1}. Let $w_r = \begin{bmatrix} w_{r,1} \\ w_{r,2} \end{bmatrix}$, $w_{g1} = \begin{bmatrix} w_{g1,1} \\ w_{g1,2} \end{bmatrix}$.

Then $w_{r,1} w_{g1,2} = w_{r,2} w_{g1,1}$. In terms of the whole state, this is equivalent to

$$[x_c^T \quad w^T] W_{rp} \begin{bmatrix} x_c \\ w \end{bmatrix} = 0 \tag{21}$$

whereW_{rp} is given by

$$W_{rp} = \begin{bmatrix} 0_3 & 0 & 0 & 0 \\ 0 & 0_2 & 0 & \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \\ 0 & 0 & 0_{2(N-1)} & 0 \\ 0 & \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} & 0 & 0_2 \end{bmatrix}$$

Now we can use the quadratic inequalities (19)-(21) to improve the evaluation of the tracking error.

Consider the closed loop system (14). Suppose that the THD of the grid voltage is less than ε_0 and $i_{g,ref}(t)= rv_{g1}(t)$ with $r \le r_{max}$. For $\gamma>0$, if there exist a positive definite matrix $P=P^T \in R^{(2N+5)\times(2N+5)}$, and number η, $\alpha_1,\alpha_2 \ge 0$, $\alpha_3 \in R$ such that

$$C^T C \le P \tag{22}$$

$$A_L(K_1,K_2)^T P + PA_L(K_1,K_2) \le -\eta\left(P - \gamma \begin{bmatrix} 0 & 0 \\ 0 & I_{2N+2} \end{bmatrix}\right) - \alpha_1 W_{THD} - \alpha_2 W_{rm} - \alpha_3 W_{rp} \tag{23}$$

whereI_{2N+2} is a$(2N+2)$ identity matrix, then for any initial condition $x_c(0)$ and $w(0)$, $x_c(t)$ and $e(t)$ will converge to a bounded set. Moreover, the tracking error e at steady state is bounded by

$$|e(t)| \le \gamma^{\frac{1}{2}} (b_1^2 + \cdots + b_N^2 + r^2 b_1^2)^{\frac{1}{2}}$$

For given feedback gain K_1,K_2, the magnitude of the tracking error e can be evaluated by solving an optimization problem with matrix inequality constraints.The constraint (23) is less restrictive than the corresponding condition (17) due to the additional parameters $\alpha_1,\alpha_2,\alpha_3$ in the terms $-\alpha_1 W_{THD}$, $-\alpha_2 W_{rm}$ and $-\alpha 3 W_{rp}$, which result from the structural information.

4. Design of state-feedback law via LMI-based optimization

The analysis problem in the previous section can be readily turned into a design problem by considering K_1,K_2as additional optimization parameters. Putting everything together, we have the following optimization problem

$$\inf_{P,K_1,K_2,\eta,\alpha 1,\alpha 2,\alpha 3} \gamma \tag{24}$$

s.t $a)$ $C^T C \le P$

$$b) A_L(K_1,K_2)^T P + P A_L(K_1,K_2) \le -\eta \left(P - \gamma \begin{bmatrix} 0 & 0 \\ 0 & I_{2N+2} \end{bmatrix} \right) -\alpha_1 W_{THD} - \alpha_2 W_{rm} - \alpha_3 W_{rp}$$

$c)$ $P > 0$, α_1,α_2, $\eta > 0$

When K_1 and$K_2$are considered as optimization parameter, the above optimization problem has bilinear termsin constraint b). We may use the path-following method as used in [39] to find the optimal or sub-optimalsolution.

5. Computation and simulation results

The Simulinkmodel was constructed for the inverter using SimPower in Matlab. The model is included a transistor bridge controlled by PWM signals. The parameters for the circuit in Fig. 1 are givenas follows: L_f = 150 μH, L_g = 450 μH, C_f= 22 μF, R_f = 0.02 Ω, R_g = 0.02 Ω, R_d = 1 Ω.

