

Concentration and Temperature Profile of LiBr Aqueous Solution Flowing Over Horizontal Tube with Film Redistribution

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Abstract:- The paper presents a numerical investigation on water vapor absorption by an aqueous Lithium Bromide solution flowing over horizontal tubes with film redistribution. The model is developed based on continuity, momentum, energy and mass diffusion equations for certain boundary conditions and assumptions. Dimensionless variables are used to transform the calculation domain into a rectangular grid that normalizes the spatial derivatives in x and y directions. Profile for variation of Lithium Bromide mass fraction and temperature across the solution film thickness are generated. Concentration profile across film thickness reveals that the mass fraction boundary layer is thin at interface and does not penetrate much into the film. To enhance absorption in such state, redistributors approach is proposed. Simulation results show an increment in absorption of vapor. Temperature profile shows non uniformity for larger value of θ , probably due to dominating gravitational force in this region which leads to flow separation.

Keywords: Film flow; Horizontal tube; Absorption; Redistribution; Mathematical Model

Nomenclature

C_p	specific heat ($J\ kg^{-1}K^{-1}$)
D	mass diffusivity (m^2s^{-1})
g	gravitational acceleration ($m\ s^{-2}$)
h_a	heat of absorption ($J\ kg^{-1}$)
h_{cw}	cooling water side heat transfer coefficient ($W\ m^{-2}K^{-1}$)
k	thermal conductivity ($W\ m^{-1}K^{-1}$)
m	mass (kg)
P	pressure (Pa)
r	radius (m)
T	Temperature (K)
U	overall heat transfer coefficient between cooling water and tube ($Wm^{-2}K^{-1}$)
v_x	velocity in flow direction ($m\ s^{-1}$)
v_y	velocity in film thickness direction ($m\ s^{-1}$)
X	LiBr mass fraction (%)
x	flow direction coordinate
y	coordinate perpendicular to flow direction
α	thermal diffusivity ($m^2\ s^{-1}$)
δ	film thickness(m)
Γ	specific film flow rate ($kg\ m^{-1} s^{-1}$)
μ	dynamic viscosity ($kg\ m^{-1} s^{-1}$)
γ	kinematic viscosity ($m^2\ s^{-1}$)
ρ	density ($kg\ m^{-3}$)
θ	angle (radian)
φ	non dimensional coordinate in x-direction
ψ	non dimensional coordinate in y-direction
Subscripts	
i	inlet or inner
if	interference
o	outer
t	tube

1. INTRODUCTION

Increased global warming and environmental effect of chlorofluorocarbon has stimulated interest in development of vapor absorption systems to generate cooling effect. In such systems, water–lithium bromide (H_2O –LiBr) is most widely used working fluid pair. Vapor absorption cycle for the system is explained in fig. 1. In this cycle, vapor evaporates and separates from aqueous LiBr solution in generator by gaining heat from external source. These vapors shift towards condenser and condensed by releasing heat to cooling water. The condensed water is throttled to evaporator at low pressure, where it changes its phase from liquid to vapor by gaining latent heat of vaporization from surroundings that generates cooling effect. These vapors are then absorbed by aqueous LiBr solution supplied from generator in the absorber. The absorption of water vapor is an exothermic process and released heat is passed to the cooling water. From exit of absorber, solution with reduced concentration of LiBr is pumped to generator at higher pressure and continues the cycle.

