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Constrained Lot Sizing Problem for Continuous Demand Supply Chain

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***Abstract*— Lot sizing is of a prime importance in determining the performance of a supply chain especially in constraint environment due to internal and external factors. In the present work, a mathematical model is proposed to determine the optimum constrained lot sizing for a supply chain that includes a supplier, manufacturer, and retailer. Integer Nonlinear Programming (INLP) was used to solve centralized control supply chain for optimum profit. The results show that, for coordinated supply chain, the constraints have major role in determining the lot or batch size at each echelon. For different supply chain parameters, the manufacturer's echelon dominates the lot sizing decisions while other downstream supply chain members have least effect on the decisions. The research extends to the study of the effect of different parameters on lot sizing decisions. Parameters studied are holding costs, order/set-up costs, demand, production rate, and material percent defective. For centralized case of a supply chain, it was proved that the cost elements along with the throughput production quality, considerably affect the lot size at each echelon.**

Keywords*— Lot sizing; supply chain; integer non-linear programming; finite production; integer replenishment policy.

I. INTRODUCTION

Coordinating decisions across the supply chain network represents an important issue in supply chain operation. It was proved that lot sizing decisions among coordinated supply chain members to be a win-win situation for all concerned parties. The lack of orders coordination across the supply chain results in high costs [1-4]. It is recommended to avoid the inefficient decentralized supply chain, where decisions are made individually, towards more efficient centralized supply chains, such that decisions are made centrally by the key player of the network [5-7]. System wide optimization is applicable in case the supply chain is vertically integrated and partially or jointly owned [1].

Considerable number of research work emphasizes on integrating a finite production or replenishment rate in their models. Wang et al. [7] investigated the penalty for treating the manufacturer as a buyer. They showed that the finite production rate should be included; especially, when the set-up cost largely exceeds the order cost. The production rate was considered in the supply chain inventory models as an input parameter [2, 7-10]. Eiamkanchanalai and Banerjee [11], and Sana [5,12] considered the production rate as a decision variable and the production cost as a function of the production rate. Khouja and Mehrez [13] considered the case of variable production rate and they assumed that both production cost and process quality are dependent on the production rate.

The assumption of perfect quality for lot sizing models has been modified by many researchers ever since Rosenblatt and Lee [14], who proposed a model for economic production cycles with imperfect production processes. Eroglu and Ozdemir [15] proposed an economic order quantity model with imperfect quality items. Their model extended Salameh and Jaber's model [16] by allowing shortages and maximum backorder level. Also, the effect of different percent defective on the optimal solution was examined. Chang and Ho [17], similar to Eroglu and Ozdemir [15], used the renewal reward theorem to derive the expected profit in order to obtain optimal lot size and backordering quantity.

Shortage may or may not be allowed for cases of finite production rate and imperfect quality. In case that shortage is not permitted, the basic assumption was that the number of acceptable quality items exceeds the demand [8,9]. In case of shortage, several models considered backorder [10,15,17,18].

The demand was considered and modeled in different ways. Pal et al. [19] and Taleizadeh et al. [20] considered price sensitive demand. Pal et al. [19] studied a joint price and lot-size determination problem over two cycle periods, and the retailer offers a discount to sell end of the season products. Taleizadeh et al. [20] expanded the problem to optimize the vendor's production rate when the supply chain comprises of multiple-retailers and deals with deteriorating items. Chung [21] considered stock-and-warranty dependent demand, where the selling rate depends on both the stock level at the buyer and the offered warranty period.

Kreng and Tan [9] developed a model for determining optimal replenishment decisions. They extended the models of Chung and Huang [22] and Huang [23] to allow for two-level trade credit, offered by suppliers to wholesalers and wholesalers to customers, while including finite replenishment rate. Su [10] relaxed the assumption of not permitting shortages and considered any shortages to be fully backlogged.

Sana [5, 12] compared between the Stakelberg approach (backward induction method) and the collaborating system approach for lot size determination. Optimal solutions obtained by collaborating system approach proved to provide better results than that obtained by Stakelberg. The same conclusion was confirmed by Sana et al. [24] and it was further implemented on a more complex network structure. Wang et al. [7] also compared decentralized lot-sizing decisions with centralized (coordinated/integrated) decisions. They studied a supply chain with price-sensitive demand and investigated the effect of supplier's finite production rate on the pricing and lot-sizing decisions. Results showed that in a centralized problem optimal quantities are sensitive to the production rate, while its effect on the optimal retail price is very small.

Lot splitting along the supply chain (e.g., integer number of batches or partial production lots are transported between supply chain members) reduces average inventory in the system [25]. It helps in implementing the time-based strategy when integrated with lot streaming techniques [26]. Different optimization techniques were used when lots are taken as integer multiplier of the number of batches (i.e., integer replenishment policy). Differential calculus is the most sought after solution technique for obtaining analytical solutions. It is used for the determination of the optimum lot/batch size for the downstream member. This value is constrained by the system other variables and parameters, so other algorithms are needed to obtain the optimum values for the correlated variables [5,7, 10]. When it comes to stipulating integer number of batches, rounding up or down the differentiation results is a matter for the optimization to decide. Integrating integer constraints turns the objective function to a discontinuous function; but when the objective cost function is proved to be convex with respect to the integer decision variables (i.e., continuous real domain is assumed), the two rounding values can be checked for optimization [4].

Despite the popularity of the lot sizing problems, when infinite planning horizon is assumed the majority of the research articles addressed the problem as an inventory system management. The operational limitations and constrains controlling the production facility in supply chain was given little attention in the research work. Researchers who combined finite production rate with integer replenishment policy in their models have given almost no attention to the constraints that simultaneously affect both the number and size of batches. One of the frequently discussed reasons for stock shortage is the withdrawal of imperfect items from inventory; since the occurrence of imperfection is assumed random. The present work proposes a mathematical model for a lot sizing problem that is implemented on a three-echelon supply chain that consists of a supplier, manufacturer, and retailer. Coordinated delivery -production- replenishment decisions are made, so as the total profit of the chain is maximized and demands are met. Both unconstrained and constrained strategies are addressed. Shortage due to production limitations is allowed. A constrained Integer Non-Linear Program (INLP) was used to optimize the problem and to obtain optimal solutions. The behavior of the optimal solution was explored against system parameter changes.

