Computation of Stress Intensity Factor and Critical crack length of ASTM A36 steel using Fracture Mechanics

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Abstract

Arising from the manufacturing process, interior and surface flaws are found in all metal structures. Not all such flaws are unstable under service conditions. Cracks often develop in the structural member due to high stress concentration factor. If one can calculate the critical crack length, an engineer can schedule inspection accordingly and repair or replace the part before failure happens. Fracture mechanics is the analysis of flaws to discover those that are safe and those that are liable to propagate as cracks and so cause failure of the flawed structure. Ensuring safe operation of structure despite these inherent flaws is achieved through fracture mechanics. Linear elastic fracture mechanics principles might use for calculation of stress intensity factor (SIF). One of the techniques of fracture mechanics is displacement exploration technique will be adopting for stress intensity factor (SIF) calculation. This technique uses the generalized finite element software, ANSYS. The values that are obtained may compare with that of theoretical values and observe that they are in order. Critical crack length may determine for different load values; by using SIF the growth of crack is going to study. This parameter will give the possibility to analyze the possible crack growth or the possible failure if a given load is applied to the structure.

1. Introduction

Fracture mechanics is a field of solid mechanics that deals with the mechanical behavior of cracked bodies. Fracture is a problem that society has faced for as long as there have been man-made structures. The problem may actually be worse today than in previous centuries, because more can go wrong in our complex technological society. Major airline crashes, for instance, would not be possible without modem aerospace technology. The Griffith theory as described in "the phenomenon of rupture and flow of solids"(1920) assumes that material contains crack like defects and work must be performed on the material to supply the energy needed to propagate the crack by creating two new crack surfaces. Thirty years later, Irwin and Orawan [Farahmand and Glassco, 1997][1] observed that for ductile material to fracture, the stored strain energy is consumed for both the formulation of two new cracked surfaces and the work done in plastic deformation. Later a great deal of research has been carried out on fracture mechanics concept and fatigue crack growth in life assessment that are employed to predict cracking of engineering materials at high temperatures under static and cyclic loading F.D. javanroodi;K. M. Nikbin[2006][2] discussed A model for predicting creep crack growth initiation and the creep uniaxial ductility. The effects of cyclic loading on crack growth behaviour are considered and fractography evidence is shown to back a simple cumulative damage concept when dealing with creep/fatigue interaction. Y.B Yeo; E.H.Lim [2010][3] conducted in depth review on LEF evaluates of crack initiation zone by microscope technique. J. Parra-Michel; A. Martinez; and J.A. Rayas[2010][4] conducted electronic speckle pattern interferometry for computation of crack tip elastic stress intensity factor in mode I. A dual illumination beam system is used to obtain stress intensity factor .Neelakantha V Londe; T.Jaya raju; P.R.Sadhanandha rao[2010][5] conducted fracture toughness of high strength metallic materials is determined by standard test methods like ASTM E 399,ASTM E 1820 using standard specimen geometries such as compact test or single edge notched bend specimens. S. Das; R. Prasad; S. Mukhopadhyay [2011][6] carried out stress intensity factor of an edge crack in composite media expressing the displacements and stresses in plane strain conditions in terms of harmonic functions this problem is reduced by a pair of integral equations solved by the Hilbert transform technique. Yu. G. Matvienko and E.L. Muravin [2011][7] conducted numerical estimation of plastic J-integral by the load separation method for inclined cracks under tension. Case studies in numerical simulation of crack trajectories in brittle materials H. Jasarevic; S. Gagula[2012][8] statistical fracture mechanics formulation. Christian Skodborg Hansen ;Henrik Stang[2012][9] discussed fracture mechanical analysis of strengthen concrete tension members with one crack ..Shouetsu Itou[2012][10]computed stress intensity factor for two parallel interface cracks between a non homogeneous bonding layer and two dissimilar orthotropic half planes under tension Rafael G. Savioli; Claudio Ruggeri [2012][11] focuses on evaluation procedure to determine the elastic-plastic J-integral and crack tip opening displacement..A Ramachandra murthy; G S Palani; Nagesh R Iyer[2012][12]discussed two major objectives of damage tolerant evaluation, namely, the remaining life prediction and residual strength evaluation of stiffened panels.

2. Modes of Crack Extension

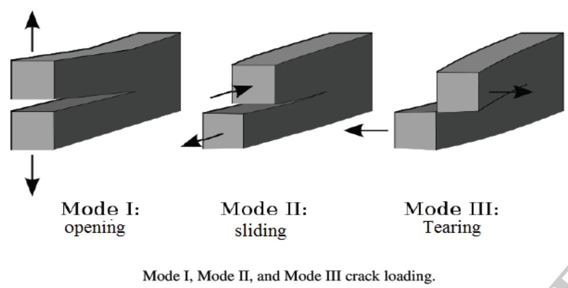


Fig:1 Modes of crack extension

Mode I:

The forces are perpendicular to the crack (the crack is horizontal and the forces are vertical), pulling the crack open. This is referred to as the opening mode. What would happen if both of the forces were pushing down on the crack? Nothing. This would close the crack.