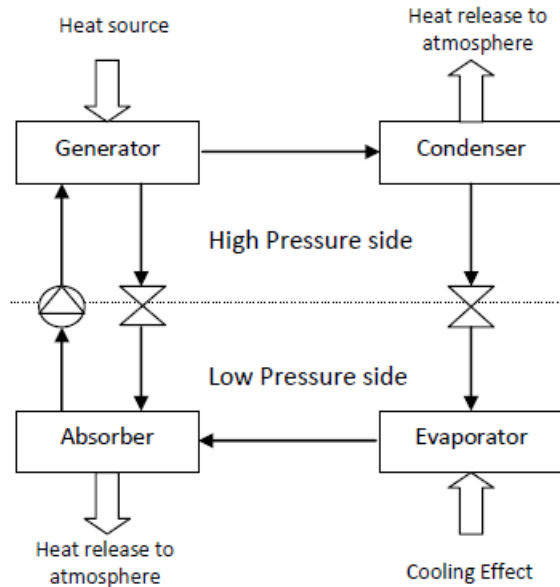


Fig.1. Schematic of a Single effect LiBr-Water absorption System.

A review of absorption refrigeration technologies was presented by Srikuhirin et al. [1]. Design and construction methodology of a LiBr–water absorption system was discussed by Florides et al. [2]. Sharma et al. [3, 4] had proposed an absorption system configuration with certain capacity and performed sensitivity analysis.

In above discussed cycle, absorption is an exothermic process and excess of heat needs to be removed from film. This simultaneous heat and mass transfer make the absorber critical and have significant effects on overall system efficiency. An experimental study on absorber with film flowing over vertical surface was performed by Kim et al. [5]. A model of water-cooled vertical plate absorber was developed by Yoon et al. [6] and by Yigit [7] on vertical tube absorber. Islam et al. [8] discussed a linearized coupled model for falling film absorbers. In water cooled horizontal tube type configuration, absorption phenomenon depends on solution properties and its flow rate. Other contributing factors which govern flow around and between the tubes are tube diameter, tube spacing and surface wetting. Oxygen gas absorption on completely wetted horizontal tubes was experimentally investigated by Nosoko et al. [9]. A detailed review of heat and mass transfer models for falling film absorption was done by Killion et al. [10]. Wassenaar [11] had conducted experimental work on the performance of horizontal tube absorber. Absorption by a wavy film flow was discussed in the model of Patnaik et al. [12].

The aim of present study is to simulate the heat and mass transfer process in water cooled horizontal tube absorber. Two-dimensional governing equations were formulated and solved for the falling film regime over a smooth horizontal tube. Using finite difference method temperature and concentration distributions were found along the film thickness. A redistribution approach had been proposed to enhance the absorption of vapor by aqueous LiBr solution.

2. PROBLEM GEOMETRY AND ASSUMPTION

Geometry of the problem and flow regime on a representative absorber tube is shown in fig. 2. The LiBr solution is injected from a distributor at the absorber inlets on the first tube and flows down by gravity forming a thin falling film over horizontal tube. The vapors present in surroundings of the tube are absorbed at the interface of the falling film and reduces the concentration of lithium bromide in the solution. Release of absorption heat at film interface is transferred to the cooling water flowing inside the tube. At bottom of tube, film breaks into drops and falls on the next tube.

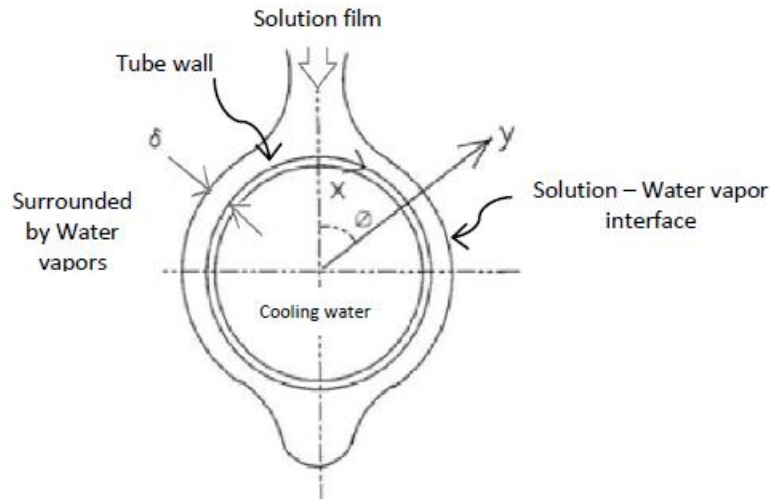


Fig.2. Solution flow over water cooled horizontal tube.