II. NOTATION AND ASSUMPTIONS

A. Notation

Parameters

D : customer demand per period

S_P : unit selling price

U_m : production capacity, in hours, per period

d : demand rate, $d = D/U_m$

t_m : production time

P : production rate, $P = 1/t_m$

C_R : unit raw material cost

C_m : hourly production cost

C_L : unit shortage cost in case that $(1-\gamma_m)P<d$

C_h : unit holding cost of perfect quality raw material per period

C_H : unit holding cost of perfect quality end product at the manufacturer per period

C_{Hr} : unit holding cost of end product per period at the retailer's warehouse

C_d : unit transportation cost between the supplier and the manufacturer

C_D : Unit transportation cost between the manufacturer and the retailer

C_O : order cost per batch

C_S : set-up cost per run

C_{sh} : shipment cost per replenished batch

γ_s : order defective percentage $(0\leq \gamma_s<1)$

γ_m : end product defective percentage $(0\leq \gamma_m<1)$

B_s : maximum supplier's batch size (units/shipment)

B_m : maximum manufacturer's lot size (units/production run)

B_r : maximum retailer's batch size (units/shipment)

U_i : raw material inventory capacity

U_I : retailer's inventory capacity

Decision Variables

Q_{mr} : number of products per replenishment shipment transported between the manufacturer and the retailer

x : number of order batches per production run (positive integer)

z : number of replenishment shipments received by the retailer per production run (positive integer)

Dependent Variables

Q_s : number of units per order shipment (i.e., economic order quantity)

$$Q_s = \frac{zQ_{mr}}{x\,(1-\gamma_s)(1-\gamma_m)} = \frac{Q_m}{x\,(1-\gamma_s)}$$

Q_m : number of units per production run (i.e., economic production quantity)

$$Q_m = \frac{zQ_{mr}}{(1-\gamma_m)} = x\,(1-\gamma_s)Q_s$$

T : production cycle time; it includes the time for pure consumption

$$T = \frac{zQ_{mr}}{\left[(1-\gamma_m)P,d\right]^{-}} = \frac{(1-\gamma_m)Q_m}{\left[(1-\gamma_m)P,d\right]^{-}} = \frac{x\,(1-\gamma_s)(1-\gamma_m)Q_s}{\left[(1-\gamma_m)P,d\right]^{-}}$$

y : number of production runs per period

$$y = \frac{U_m}{T} = \frac{U_m\left[(1-\gamma_m)P,d\right]^{-}}{zQ_{mr}}$$

X : total number of order shipments per period

$X = xy$

Z : total number of replenishment shipments per period

$Z= zy$

B. *Model Assumptions*

- The supply chain consists of a single-supplier, single-manufacturer, and single-retailer for single-item production.
- The planning horizon is infinite.
- The supplier has unlimited capacity, infinite production rate, and order replenishments are instantaneous.
- The screening of the raw material is done in the marshaling area with no additional cost, only the good quality material is stored at the manufacturer's raw material inventory. The defective parts are scraped without additional cost ($\gamma_s Q_s$).
- Production rate of the manufacturer equal to the consumption rate of the raw material.
- The manufacturer has a finite production rate with a percent defective (γ_m).
- Defective finished products are detected instantaneously during production and discarded.
- Coordination mechanism of an equal cycle time is assumed between the supply chain members.
- At each production run, the manufacturer produces a lot (Q_m) that is ordered and delivered on integer number of equal-sized shipments. Number of order/replenished shipments per production run are decision variables.
- The first batch is shipped immediately after being produced. That is, the first shipment to the retailer is allowed to be made before the whole production lot is produced. The succeeding batches are continuously produced and every batch is shipped right after the retailer depletes his inventory (the consumption of the preceding batch), see *Fig. 1*.
- The production run starts after the manufacturer has depleted its excess inventory from the preceding cycle.
- The inventory cost for the defective raw material and finished products are negligible.
- The product consumption rate at the retailer equal to the retailer's demand rate.
- Cost of idle times at the manufacturer is not considered.
- Lead time is negligible at different echelons.

III. PROBLEM FORMULATION

The following mathematical model provides the order/replenishment batch sizes, the production lot size, and their period frequencies that maximize the total profit of the supply chain.

$$Max.\, f(Q_{mr},x,z) = U_m\left[(1-\gamma_m)P,d\right]^- S_P -$$

$$\frac{U_m\left[(1-\gamma_m)P,d\right]^-}{(1-\gamma_m)(1-\gamma_s)}C_R - \frac{U_m\left[(1-\gamma_m)P,d\right]^- t_m}{(1-\gamma_m)}C_m -$$

$$\frac{0.5z\,(Q_{mr})t_m\left[(1-\gamma_m)P,d\right]^-}{x\,(1-\gamma_m)^2}C_h -$$

$$\frac{Q_{mr}}{2}\left((z-1)-\frac{(z-2)t_m}{(1-\gamma_m)}\left[(1-\gamma_m)P,d\right]^-\right)C_H -$$

$$0.5\,\frac{Q_{mr}}{d}\left[(1-\gamma_m)P,d\right]^- C_{Hr} -$$

$$\frac{x\,(U_m)\left[(1-\gamma_m)P,d\right]^-}{zQ_{mr}}C_O - \frac{U_m\left[(1-\gamma_m)P,d\right]^-}{zQ_{mr}}C_S -$$

$$\frac{U_m\left[(1-\gamma_m)P,d\right]^-}{Q_{mr}}C_{sh} - \frac{U_m\left[(1-\gamma_m)P,d\right]^-}{(1-\gamma_m)(1-\gamma_m)}C_d -$$

$$U_m\left[(1-\gamma_m)P,d\right]^- C_D -$$

$$\left(D-\left(U_m\left[(1-\gamma_m)P,d\right]^-\right)\right)C_L \qquad (1)$$

subject to

$$\frac{zQ_{mr}}{(1-\gamma_m)} \le B_m\,, \qquad (2)$$

$$\frac{zQ_{mr}}{x\,(1-\gamma_m)(1-\gamma_s)} \le B_s\,, \qquad (3)$$

$$\frac{zQ_{mr}}{x\,(1-\gamma_m)} \le U_i\,, \qquad (4)$$

$$Q_{mr} \le \left[B_r, U_I\right]^-, \qquad (5)$$

$Q_{mr}, x, z \ge 1$, and integer. (6)