Mode II:

The forces are parallel to the crack. One force is pushing the top half of the crack back and the other is pulling the bottom half of the crack forward, both along the same line. This creates a shear crack: the crack is sliding along itself. It is called in-plane shear beacuse the forces are not causing the material to move out of its original plane.

In this case, what would happen if both the forces were moving in the same direction, both forward or both backward? This would not cause the crack to grow, since all of the material would be moving in the same direction.

Mode III:

The forces are perpendicular to the crack (the crack is in front-back direction, the forces are pulling left and right). This causes the material to separate and slide along itself, moving out of its original plane (which is why its called out-of-plane shear). The forces could also be *pushing* left and right and the same effect would occur. But the forces have to be moving in opposite directions in order to grow the crack

3.Methodologies for Fracture Mechanics

Methodologies generally used for computation of SIF by using the results of finite element analysis are:

- Displacement correlation/extrapolation method,
- Nodal force approach,
- Strain Energy Release Rate technique,
- Virtual crack extension (VCE) technique,
- Modified crack closure integral (MCCI)
- Technique
- J-integral method

Among all the methods listed above, displacement correlation/extrapolation technique is a simple and efficient technique. In the present study, it is proposed to adopt this technique. Brief description about the technique is given below.

The displacement correlation/extrapolation method makes use of the crack opening displacements (COD) directly behind the crack tip. In the displacement correlation method, the FE displacements at the nodes directly behind the crack tip at which SIF are desired can be used by equating with the corresponding analytical expression for the displacement variation along the rays emanating from the crack tip.]. This provides a system of linear algebraic equations in terms of SIF. Alternatively, in the displacement extrapolation method, several points behind the crack tip is used along the line θ=180°. At each point the SIF is obtained as a function of distance behind the crack tip under study. Linear regression is then performed to obtain SIF value at the crack tip. The displacement correlation/extrapolation method is most often employed with quarter-point elements. The displacements near the crack front could be employed from any neighboring elements. It is observed that along the line θ=180°, the SIF obtained by using displacement extrapolation method is more accurate than other radial rays.

3.1 Stress intensity factor – What It Is:

The model referred to above is called the linear elastic fracture mechanics model and has found wide acceptance as a method for determining the resistance of a material to below-yield strength fractures. The model is based on the use of linear elastic stress analysis; therefore, in using the model one implicitly assumes that at the initiation of fracture any localized plastic deformation is small and considered within the surrounding elastic stress field. Application of linear elastic stress analysis tools to cracks of the type shown in Figure 2 shows that the local stress field (within r < a/10) is given by

$$\sigma_x = \frac{K}{\sqrt{2\Pi r}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$
$$\sigma_y = \frac{K}{\sqrt{2\Pi r}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \qquad \text{Eq….1}$$
$$\sigma_{xy} = \frac{K}{\sqrt{2\Pi r}} \sin\frac{\theta}{2}\left[\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right]$$

The stress in the third direction are given by $\sigma_z = \sigma_{xz} = \sigma_{yz} = 0$ for the plane stress problem, and when the third directional strains are zero (plane strain problem), the out of plane stresses become $\sigma_{xz} = \sigma_{yz} = 0$ and $\sigma_z = \nu (\sigma_x + \sigma_y)$. While the geometry and loading of a component may change, as long as the crack opens in a direction normal to the crack path, the crack tip stresses are found to be as given by Equations.1.

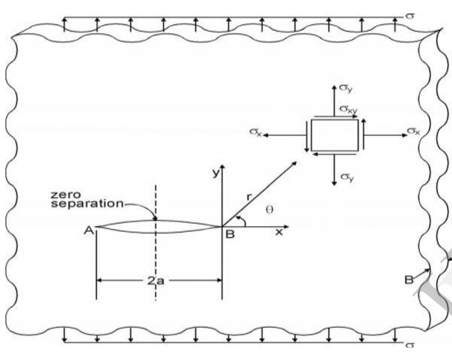


Figure 2. Infinite Plate with a Flaw that Extends Through Thickness

The parameter K, which occurs in all three stresses, is called the stress intensity factor because its magnitude determines the intensity or magnitude of the stresses in the crack tip region. The influence of external variables, i.e. magnitude and method of loading and the geometry of the cracked body, is sensed in the crack tip region only through the stress intensity factor. Because the dependence of the stresses (Equation.1) on the coordinate variables remain the same for different types of cracks and shaped bodies, the stress intensity factor is a single parameter characterization of the crack tip stress field. The stress intensity factors for each geometry can be described using the general form:

$$K = \sigma\beta\sqrt{\pi a}$$

$$\beta = (1 - 0.025.\alpha^2 + 0.06.\alpha^4).\sqrt{\sec\frac{\alpha\pi}{2}} \qquad \text{Eq..2}$$

Where,

KI = Stress intensity factor
β = Geometry factor
α = 2a/W
2a = Crack length
W = Width of the plate
σ = Force applied

Where the factor β is used to relate gross geometrical features to the stress intensity factor

4. Methodology

SIF is an important parameter for reliable prediction of Critical crack length. In order to validate the methodologies described in the previous sections, numerical studies on plate of ASTM A36 STEEL with a centre crack have been carried out. SIF has been calculated by using generalized finite element software, ANSYS [2002] [6]. Displacement extrapolation technique has been adopted for SIF calculation. The values obtained by using ANSYS have been compared with that of theoretically calculated values. Further, critical crack length have been predicted by using SIF values using residual strength diagram.