In above geometry, redistributors are placed in form of small diameter wire at specific positions along the length of horizontal tube and perpendicular to the flow direction of falling film. This agitates the falling film and homogenizes LiBr concentration throughout the film thickness and afterward provides a fresh surface for absorption of vapor at the interface. Considering gravitational effect on falling film, redistributors are positioned on upper half of the tube. Exposed surface area of film and time of exposure at interface are critical for absorption of vapors and influence the number of redistribution positions. Hence for illustration purpose, four redistributors position symmetrically placed at 30° and 60° had been selected (Fig.3). Properties of solution were referred from ASHRAE handbook of fundamentals [13].

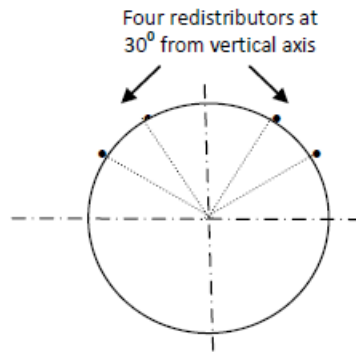


Fig.3. Redistribution positions over horizontal tube.

3. MATHEMATICAL FORMULATION

3.1 Governing equations

Under following assumptions;

- The aqueous LiBr solution is incompressible.
- The solution is Newtonian.
- There is no pressure variation in the direction of film thickness.
- The flow of falling film is laminar
- The thermo physical properties of solution are constant.

Mass balance on elemental control volume of falling film represented in fig. 2 yields the mass continuity equation.

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \tag{1}$$

Resolving the forces acting on fluid element for horizontal tube at an angle θ from vertical axis yield; $\mu \frac{\partial^2 v_x}{\partial y^2} = -\rho g \sin\theta$

$$\tag{2}$$

Neglecting pressure gradients, viscous dissipation, heat conduction in the direction of flow and transport of energy by mass diffusion, the energy analysis of elemental volume reduces energy equation to;

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

Change of velocity in film thickness direction due to diffusion can be neglected because of small absorption rate. By ignoring diffusion in flow direction and assumption of no chemical reaction, the mass transfer equation in terms of LiBr concentration is expressed as;

$$v_x \frac{\partial X}{\partial x} + v_y \frac{\partial X}{\partial y} = D \frac{\partial^2 X}{\partial y^2} \quad (4)$$

As the tube wall is impermeable, boundary condition for concentration at wall became;

$$\left. \frac{\partial X}{\partial y} \right|_{y=0} = 0 \quad (5)$$

3.2 Derived equations

Integrating Eq. (2) and determining constants with following boundary conditions that;

$$at, y = 0, v_x = 0 \quad \text{and} \quad at, y = \delta, \frac{\partial v_x}{\partial y} = 0 \quad (6)$$

Velocity profile in film flow direction is derived in term of y/δ as follows;

$$v_x = \frac{g\delta^2}{2\gamma} \sin\theta \left[2 \left(\frac{y}{\delta} \right) - \left(\frac{y}{\delta} \right)^2 \right] \quad (7)$$

Using continuity Eq. (1) and boundary condition of no shear stress exerted at the interface of solution and vapor;

$$\left. \frac{\partial v_x}{\partial y} \right|_{y=\delta} = 0 \quad (8)$$

Velocity profile in film thickness direction is given as;

$$v_y = -\frac{g\delta^3}{r_o\gamma} \cos\theta \left(\frac{1}{2} \left(\frac{y}{\delta} \right)^2 - \frac{1}{6} \left(\frac{y}{\delta} \right)^3 \right) - \frac{g\delta^2}{2\gamma} \sin\theta \frac{d\delta}{dx} \left(\frac{y}{\delta} \right)^2 \quad (9)$$