Objective function (1) includes the total income from which the total cost is deducted. The income is the product of the selling price per unit and the total amount delivered to the customer, which is the minimum amount of both the required and available. The total cost includes the following cost elements, respectively.

Raw material cost: this is the cost of ordered raw material per period. It is the product of the number of order batches per period, order batch size, and the unit price (xyQ_sC_R).

Production cost: this is the cost of production of the delivered and scraped amount per period. It is the product of the number production runs per period, the production lot size, the product's standard production time, and the hourly production cost ($yQ_mt_mC_m$).

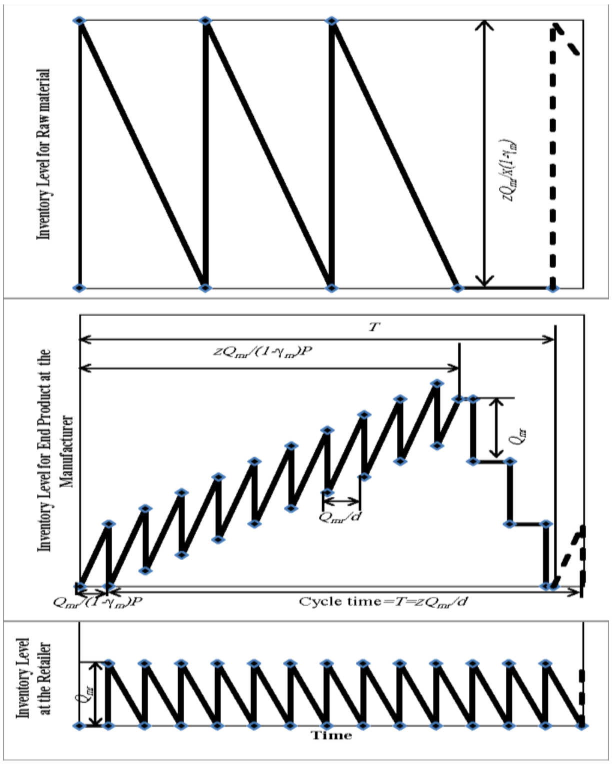

Fig. 1. Representation of inventory levels for the proposed model

Raw material inventory holding cost: this is the cost of holding the ordered material that confirms to specifications in inventories at the manufacturer. It is the product of holding cost per unit per period and the average quantity per period. From *Fig. 1*, the average quantity stored per period can be detailed as follows:

$$\text{Average quantity } = x0.5\,(1-\gamma_s)^2 Q_s{}^2 t_m \frac{\left[(1-\gamma m)P, d\right]^{-}}{zQ_{mr}}$$

End product inventory holding cost at the manufacturer: this is the cost of holding the manufactured products that confirm to specifications in inventories at the manufacturer. It is the product of holding cost per unit per period and the average quantity per period. For detailed proof, please see Appendix A.

End product inventory holding cost at the retailer: this is the cost of holding the products in inventories at the retailer. It is the product of holding cost per unit per period and the average quantity per period ($\frac{0.5zQ_{mr}{}^2}{d\,(T)}C_{Hr}$).

Ordering cost : this is the cost incurred by ordering batches from the supplier. It is the product of the total number of orders per period and the order cost (xyC_O).

Set-up cost: this is the cost incurred by setting-up batches during production. It is the product of total number of production runs per period and the set-up cost per run (yC_S).

Replenishment cost: this is the cost incurred by delivering batches to the retailer. It is the product of the total number of replenishment shipments per period and the replenishment shipment cost (zyC_{sh}).

Transportation cost: It is the cost of transportation of material from the supplier to manufacturer and the transportation of products from the manufacture to the retailer ($xyQ_sC_d + zyQ_{mr}C_D$). Transportation cost is charged on the incoming material and the outgoing products regardless of their quality.

Shortage cost: In case the demand exceeds the production, unmet demand incurs a shortage cost. The unmet demand is assumed to be lost. This cost is the product of the difference between the customer demand per period and the total amount delivered to the retailer and the shortage cost per unit (lost sales cost $(D - zyQ_{mr})C_L$).

Constraint (2) restricts the amount of manufactured products per production run (Q_m). Constraint (3) limits the order shipment size (Qs). Constraint (4) limits the raw material inventory level ($((1-\gamma_s)Q_s$) to storage capacity. Constraint (5) limits the replenishment shipment size. Constraint (6) prevents division by zero and ensures integer number of shipments and integer shipment size.

IV. RESULTS AND ANALYSIS

Decision variables in a centralized supply chain include economic order/production quantities and number of shipments delivered to the downstream members. These decision variables are correlated differently with each other depending on problem configuration and the parameters considered. The present work is concerned with investigating the effect of supply chain parameters and constraints on the lot sizing at optimal solutions.

The results are obtained using FICO-Xpress software v7.8. Different software modules were used to verify the obtained results and to ensure the solution convergence. Brute-force search, as a problem-solving technique, was used after narrowing down the set of candidate solutions using an INLP optimization module. Values of considered parameters are given in Table (1).

An initial analysis of the present mathematical model may lead to the following observations:

- Fewer orders per period are placed with increased order quantity at higher ordering cost.
- Lot size decreases with the increase in the holding cost.
- Lot size may increase with production time.
- Larger production lot sizes yield optimal solutions with increased demand.