The switching frequency is 20KHz and the DC voltage supply is 12V. The state feedback is processedby a low pass filter$\frac{1}{1+s\frac{1}{3000}}$before sent to drive the transistor bridge.

The grid voltage v_g was measured via a transformer and its harmonic components were analyzedinMatlab.

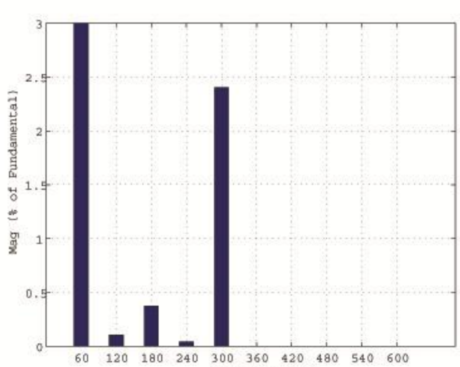

Figure 2.Hamonic components of grid voltage

Fig. 2 is the grid voltage harmonic components. Its fundamental frequency is β_0 = 60Hz. The total harmonic distortion (THD) is 2.434%. For simulation, only the first 5 harmonics are kept. The resulting v_g used for simulation is

$v_g(t)$ = 7.9554sin($\beta_0 t$ -0.4868) + 0.0084sin(2$\beta_0 t$ -0.525) + 0.0299sin(3$\beta_0 t$ + 2.67) + 0.0032sin(4$\beta_0 t$ -1.1385) + 0.1911sin(5$\beta_0 t$ + 0.3363)

The reference current is $i_{g,ref}$= r7.9554$sin(\beta_0 t$ − 0.4868), with r is the proportional factor. The output current i_g will be created in proportion with $i_{g,ref}$.

By choosing different weighting for K, and solving the optimization problem, we obtained the feedbackgain K.

K = [-0.2005 7.1971 -72.8533 0.2944 -0.0012 0.2942 -0.0023 0.2940 -0.0035 0.2936 -0.0046 0.2932 -0.0058 1.3568 0.0545]

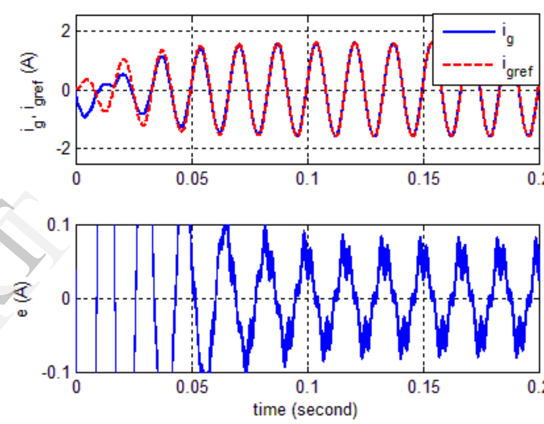

Figure 3a. Grid current i_g and tracking error with r=0.2

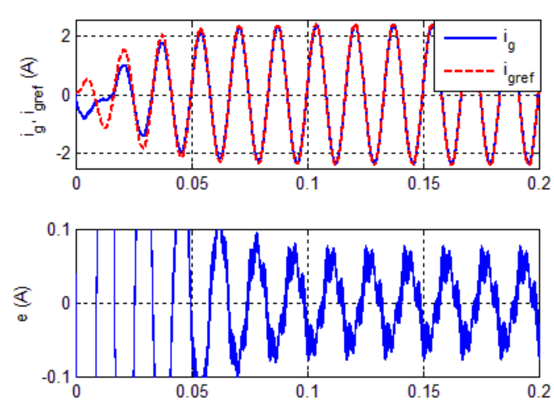

Figure 3b. Grid current i_g and tracking error with r=0.3

Fig. 3a and 3b show the tracking performance by the PWM model with r=0.2 and 0.3 respectively. As expected, the output current i_g tracks the reference i_{gref}, which is proportional with the first harmonic of grid voltage v_g, $i_g \approx i_{gref} = r v_{g1}$. With r=0.2 and r=0.3, the magnitude of i_g will be about 1.6 and 2.4,and the THD value for i_g is 0.9369% and 6798%respectively.The magnitude of tracking error at stable state is about 0.08(A). Fig. 4a and 4b show the harmonic components of the current output i_g.