Under assumption that, falling film completely wets tube wall, integrating velocity profile Eq. (7) from 0 to δ , film thickness over horizontal tube is derived as;

$$\delta = \left(\frac{3\gamma r}{\rho g \sin\theta} \right)^{1/3} \quad (10)$$

Differentiating Eq. (10) with respect to x, the rate of change of film thickness in x direction become;

$$\frac{d\delta}{dx} = - \left(\frac{\gamma r}{9\rho g} \right)^{1/3} \frac{1}{r_o \sin^{1/3}\theta \tan\theta} \quad (11)$$

Heat transfer through conduction at the copper tube wall;

$$k \left. \frac{\partial T}{\partial y} \right|_{y=0} = U(T_{y=0} - T_{cw}) \quad (12)$$

The overall heat transfer coefficient (U) in Eq. (12) is defined as;

$$U = \frac{1}{\frac{r_o}{h_{cw}r_i} + \frac{r_o \ln(r_o/r_i)}{k_t}} \quad (13)$$

Energy released at solution-vapor interface due to absorption of vapor is equal to the product of absorbed mass flux and heat of absorption. Balancing the heat transfer, following relation is established.

$$-k \left. \frac{\partial T}{\partial y} \right|_{y=\delta} = h_a \left. \frac{\rho D}{X_{y=\delta}} \frac{\partial X}{\partial y} \right|_{y=\delta} \quad (14)$$

Heat transferred between vapor and falling film is negligible. Thermodynamic equilibrium exists at the vapor - solution interface. Following equilibrium condition at interface is given in ASHRAE [13].

$$X_{y=\delta} = f(P, T_{y=\delta}) \quad (15)$$

3.3 Transformation of derived and governing equations

To normalize the spatial derivatives in x and y directions, following dimensionless variables were formed considering that the film thickness is much smaller than circumference of tube.

$$\varphi = \frac{\theta}{\pi} = \frac{x}{\pi r_o} \quad (16)$$

$$\psi = \frac{y}{\delta(x)} \quad (17)$$

This transforms the calculation domain into a rectangular grid and reduces truncation error in finite difference representation. Accordingly transformation of equations (7), (9), (10) and (11) results in following form;

$$v_x(\varphi, \psi) = \frac{g}{\gamma} \sin(\pi\varphi) \delta^2 \left(\psi - \frac{\psi^2}{2} \right) \quad (18)$$

$$v_y(\varphi, \psi) = -\frac{g\delta^3}{r_o\gamma} \cos(\pi\varphi) \left(\frac{\psi^2}{2} - \frac{\psi^3}{6} \right) - \frac{g\delta^2}{2\pi r_o\gamma} \sin(\pi\varphi) \frac{d\delta}{d\varphi} \psi^2 \quad (19)$$

$$\delta(\varphi) = \left(\frac{3\gamma\Gamma}{\rho g \sin(\pi\varphi)} \right)^{1/3} \quad (20)$$

$$\frac{d\delta}{d\varphi} = - \left(\frac{\gamma\Gamma}{9\rho g} \right)^{1/3} \frac{\pi}{\sin^{1/3}(\pi\varphi) \tan(\pi\varphi)} \quad (21)$$

Similarly, the energy Eq. (3) and mass diffusion Eq. (4) are transformed as;

$$\frac{\partial T}{\partial \varphi} + \left(\frac{v_y}{v_x} \frac{\pi r_o}{\delta} - \frac{\psi}{\delta} \frac{d\delta}{d\varphi} \right) \frac{\partial T}{\partial \psi} = \frac{\alpha}{\delta^2} \frac{\pi r_o}{v_x} \frac{\partial^2 T}{\partial \psi^2} \quad (22)$$

$$\frac{\partial X}{\partial \varphi} + \left(\frac{v_y}{v_x} \frac{\pi r_o}{\delta} - \frac{\psi}{\delta} \frac{d\delta}{d\varphi} \right) \frac{\partial X}{\partial \psi} = \frac{D}{\delta^2} \frac{\pi r_o}{v_x} \frac{\partial^2 X}{\partial \psi^2} \quad (23)$$