A. *Sensitivity analysis*

A series of experiments are conducted to study the effect of changing the model parameters on the joint total profit (*JTP*) and the optimal decision variables. The analysis is made by changing one of the parameters (D, t_m, C_h, C_H, C_{Hr}, C_O, C_S, C_{sh}, γ_m, γ_s, S_P, C_m, C_R, C_D, C_d) by ±%50, ±30%, and ±10%from its assumed nominal value, while the rest of the parameters remain unchanged. The results are given in Tables (2,3) and profit percentage change is represented by radar charts in *Figs. (2*,3). The profit percentage change (*PPC*) is the percentage increase or decrease of the total profit (*JTP*) compared to the optimal profit at the nominal values of (*JTP**), where $PPC = \left(\frac{JTP - JTP^*}{JTP^*}\right) x\,100$.

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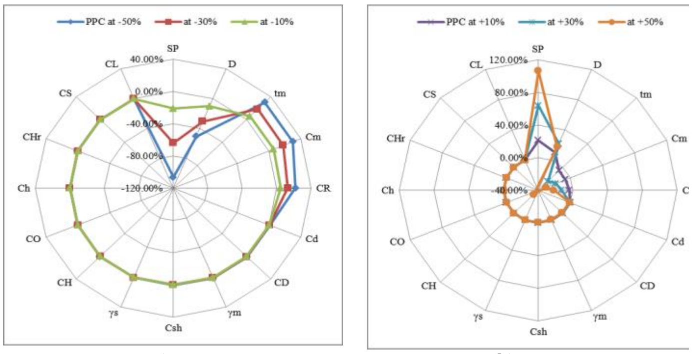
TABLE I. ASSUMED VALUES OF PROBLEM PARAMETERS

Parameter	Value	Dimension
Customer Demand (D)	1920	units/period
Manufacturer's production capacity (U_m)	1920	hrs/period
Production Time (t_m)	0.8	hrs/unit
Holding cost of raw material (C_h)	3	\$/unit per period
Holding cost of finished product at the manufacturer (C_H)	5	\$/unit per period
Holding cost of finished product at the retailer (C_{Hr})	4	\$/unit per period
Ordering cost (C_O)	80	\$/shipment
Set-up cost (C_S)	140	\$/run
Replenishment cost (C_{sh})	90	\$/shipment
Defective percentage at the manufacturer (γ_m)	0-2	%
Defective percentage from the supplier (γ_s)	0-2	%
Selling price (S_P)	100	\$/unit
Material cost (C_R)	20	\$/unit
Production cost (C_m)	35	\$/hr
Shortage Cost (C_L)	10	\$/unit
Transportation cost (C_d, C_D)	1	\$/unit
Max. allowed production run size (B_m)	1500	units/run
Max. supplier's shipment size (B_s)	500	units/shipment
Max. retailer's shipment size (B_r)	400	units/shipment
Raw material inventory capacity (U_i)	1000	units
Retailer's inventory capacity (U_I)	1000	units

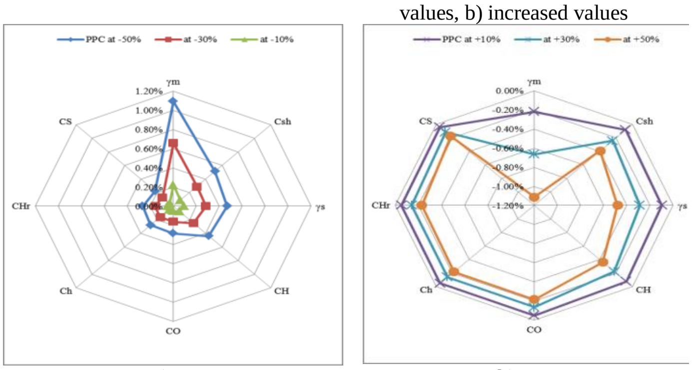
It is worth knowing that the selling price, material cost, production cost, and transportation cost affect the optimum profit but do not affect the optimal solution as shown in Table (3). The Shortage Cost does not affect both the profit and the decision variables, since the nominal values represent the case where the production volume exceeds the demand.

Table (2) shows that the cost elements and the throughput production quality considerably affect the lot size at each echelon.



a) b)

Fig. 2. Profit% change against different parameters. a) reduced parameter values, b) increased values



a) b)

Fig. 3. Profit% change with different parameters after hiding the dominating parameters. a) reduced values, b) increased values

It is evident from the model and experimental results that the profit has an inverse relation with C_R, C_m, C_D, C_d, C_h, C_H, C_{Hr}, C_O, C_S, C_{sh}, γ_m, γ_s, t_m. It has direct relation with the S_P, and D except in the case when the demand exceeds the available production capacity and consequently the shortage cost increases. Therefore, it is evident that the profit does not change with changing the C_L provided that the demand does not exceed the available production capacity. The relation governing the available production with problem parameters is as follows: Max. production per period $= \frac{(1-\gamma_m)}{t_m} U_m$.

The degree of profit sensitivity is affected by the values of different parameters under consideration. It depends on the model assumptions, the way costs are incurred, and the parameter relative values. For instance, the profit has an absolute importance relation with S_P, D, t_m, and C_R as shown in *Figs. 2(a,b)*. *Figs. 3(a,b)* show that the profit is affected largely by γ_m, and moderately by C_{sh}, γ_s, C_H. and weakly by C_O, C_h, C_{Hr}, and C_S.

Effect of percent defective at the manufacturer: Manufacturing percent defective highly affects the lot size at the manufacturer (*Fig. 4*). It has lower effect on the supplier's batch size and minor effects on the retailer's batch size (*Fig. 5*). Optimizing supplier' batch size follow certain pattern; it generally increases up to a certain value then decreases and resumes its increase as the percent defective increases. This is because, as shown in *Fig. 6*, the integer number of shipments per production run increases as the percent defective increases.