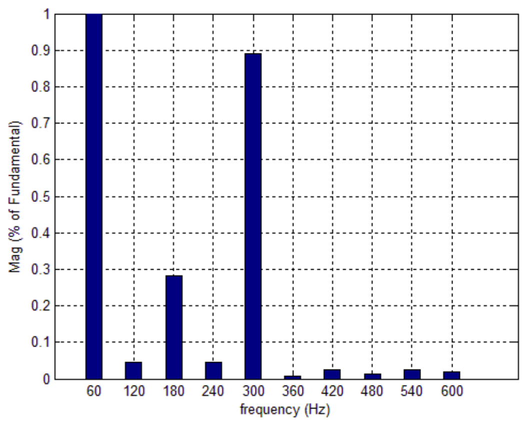

Figure 4a. Harmonic components of the current output $i_a$with r=0.2

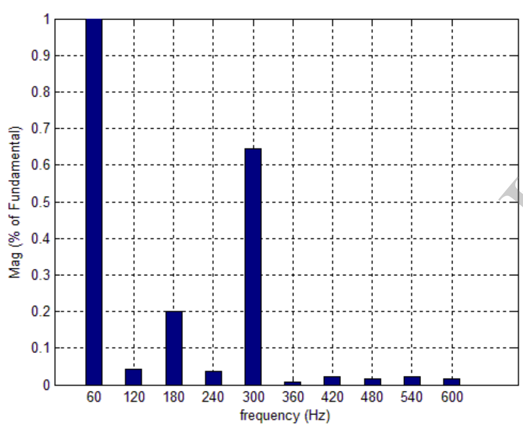

Figure 4b. Harmonic components of the current output $i_g$with r=0.3

6. Conclusions

This paper developed a Lyapunov approach to modelling and design the state-feedback control for grid-connected inverters. The method was applicable to inverters connected to the grid via a transformer. The model ensures the internal stability and makes efficient use of harmonic information of the grid voltage, and the magnitude/phase of the reference current. The effectiveness of this design was then validated by SimPower simulation. The results show the robustness of the design, the output of the inverter can be fed to the grid with low THD.

7. References

[1] S. Kjaer, J. Pedersen and F. Blaabjerg, ”A Review of Single-Phase Grid-Connected Inverters for Photo-voltaic Modules,” *IEEE Trans. on Industry Applications*, vol. 41, no. 5, pp. 1292 -1306, 2005.

[2] Y. Ye, B. Zhang, K. Zhou, D. Wang and Y. Wang, ”High-performance cascade-type repetitive controller for CVCF PWM inverter: analysis and design,” *IET Electric Power Applications*, vol. 1, no. 1, pp. 112-118, January 2007.

[3] Y. Ye, B. Zhang, K. Zhou, D. Wang and J. Wang, ”High-Performance Repetitive Control of PWM DCAC Converters With Real-Time Phase-Lead FIR Filter,” *IEEE Transactions on Circuits and Systems*, vol. 53, no. 8, pp. 768-772, August 2008.

[4] S. Chen Y.M. Lai S.-C. Tan C.K. Tse, ”Analysis and design of repetitive controller for harmonic elimination in PWM voltage source inverter systems,” *IET Electric Power Applications*, vol. 1, no. 4, pp. 497-506, December 2008.

[5] Y.Y. Tzou, S.L. Jung, and H.C. Yeh, ”Adaptive Repetitive Control of PWM Inverters for Very Low THD AC-Voltage Regulation with Unknown Loads,” *IEEE Transactions on Power Electronics*, vol. 14, no. 5, pp. 973-981, September 1999.