4. NUMERICAL METHODOLOGY

Eq. (22) and Eq. (23) are parabolic differential equations in following form;

$$\frac{\partial T}{\partial \varphi} + K1 \frac{\partial T}{\partial \psi} = K2 \frac{\partial^2 T}{\partial \psi^2} \quad (24)$$

$$\frac{\partial X}{\partial \varphi} + K1 \frac{\partial X}{\partial \psi} = K3 \frac{\partial^2 X}{\partial \psi^2} \tag{25}$$

Here, $K1 = \left(\frac{v_y}{v_x} \frac{\pi r_o}{\delta} - \frac{\psi}{\delta} \frac{d\delta}{d\varphi} \right)$, $K2 = \frac{\alpha}{\delta^2} \frac{\pi r_o}{v_x}$ and $K3 = \frac{D}{\delta^2} \frac{\pi r_o}{v_x}$

Above set of equations were solved by using implicit finite difference structure based on Crank-Nicolson methodology.

$$\frac{\partial T}{\partial \varphi} = \frac{T_{j,n+1} - T_{j,n-1}}{2\Delta\varphi} \tag{26}$$

$$\frac{\partial T}{\partial \psi} = \frac{T_{j+1,n} - T_{j-1,n}}{2\Delta\psi} \tag{27}$$

$$\frac{\partial^2 T}{\partial \psi^2} = \frac{T_{j+1,n} - 2T_{j,n} - T_{j-1,n}}{\Delta\psi^2} \tag{28}$$

And,

$$\frac{\partial X}{\partial \varphi} = \frac{X_{j,n+1} - X_{j,n-1}}{2\Delta\varphi} \tag{29}$$

$$\frac{\partial X}{\partial \psi} = \frac{X_{j+1,n} - X_{j-1,n}}{2\Delta\psi} \tag{30}$$

$$\frac{\partial^2 X}{\partial \psi^2} = \frac{X_{j+1,n} - 2X_{j,n} - X_{j-1,n}}{\Delta\psi^2} \tag{31}$$

The tridiagonal matrix systems thus obtained is worked out by Thomas algorithm. Discretization of the solution area is shown in Fig. 4. Flow chart for solving equations has been given in Fig.5.