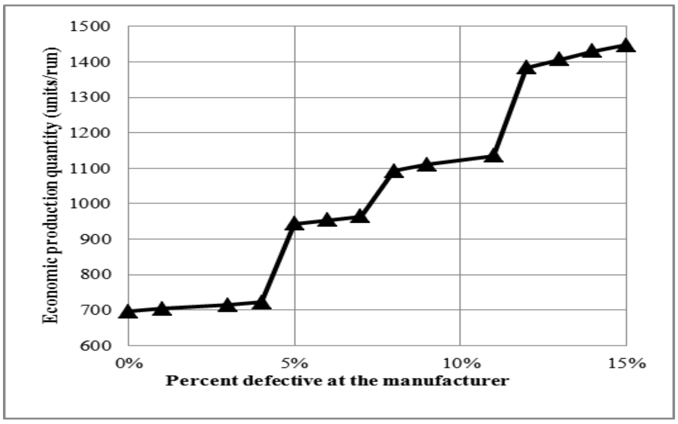


Fig. 4. The effect of percent defective on the economic production quantity

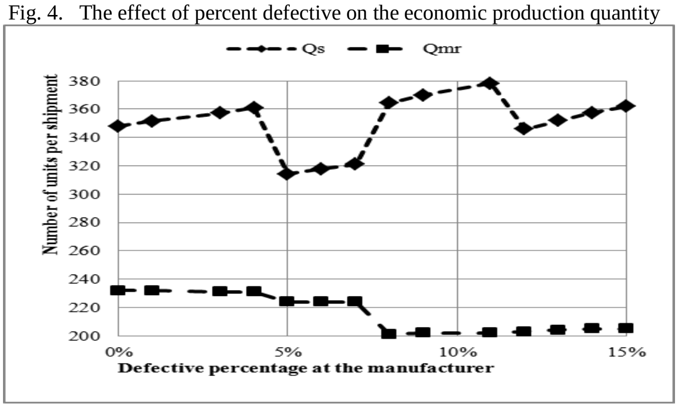


Fig. 5. The effect of percent defective on economic shipment sizes

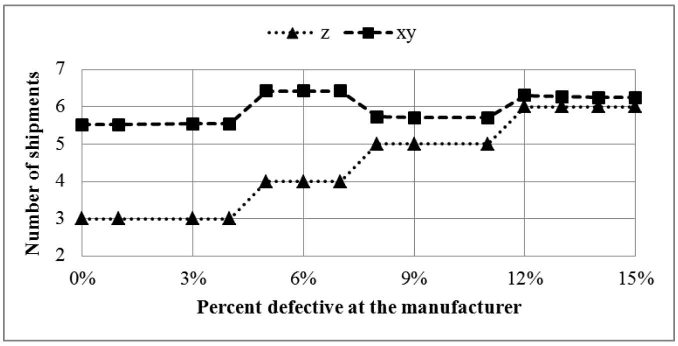


Fig. 6. The effect of percent defective on number of shipments

B. *Effect of Integrating Different Constraints with the proposed Lot Sizing Problem*

Studying the effect of integrating functional constraints with the lot sizing problem is important for obtaining practical optimal solutions. The proposed mathematical model is optimized for profit while considering different constraints.

Effect of Batch/Lot Size Constraints: Fulfilling different constraints causes extra costs, see *Fig. 7*. Comparing *Figs. 8-10* shows that the lot size constraint at the manufacturer dominates the change in both the supplier and retailer's batch sizes. The supplier's batch size is highly affected by manufacturing lot size constraint when compared with that of the retailer, since the optimal supplier batch size is controlled by the optimal values of both integer number of shipments *x* and *z*.

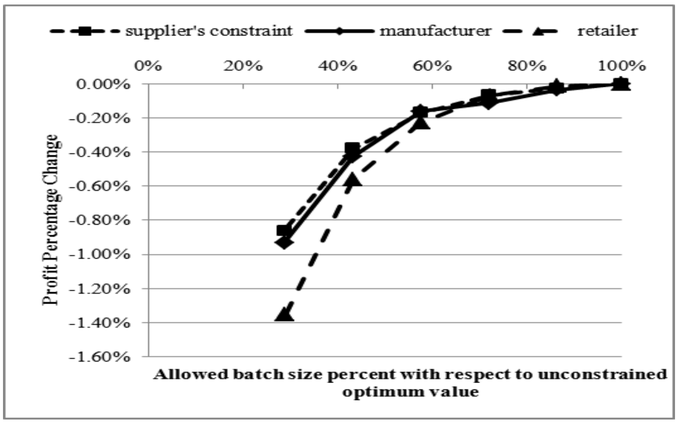


Fig. 7. The effect of echelon constraints on the profit

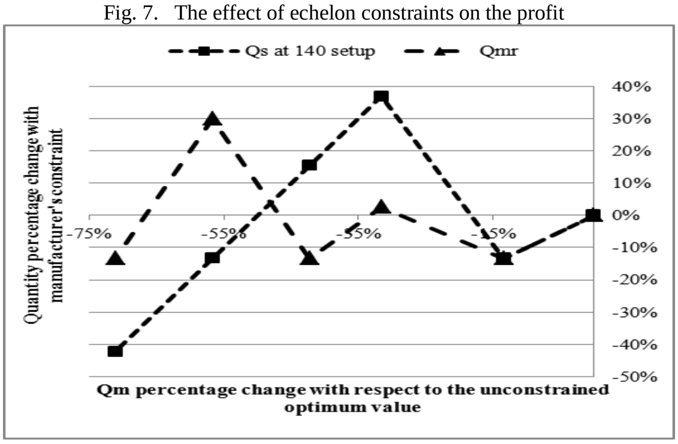


Fig. 8. The effect of manufacturer's related constraint on the batch optimal decisions

