Contribution To The Readjustment Of Mesh On 2D Structures

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Abstract: ***Before any achievement of a piece, we start a design on a computer. At this stage, we numerically solved the mechanics milieu equations using the most common resolution technique known as finite element method. This technique is an approach method set up with some approximation errors which are important to analyze, for the best feet of the solution. This contribution is a presentation of one of the analysis techniques of these errors including the r-adaptation and its effectiveness through its application to two structures in two dimensions.***

Key words: structures, calculation finite element method mesh, errors analysis.

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I.INTRODUCTION

The r-adaptation of mesh (gearing) aim is to improve the accuracy of approximate solutions obtained by the finite element method [1] In principle; this method helps assessing the gap between the results of the simulations and the correct answer. Therefore, the control of the discrepancies between the results of simulations and reality remains a crucial issue in all branches of engineering sciences. The hypothesis, the error decreases if the step of mesh (gearing) decreases, the adaptation of mesh is to refine or not meshing locally in order to meet the desired criteria. Two methods are commonly used: The increase in the number of nodes (knots) in the mesh, the h-adaptation and the increase of the degree of the polynomial form functions, p-adaptation [2, 3]. In this contribution, we propose to study a third adaptation of mesh technology called r-adaptation, which allows changes in the position of knots of the mesh. While maintaining the same number of degrees of freedom [4, 5, 6]. This strategy of mesh adaptation driven by an estimator of error of type ZZ will be presented and applied to two different structures in order to choose for each structure the optimal mesh where the errors of discretization in areas of interest will be decreased.

II MATERIALS AND METHODS

To solve a problem in mechanics of continuous milieu we have to determine the unknown moving field $\vec{u}$ which components are u_i, and the unknown field of stress $\bar{\bar{\sigma}}$ with its components σ_{ij}.Under the effect of applied strength $\vec{F}^d$ (area strength) and $\vec{f}$ (volume strength).

These sizes verified the equilibrium equations [7]

$$\sigma_{ij,j} + f_i = 0 \text{ in } \Omega \qquad (1)$$

$$\sigma\ \vec{n} = \vec{F}^d \text{ on } \partial_F \Omega \qquad (2)$$

with $\vec{n}$ the normal external vector $\partial_F \Omega$.

The comportment relations:

$$\sigma_{ij} = C_{ijkl}\ \varepsilon_{kl} \qquad (3)$$

The discretized problem by finite element method consist of finding (u_k, σ_k) solutions of

$$(u_k) = [N](u_k^e) \qquad (4)$$

Where:

(u_k) are the approached shifts;

(u_k^e) are the known shifts in levels of mesh knots

$[N]$ are interpolation functions which verified the relation:

$$[K]\{U\} = (F) \qquad (5)$$

[K], being the stiffness matrix;

$\{U\}$, the stiffs vector (of displacements);

(F), the external strength vector.

The gap between the approached solution and the exact solution is determined by the method based on consistency flaw called smoothing method [8, 9].

II.1.Evaluation of errors

In general, the exact solution is not known. However, an estimate of the error .can be calculated Several estimators exist but we used an estimator simple to implement called ZZ1.The relative global error ξ equal to [5]:

$$\xi^2 = \frac{\left\|u^{ex}\right\|^2 - \left\|u^k\right\|^2}{\left\|u^{ex}\right\|^2} = \frac{a(u^{ex}, u^{ex}) - a(u^k, u^k)}{a(u^{ex}, u^{ex})}$$

(6)

with

$$a(u^{ex}, u^{ex}) = \int_\Omega \varepsilon^T(u^{ex})[C]\varepsilon(u^{ex})d\Omega \qquad (7)$$

The exact stress σ^{ex} is unknown but the idea is to substitute the exact stress by smoothing stress $\tilde{\sigma}$.This new approximation of stress $\tilde{\sigma}(u^k)$ is based on shape functions [8, 9 10]. If Σ Is a vector which contains stress at mesh knots, then $\tilde{\sigma}$ is equal to:

$$(\tilde{\sigma}) = [N]^T(\Sigma) \qquad (8)$$

The stress (Σ) are determined by less square method using the formulation:

$$J = \frac{1}{2}\int_\Omega (\tilde{\sigma} - \sigma^k)^2 d\Omega \qquad (9)$$

Hence:

$$J = \frac{1}{2}\Sigma^T A \Sigma - \Sigma^T b + c \qquad (10)$$

The matrix A and the vector b are numerically

Calculated by the finite element method Strategy, using elements of reference, points of Gauss and assembling matrix technical. The minimization of the latter expression returns to solve the following system of equations:

$$A\Sigma = b \qquad (11)$$

The component of the smooth stress $\tilde{\sigma}_{xx}$ is therefore:

$$(\tilde{\sigma}_{xx}) = [N]^T(\Sigma_{xx}) \qquad (12)$$

An estimation of relative error on these stresses called θ is evaluated by the relation:

$$\theta^2 = \frac{\int_\Omega (\tilde{\sigma})^T[C]^{-1}(\tilde{\sigma})d\Omega - \int_\Omega (\varepsilon)^T[C](\varepsilon)d\Omega}{\int_\Omega (\tilde{\sigma})^T[C]^{-1}(\tilde{\sigma})d\Omega}$$

(13)

This relation can also be written as:

$$\theta^2 = \sum_{e=1}^{n\,elts} \theta_e^2 \qquad (14)$$

The quantity θ_e represent the contribution of the element e to the relative global .error The calculation is repeated by adapting the mesh (gearing) until the error becomes less

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than the limit fixed error by wished precision. In our study we have used the strategy called r-adaptation of mesh [9, 11]. Here the adaptation is done by reducing the waist of elements of the interested zone. The fineness of the mesh element is obtained by the stiff of mesh knots. The degree of freedom of the interpolation functions remains the same, but the position of knots is optimized. [9.]