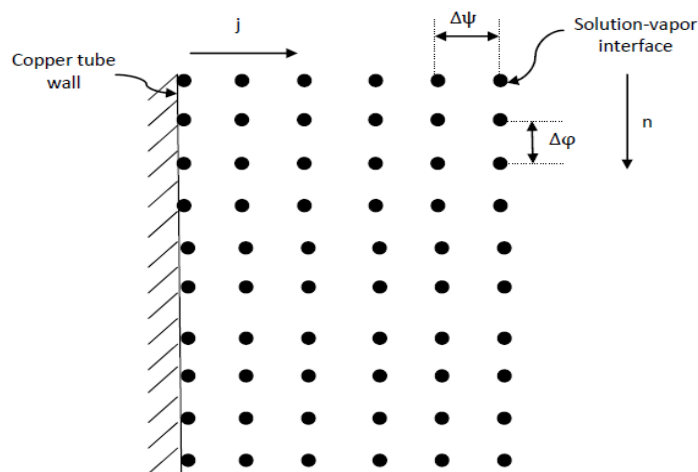


Fig.4. Discretization of the solution area

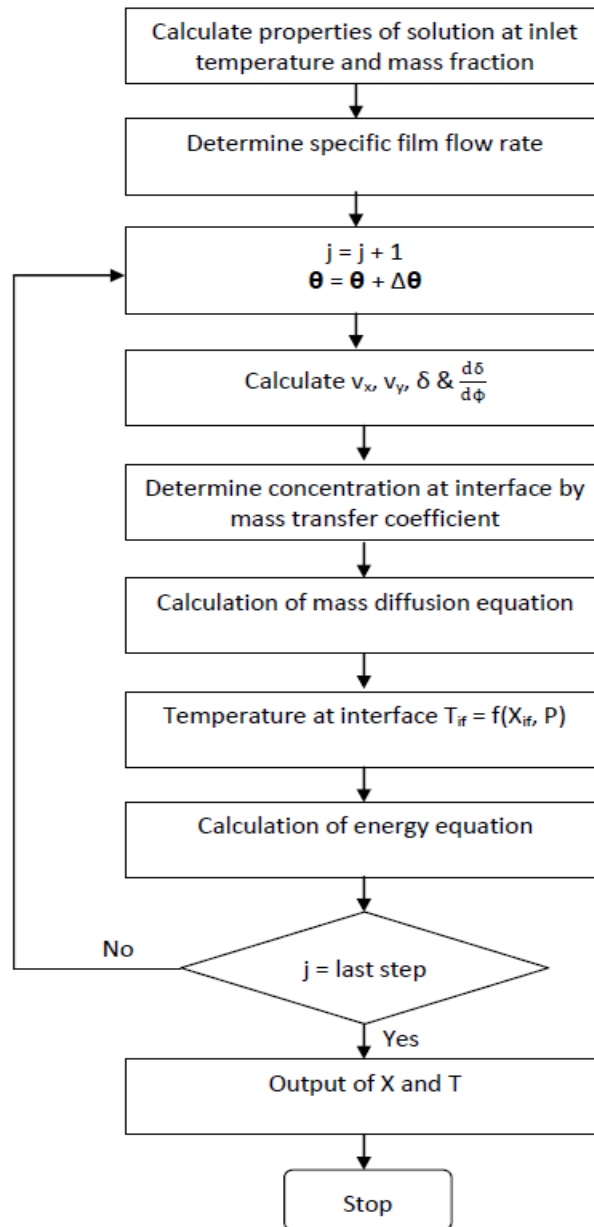


Fig.5. Flow chart for calculation

5. RESULTS AND DISCUSSION

The study is conducted for a water-cooled horizontal tube type absorber where aqueous LiBr solution flow over tube having 15.74 falling film Reynolds number. A two-dimensional numerical simulation of water vapor absorption by falling film has been performed. The temperature and LiBr concentration profile across the film thickness are developed. A film redistribution approach is proposed and simulated results are generated.

Concentration profile without redistribution being plot for positions from $y/\delta = 0$ (tube wall) to $y/\delta = 1$ (solution – vapor interface) is given in Fig. 6. With increase in θ , as the flow passes over the tube, mass fraction of LiBr in the solution at the interface decreases due to absorption of vapors. The graph indicates that the variation in concentration of LiBr does not penetrate much in the film. It is observed that the change in concentration nullifies for $y/\delta < 0.4$. The bulk concentration of LiBr in the solution at $\theta = 170^\circ$ is calculated to be 59.19 %. Simulated concentration profile shows similar behavior of concentration in aqueous solution falling film being presented by Rogdakis et al. [14] and Babadi et al. [15]. In the model presented by Kyung et al. [16] the mass fraction boundary layer is even smaller. Papaefthimiou et al. [17] have validated similar results with existing measurement data in the non-wavy flow regime. This small mass fraction boundary layer is of prime focus of efforts to enhance absorption by redistribution.