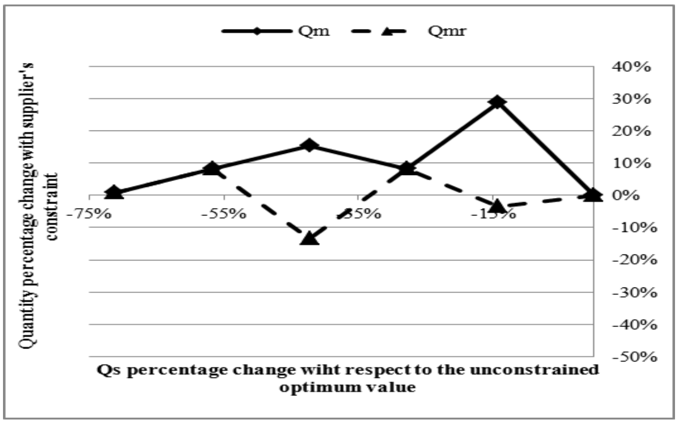


Fig. 9. The effect of supplier's related constraint on the manufacture's economic production quantity and on the retailer's optimal shipment size

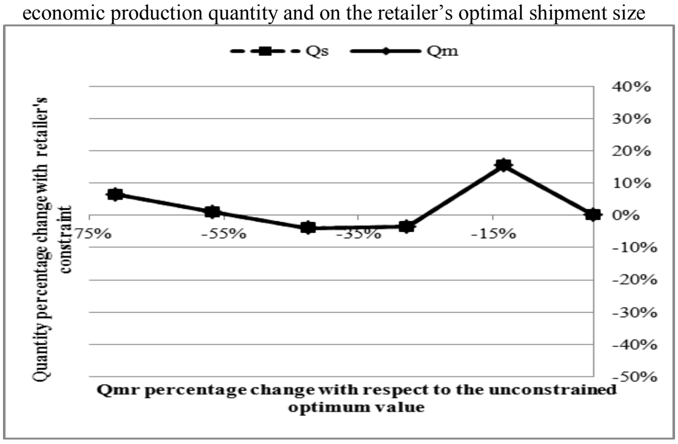


Fig. 10. The effect of retailer's related constraint on the manufacture's economic order and production quantities

Effect of Production Capacity and Different Production Rates: Constrained capacity of the manufacturer drastically changes the production lot size. It has a consequent effect on supplier's batch size more than that on the retailer's batch size. In other words, the retailer may be the least affected echelon with regard to lot sizing within the supply chain for constrained conditions (*Fig. 11*). Also, increasing the production rate (or decreasing the production time) considerably affects the lot size at the manufacturer. The higher the production rate, the lower the lot size (*Fig. 12*). The batch size at the supplier and the retailer slightly changes; however, the change at the supplier is higher than that at the retailer. The reason for this is that the higher the production rate, the higher the products' inventory holding cost at the manufacturer and the lower the raw material inventory holding cost.

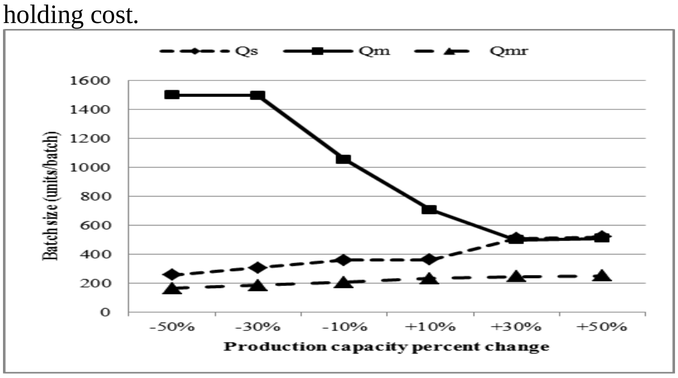


Fig. 11. The effect of production capacity on the optimal lot/batch size

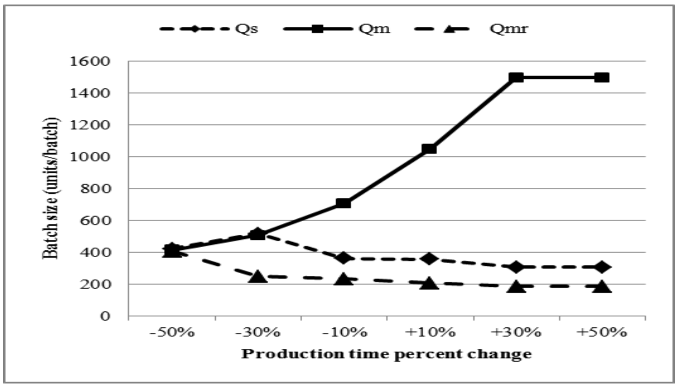


Fig. 12. The effect of production time on the optimal lot/batch size

V. CONCLUSION

The goal of the lot sizing problem is to establish a policy that would maximize profit or minimize relevant costs when implemented. The optimal policy depends on the assumptions made about the way costs are incurred, how demand is satisfied, and any limitations and constraints that face the supply chain's operations. Calculus differentiation is suitable to obtain exact solutions for shipment sizes in non-constrained system. In real industrial problems, lot sizing decisions are made under certain constrains and limitations. There might be constrains on the size and number of shipments, size and number of production lots, space and monetary limitations, etc. The present proposed model considers constrained situations seeking more feasible and practical decisions. It considers a multi-echelon supply chain where ordering and inventory management decisions have to be made in multiple locations. The demand occurs continuously at a constant and known rate. Integer non-linear programming was used to optimize the problem

The results show that, for constrained model, constraints at any echelon affect the lot sizing decisions at the other echelons. Increased demand rate; generally, increases the total number of shipments per period and consequently, the order and replenishment shipment quantities are determined. However, it does not always guaranty an increase in the number of production runs per period. Meanwhile, any increase in the number of shipments per production run results in a decrease of number of production runs per period. It was also found that the manufacturer is the key decision maker when maximizing the total profit of the system.

The present work can be extended for soft constraints, stochastic parameters, and more complex networks.