[6] K. Zhou, D. Wang, B. Zhang, and Y. Wang, ”Plug-In Dual-Mode-Structure Repetitive Controller for CVCF PWM Inverters,” *IEEE Trans. on Industrial Electronics*, vol. 56, no. 3, pp. 784 -791, March. 2009.

[7] G. Weiss, Q.C. Zhong, T.C. Green, and J. Liang, ”H^{∞} Repetitive Control of DC-AC Converters in Microgrids,” *IEEE Trans. on Power Electronics*, vol. 19, no. 1, pp. 219 -230, Jan. 2004.

[8] T. Hornik and Q-C Zhong, ”A Current-Control Strategy for Voltage-Source Inverters in Microgrids Based on H^{∞} and Repetitive Control,” *IEEE Transactions on Power Electronics*, vol. 26, no. 3, pp. 943-952, 2011.

[9] D. Chen, J. Zhang, and Z. Qian, ”An Improved Repetitive Control Scheme for Grid-Connected Inverter With Frequency-Adaptive Capability,” *IEEE Trans. on Industrial Electronics*, vol. 60, no. 2, pp. 814 -823, Feb. 2013.

[10] A. Garca-Cerrada, O. Pinzn-Ardila, V. Feliu-Batlle, P. Roncero-Snchez, and P. Garca-Gonzlez, ”Application of a Repetitive Controller for a Three-Phase Active Power Filter,” *IEEE Transactions on Power Electronics*, vol. 22, no. 1, pp. 237-246, 2007.

[11] R. Costa-Castello, R. Grino, R. CardonerParpal, and E. Fossas, ”High-Performance Control of a Single-Phase Shunt Active Filter,” *IEEE Trans. on Control systems Technology*, vol. 17, no. 6, pp. 1318-1329, Nov. 2009.

[12] M. Routimo, M. Salo, and H. Tuusa, ”Comparison of Voltage-Source and Current-Source Shunt Active Power Filters,” *IEEE Transactions on Power Electronics*, vol. 22, no. 2, pp. 636 -643, Mar. 2007.