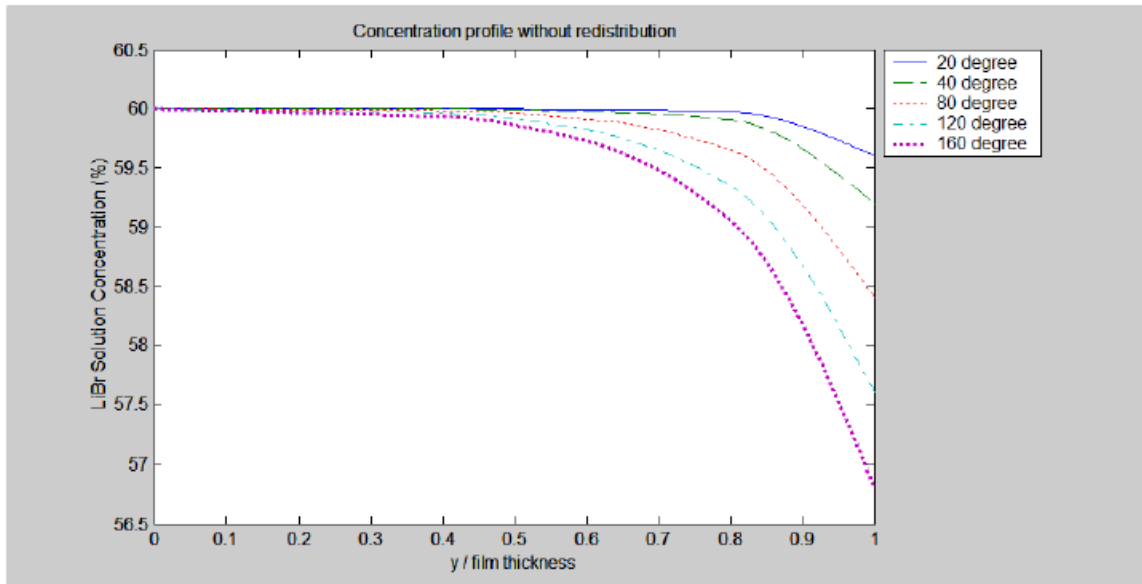


Fig. 6 Concentration Profile without redistribution.

At redistribution positions of 30° and 60° , it is assumed that LiBr concentration in aqueous solution became homogenous and provide fresh surface for absorption afterwards. Fig. 7 illustrates the concentration profile with redistribution. Through this approach, bulk concentration at $\theta = 170^\circ$ is determined to be 58.97 %. This is less than 0.22 % of the value being determined without redistribution for the same region. Difference of concentration shows that through redistribution approach more vapours are being absorbed by the falling film.