TABLE II. RESULTS OF SENSITIVITY ANALYSIS ON DIFFERENT PARAMETERS

Parameter	-50%							-30%							-10%						
	PPC	**T**	Q_s	Q_m	Q_{mr}	**xy**	**zy**	**PPC**	**T**	Q_s	Q_m	Q_{mr}	**xy**	**zy**	**PPC**	**T**	Q_s	Q_m	Q_{mr}	**xy**	**zy**
D	**-50.56%**	570	297	291	285	3.4	3.4	**-30.42%**	594.3	433	424	208	3.2	6.5	**-10.17%**	726.7	340	667	218	5.3	7.9
t_m	**30.68%**	404	421	412	404	4.8	4.8	**18.33%**	494	514	504	247	3.9	7.8	**6.09%**	690	359	704	230	5.6	8.3
γ_m	**1.09%**	696	359	703	232	5.5	8.3	**0.66%**	696	360	706	232	5.5	8.3	**0.22%**	696	362	709	232	5.5	8.3
C_{sh}	**0.51%**	750	390	765	150	5.1	12.8	**0.29%**	732	381	747	183	5.2	10.5	**0.09%**	776	404	792	194	4.9	9.9
γ_s	**0.47%**	696	359	710	232	5.5	8.3	**0.28%**	696	360	710	232	5.5	8.3	**0.09%**	696	362	710	232	5.5	8.3
C_H	**0.44%**	1028	357	1049	257	5.6	7.5	**0.25%**	968	336	988	242	6.0	7.9	**0.08%**	708	369	722	236	5.4	8.1
C_O	**0.28%**	699	243	713	233	8.2	8.2	**0.16%**	848	294	865	212	6.8	9.1	**0.05%**	681	355	695	227	5.6	8.5
C_h	**0.28%**	872	454	890	218	4.4	8.8	**0.16%**	844	439	861	211	4.5	9.1	**0.05%**	702	365	716	234	5.5	8.2
C_{Hr}	**0.27%**	753	392	768	251	5.1	7.6	**0.16%**	723	376	738	241	5.3	8.0	**0.05%**	702	365	716	234	5.5	8.2
C_S	**0.22%**	651	339	664	217	5.9	8.8	**0.13%**	666	347	680	222	5.8	8.6	**0.04%**	687	358	701	229	5.6	8.4

Parameter	+10%							+30%							+50%						
	PPC	**T**	Q_s	Q_m	Q_{mr}	**xy**	**zy**	**PPC**	**T**	Q_s	Q_m	Q_{mr}	**xy**	**zy**	**PPC**	**T**	Q_s	Q_m	Q_{mr}	**xy**	**zy**
D	**10.24%**	977.3	373	1097	215	5.9	9.8	**21.61%**[a]	1200	383	1500	210	6.4	11.2	**17.41%**[a]	1200	383	1500	210	6.4	11.2
t_m	**-6.03%**	1025	356	1046	205	5.6	9.4	**-24.03%**[a]	1553.6	305	1494	183	6.2	9.9	**-47.08%**[a]	1792.7	305	1494	183	5.4	8.6
γ_m	**-0.22%**	693	362	709	231	5.5	8.3	**-0.66%**	690	361	708	230	5.6	8.3	**-1.11%**	690	363	711	230	5.6	8.3
C_{sh}	**-0.08%**	711	370	726	237	5.4	8.1	**-0.24%**	741	386	756	247	5.2	7.8	**-0.39%**	768	400	784	256	5.0	7.5
γ_s	**-0.10%**	696	363	710	232	5.5	8.3	**-0.29%**	696	365	710	232	5.5	8.3	**-0.48%**	696	366	710	232	5.5	8.3
C_H	**-0.08%**	678	353	692	226	5.7	8.5	**-0.22%**	651	339	664	217	5.9	8.8	**-0.36%**	627	326	640	209	6.1	9.2
C_O	**-0.05%**	702	365	716	234	5.5	8.2	**-0.14%**	828	431	845	207	4.6	9.3	**-0.22%**	844	439	861	211	4.5	9.1
C_h	**-0.05%**	681	355	695	227	5.6	8.5	**-0.14%**	864	300	882	216	6.7	8.9	**-0.22%**	844	293	861	211	6.8	9.1
C_{Hr}	**-0.05%**	681	355	695	227	5.6	8.5	**-0.14%**	772	402	788	193	5.0	9.9	**-0.23%**	752	392	767	188	5.1	10.2
C_S	**-0.04%**	702	365	716	234	5.5	8.2	**-0.12%**	920	319	939	230	6.3	8.3	**-0.18%**	1040	361	1061	208	5.5	9.2

[a] Case of (1-γm)/tm<d

TABLE III. RESULTS OF SENSITIVITY ANALYSIS ON DIFFERENT PARAMETERS

Parameter	Profit Percentage Change						Decision variables	
	-50%	-30%	-10%	+10%	+30%	+50%		
S_P	-106.57%[b]	-63.94%	-21.31%	21.31%	63.94%	106.57%	T	696
C_m	30.45%	18.27%	6.09%	-6.09%	-18.27%	-30.45%	Q_s	362
C_R	22.19%	13.32%	4.44%	-4.44%	-13.32%	-22.19%	Q_m	710
C_d	1.11%	0.67%	0.22%	-0.22%	-0.67%	-1.11%	Q_{mr}	232
C_D	1.07%	0.64%	0.21%	-0.21%	-0.64%	-1.07%	xy	5.5
C_L	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	zy	8.3

[b] Income < Total Cost, minimum selling price is 53% of the nominal value.