[13] T. Fujii and T. Yokoyama, ”FPGA based Deadbeat

Control with Disturbance Compensator for Single Phase PWM Inverter," *Proc. IEEE Power Electronics Specialists Conference*, pp. 1-6, Jun. 2006.
[14] P. Mattavelli, "An improved deadbeat control for UPS using disturbance observers," *IEEE Transactions on Industry Electronics*, vol. 52, no. 1, pp. 206-212, Feb. 2005.
[15] Q.C. Zhong, "Harmonic Droop Controller to Reduce the Voltage Harmonics of Inverters," *IEEE Transactions on Industry Applications*, vol. 60, no. 3, pp. 936-945, Mar. 2013.
[16] R. Teodorescu, F. Blaabjerg, M. Liserre and P.C. Loh, "Proportional-resonant controllers and filters for grid-connected voltage-source converters," *IEE Proc. Electric Power Applications*, vol. 153, no. 5, pp. 750-762, September 2006.
[17] C. Lascu, L. Asiminoaei, I. Boldea, and F. Blaabjerg, "High Performance Current Controller for Selective Harmonic Compensation in Active Power Filters," *IEEE Transactions on Power Electronics*, vol. 22, no. 5, pp. 1826 -1835, Sep. 2007.
[18] Z. Shicheng, W. Peizhen and G. Lusheng, "Study on improving output current waveform of photovoltaic grid-connected system," *IEEE Conference on Industrial Electronics and Applications*, 2006.
[19] X. Wang and M. Kazerani, "A multicarrier modular photovoltaic grid-connected inverter with a new phase-shift rule," *Electric Power Systems Research*, pp. 754 - 760, 2007.
[20] F. Blaabjerg, R. Teodorescu, M. Liserre and A. V. Timbus, "Overview of control and grid synchronization for distributed power generation systems," *IEEE Transactions on Industry Electronics*, vol. 53, no. 5, pp. 1398-1409, 2006.
[21] J. He and G. Zhao, "The Simulation of Controlling Strategy for Output Current Tracking of Grid-connected Inverter," *IEEE Symposium on Electrical & Electronics Engineering*, pp. 171 -174, 2012.
[22] X. Jin, "Photovoltaic Grid-connected Inverter Harmonic Compensation and Grid-connected Unified Control," *Power and Energy Engineering Conference*, Asia-Pacific, pp. 1 -4, 2009.
[23] W. Haibo, C. Yanbo and Z. Lihua, "Research and Development of Photovoltaic Grid-connected Inverter Based on DSP," *International Conference on Power Electronics Systems and Applications (PESA)*, pp. 15, 2011.
[24] Y. Xiong, S. Qian and J. Xu, "Single-phase grid-connected photovoltaic system based on boost inverter," *Power and Energy Engineering Conference (APPEEC)*, pp. 1 -3, 2012.
[25] Y. Yang, K. Zhou, F. Blaabjerg, "Harmonics suppression for single-phase grid-connected PV systems in different operation modes," *Applied Power Electronics Conference and Exposition (APEC)*, pp. 889 896, 2013.
[26] T. Hu, A. R. Teel and Z. Lin, "Lyapunov characterization of forced oscillations," Automatica, pp. 1723-1735, 2005.
[27] T. Hu and Z. Lin, "Output regulation of linear systems with bounded continuous feedback," *IEEE Trans. Automatic Control*, pp. 1941-1953, 2004.
[28] A. Isidori and C. I. Byrnes, "Output Regulation of Nonlinear Systems," *IEEE Trans. on Automatic Control*, vol. 35, pp. 131-140, 1990.
[29] Y. Yao, F. Fassinou and T. Hu, "Robust Regulation of Battery Driven Boost Converter with Simple Feedback," *IEEE Trans. on Power Electronics*, pp. 2614-2626, 2011.
[30] H. Pham, H. Jung and T. Hu, "State-Space Approach to Modeling and Ripple Reduction in AC-DC Converters," *IEEE Trans. on Control Systems Technology*, vol. 21, no. 5, pp. 1949-1955, 2013.
[31] R. Erickson and D. Maksimovic, *Fundamentals of Power Electronics*, 2nd edition, Springer, 2001.
[32] G. Escobar, A.A. Valdez, J. Leyva-Ramos and P. Mattavelli, "Repetitive-Based Controller for a UPS Inverter to Compensate Unbalance and Harmonic Distortion," *IEEE Trans. Industrial Electronics*, vol. 54, no. 1, pp. 504-510, Feb. 2007.
[33] S. Fukuda and R. Imamura, "Application of a Sinusoidal Internal Model to Current Control of Three-phase Utility-Interface Converters," *IEEE Trans. on Industrial Electronics*, vol. 52, no. 2, pp. 420-426, March. 2005.
[34] K. Lee, T.M. Jahns, T.A. Lipo, V. Blasko and R.D. Lorenz, "Observer-Based Control Methods for Combined Source-Voltage Harmonics and Unbalance Disturbances in PWM Voltage-Source Converters," *IEEE Transactions on Industry Applications*, vol. 45, no. 6, pp. 2010-2021, Nov.-Dec. 2009.
[35] K. Selvajyothi and P.A. Janakiraman, "Extraction of Harmonics Using Composite Observers," *IEEE Trans. Power Delivery*, vol. 23, no. 1, pp. 31-40, 2008.
[36] B. A. Francis, "The Linear Multivariable regulator Problem," *SIAM Journal of Control and Optimization*, 15, pp. 486-505, 1977.
[37] B. Francis and W. Wonham, "The Internal Model Principle for Linear Multivariable Regulators," *Appl. Math. Optimization*, vol. 2, pp. 170-194, 1975.
[38] C. T. Chen, *Linear System Theory and Design*, 3rd edition, Oxford University Press, 1998.
[39] T. Hu, "Conjugate Lyapunov Functions for saturated Linear Systems," *Automatica*, vol. 41, no. 11, pp. 1949-1956, 2005.