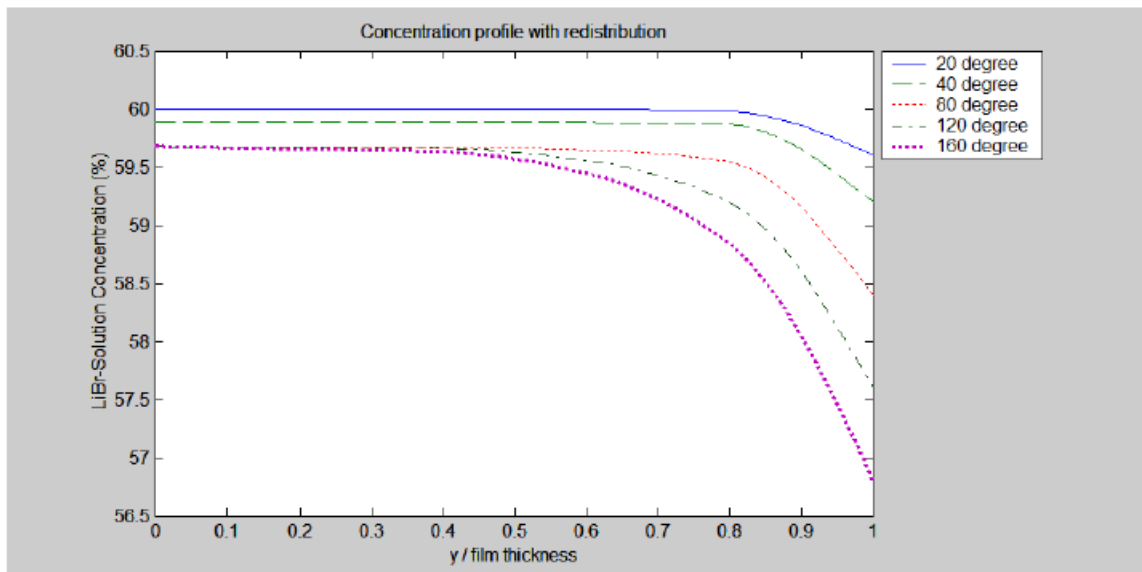


Fig. 7 Concentration profile with redistribution

Temperature profile across the film thickness is shown in Fig. 8. It is obvious that temperature of film decreases in radial inward direction due to heat being transferred to cooling water. Near the tube wall, temperature profile is found to be nonlinear. This may because of the time taken to propagate cooling effect through the film. The rate of change of temperature is practically constant at all value of θ . For larger value of θ , temperature variation is not perfectly linear. Curve starts showing non uniformity for θ greater than 100° and y/δ greater than 0.5. In this region it probably indicates that, gravitational force on solution starts dominating, which leads to flow separation over the tube. This analysis requires further investigation.

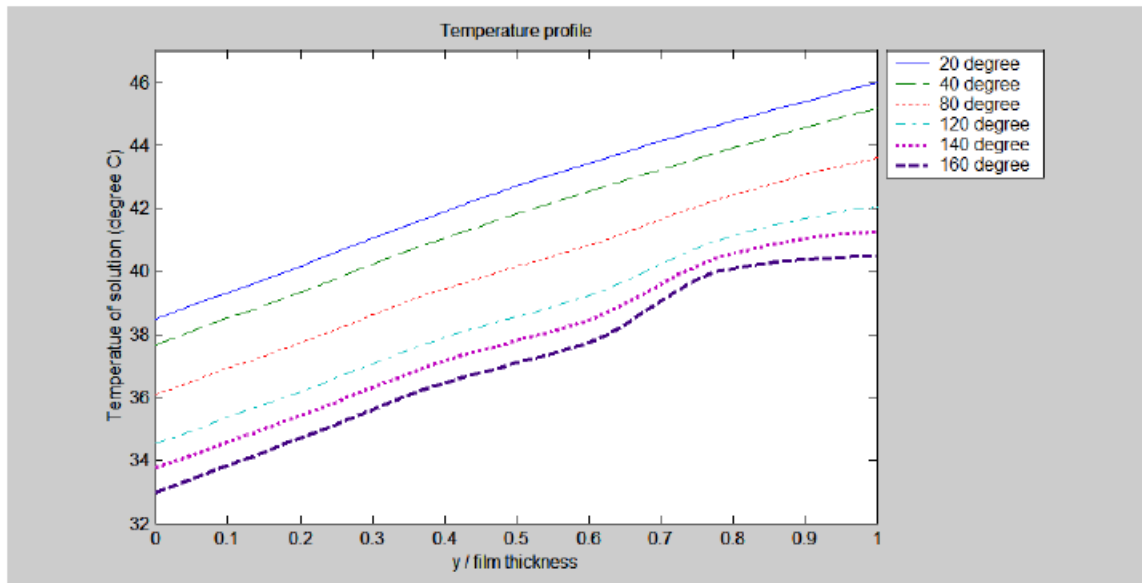


Fig. 8 Temperature profile

CONCLUSION

A physical model has been developed to study the phenomenon of vapor absorption by laminar film flowing over a water-cooled horizontal tube. An approach is proposed to enhance the absorption by agitating the falling film at specific position over the tube. Simulated results of this film redistribution show that there is an increase in absorption of vapors. Further investigation is required, for the temperature variation curve which starts showing non uniformity for θ greater than 100° and y/δ greater than 0.5.

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