III.RESULTS AND ANALYSIS

We applied this technic on two structures: one rectangular full plate and one rectangular plate drilled a hole in the middle. By these two applications, we presented the r-adaptation strategy to be used to solve a problem of concentration of relative discretization error on the interested zone element. The mesh is type Delaunay [12].The results obtained are the following:

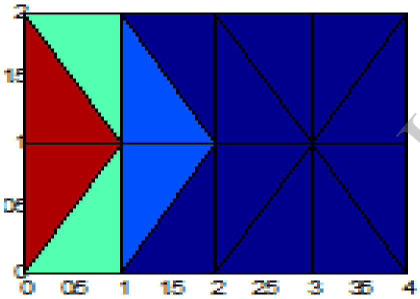

Figure1-a: Initial mesh

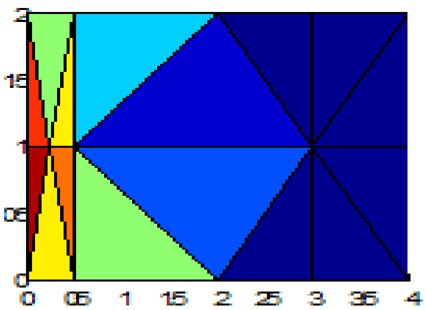

Figure1-b: Intermediary mesh

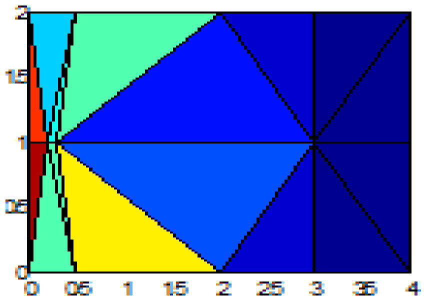

Figure 1-c: Final mesh

Figure 1: Numerically results of one full rectangular plate meshed by 16 triangular elements

Since it is possible to bring the discret continuous gap to the level fixed in advance, provided they refine enough the mesh, we moved the knots of the areas of interest so as to reduce the surface of elements in this area [4, 5]. The relatives' errors by element such as the last figure 1 shown are equidistributed in the whole of the mesh. The overall relative error changes from 9.649% before the readjustment of mesh to 8.773%.after. The maximum elementary error contributed to 10%.decrease.

The second application is a rectangular plate with a circular hole. The plate is in a state of plan stress. The radius of the hole is supposed to be small before the dimension of the plate. The problem being symmetrical about x=0 and y=0 plans, we just have to model a quarter of the piece as presented in different mesh sizes in figures 2a.-, 2b.- and 2c.

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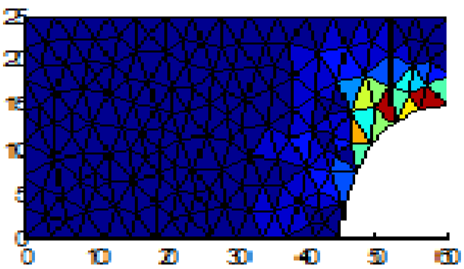

Figure2-a: Initial mesh

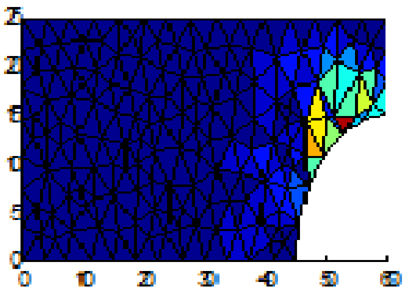

Figure 2-b: Intermediary mesh

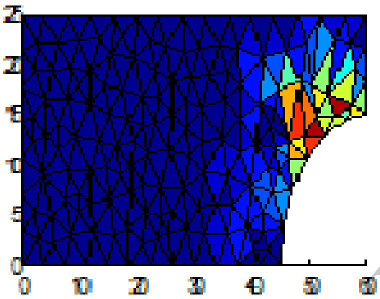

Figure 2-c: Final mesh

Figure 2: Numerically results on one rectangular plate with a circular hole meshed by 16 triangular elements.

The overall relative errors of the elements where the r-adaptation was made decreased from 7.63% to 7.56%.The effect of pollution error cause increase in errors in some elements of the mesh. After several iterations, the final or optimal mesh is retained. The results show that this mesh gives a satisfactory equi-distribution of the error by element. Indeed, we also noted that the maximum error of an element that contributes most to the overall error decreased from 6 to 4%.

IV. CONCLUSION

In this contribution, the aim was to improve the accuracy of approximate solutions obtained by adjustment of mesh on two planar structures, using the r-adaptation of mesh which is to change the position of the initial mesh knots while retaining the same number of degrees of freedom. This technic coupled with the estimator of error of type ZZ on the finite element method solutions has allowed to equi-distribute the discretization error on areas where these errors are high.

For the full plate, the overall error changes from 9.649% before the readjustment of mesh to 8.773%.For the perforated plate, the maximum error of an element that contributes most to the overall error goes from 6% to 4%.

These results suggest that for both structures, the r-adaptation allow improvement of the accuracy of solutions by reducing the overall error.

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