Appendix A. Average end product inventory per period at the manufacturer is given by the following equation : $\frac{Q_{mr}}{2}\left((z-1)-\frac{(z-2)t_m}{(1-\gamma_m)}\left[(1-\gamma_m)P,d\right]^-\right)$

The following is the general form for the total end product inventory at the manufacturer per *dominant cycle* (i.e., the cycle that is determined by the minimum rate), see *Fig. 1A.*

$$=zQ_{mr}^2\left(\frac{1}{(1-\gamma_m)P}+\frac{(z-1)}{\left[(1-\gamma_m)P,d\right]^-}\right)-\frac{z^2Q_{mr}^2}{2(1-\gamma_m)P}-\left(\frac{Q_{mr}^2}{\left[(1-\gamma_m)P,d\right]^-}\left[1+2+\ldots+(z-1)\right]\right)$$

Since the summation of an arithmetic series is as follows:

$$\sum_{k=0}^{n-1}a+kd=\frac{n}{2}\left(2a+(n-1)d\right)$$

$$\sum_{k=0}^{z-1}k=1+2+\ldots+(z-1)=\frac{z}{2}(z-1)$$

Then,

$$=zQ_{mr}^2\left(\frac{1}{(1-\gamma_m)P}+\frac{(z-1)}{\left[(1-\gamma_m)P,d\right]^-}\right)-\frac{z^2Q_{mr}^2}{2(1-\gamma_m)P}-\frac{z(z-1)}{2}\left(\frac{Q_{mr}^2}{\left[(1-\gamma_m)P,d\right]^-}\right)$$

$$=\frac{zQ_{mr}^2}{2}\left(\frac{2}{(1-\gamma_m)P}+\frac{2(z-1)}{\left[(1-\gamma_m)P,d\right]^-}-\frac{z}{(1-\gamma_m)P}-\frac{(z-1)}{\left[(1-\gamma_m)P,d\right]^-}\right)$$

$$=\frac{zQ_{mr}^2}{2}\left(\frac{(z-1)}{\left[(1-\gamma_m)P,d\right]^-}-\frac{(z-2)}{(1-\gamma_m)P}\right)$$

Manufacturer's average inventory per period

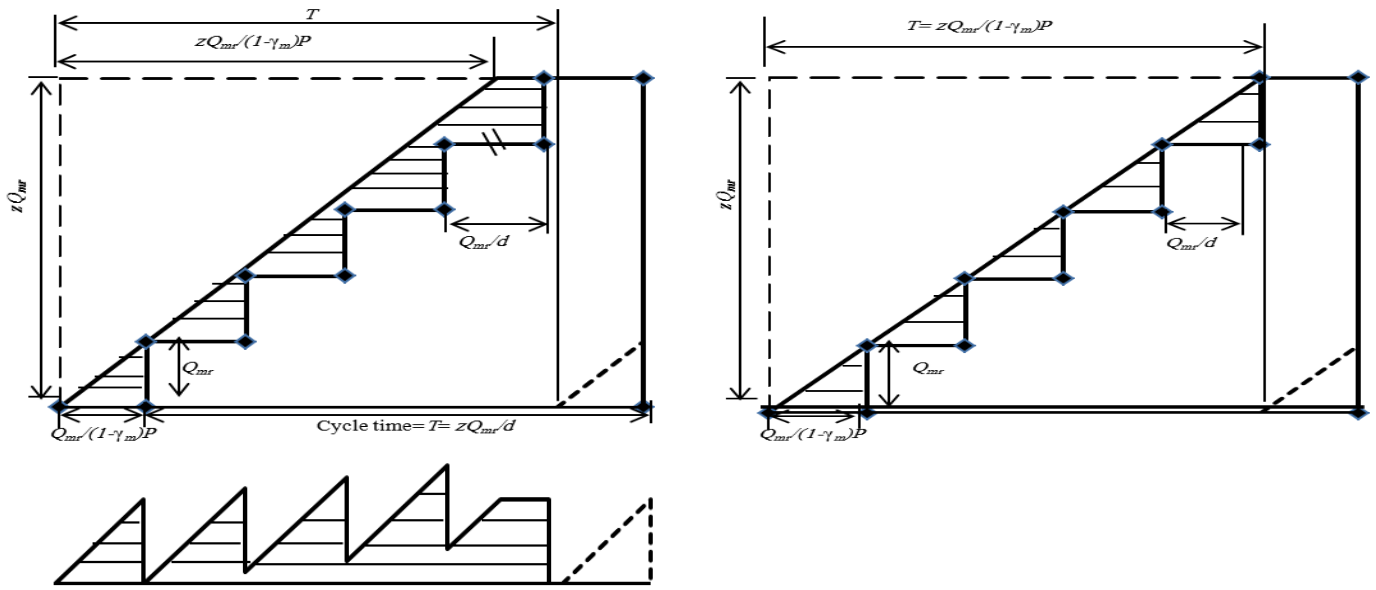
$$=\frac{zQ_{mr}^2}{2}\left(\frac{(z-1)}{\left[\frac{(1-\gamma_m)}{t_m},d\right]^-}-\frac{(z-2)t_m}{(1-\gamma_m)}\right)\left(\frac{1}{T}\right)$$

$$=\frac{zQ_{mr}^2}{2}\left(\frac{(z-1)}{\left[(1-\gamma_m)P,d\right]^-}-\frac{(z-2)t_m}{(1-\gamma_m)}\right)\left(\frac{\left[(1-\gamma_m)P,d\right]^-}{zQ_{mr}}\right)$$

$$=\frac{Q_{mr}}{2}\left((z-1)-\frac{(z-2)t_m}{(1-\gamma_m)}\left[(1-\gamma_m)P,d\right]^-\right)\#$$

In case the production exceeds the demand (*Fig. 1A,a*); the manufacturer's average inventory holding cost per period is as follows: $\frac{Q_{mr}}{2}\left((z-1)-\frac{(z-2)t_m}{(1-\gamma_m)}d\right)C_H$

In case the demand exceeds the production (*Fig. 1A,b*); the manufacturer's average inventory holding cost per period is as follows: $\frac{zQ_{mr}^2}{2(1-\gamma_m)}t_m\left(\frac{1}{T}\right)C_H=\frac{Q_{mr}}{2}C_H$.



a) b)

Fig. 1A. Accumulated inventory at the manufacturer, a) Case (1- γm)*P>d* b) Case (1- γm)*P<d*

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