ASTM A36 steel is the carbon steel that is used in general plate application when the plate will be riveted, bolted, or welded, it also available in hot-rolled steels. The hot roll process means that the surface on this steel will be somewhat rough; it will bend much more quickly.

Table 1 : Properties of Material ASTM A36

ASTM A36 steel		
Minimum Properties	Ultimate Tensile Strength, psi	58,000 - 79,800
	Yield Strength, psi	36,300
	Elongation	20.0%
	Young's Modulus	$29*10^6$psi[5]
	Poisson's ratio	0.29[5]
	Fracture Toughness	90 ksi$\sqrt{in}$[5]
Chemistry	Iron (Fe)	99%
	Carbon (C)	0.26%
	Manganese (Mn)	0.75%
	Copper (Cu)	0.2%
	Phosphorus (P)	0.04%max
	Sulfur (S)	0.05%max

4.1 Calculation of SIF using theoretical formulae

In this we will find out the stress intensity factor of a plate of dimensions 'w' width 'h' height and't' thickness. First we will find out the stress intensity factor for the crack length 20mm and then we will find out for the other crack lengths up to 54mm.

We will find out the stress intensity factor by both analysis software ANSYS and by using mathematical formulae.

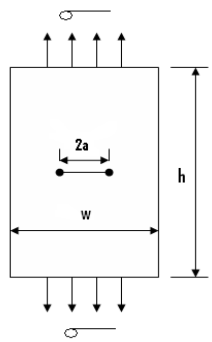


Figure 3: Plate with Centre Crack

The mathematical formulae for calculating stress intensity factor are

$$KI = \beta.\sigma.\sqrt{\pi.a}$$

$$\beta = (1 - 0.025.\alpha^2 + 0.06.\alpha^4).\sqrt{\sec\frac{\alpha\pi}{2}}$$

By using analysis package we find out the stress intensity factor values. We will compare those formulae values and analysis values we will get the stress intensity factor values.

In the present work we have taken a plate of length 200mm, width 200mm. In this we have taken a crack of length 20mm.We has applied a stress of 250N/mm^2 275N/mm^2 300N/mm^2 on the plate. We have calculated stress intensity factor for this problem by using the formula.

$$KI = \beta\sigma\sqrt{\pi a}$$

$$\beta = (1 - 0.025 * \alpha^2 + 0.06 * \alpha^4)\sqrt{\sec\frac{(\alpha\pi)}{2}}$$

Where,

$$\alpha = \frac{2a}{w} = (20/2) = 0.1$$

$$\beta = (1 - 0.025 * 0.1^2 + 0.06 * 0.1^4)\sqrt{\sec\frac{(0.1\pi)}{2}}$$

$= 0.9997578$

KI = 0.999103509*250*√ (∏*a)

= 44.306 N\mm$^{(-3/2)}$

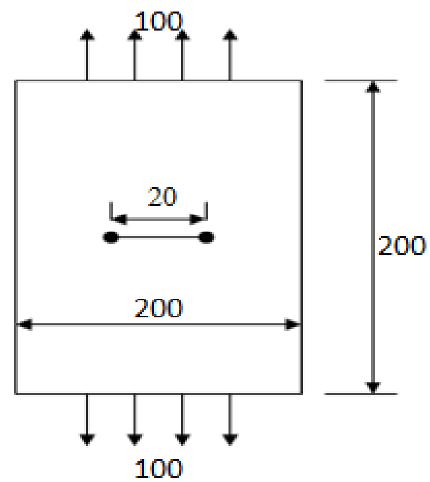


Figure 4: A plate with a crack length 20mm

By using this formula we have calculated stress intensity factor for other crack lengths ranging from 10mm to 40mm.

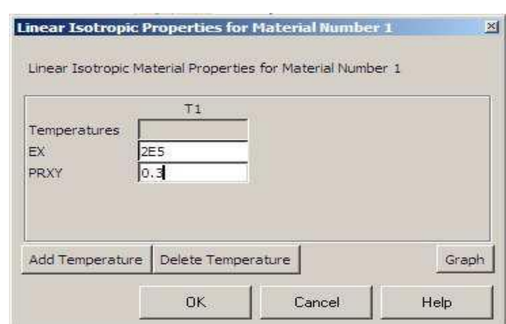
4.2 SIF calculation using displacement extrapolation technique

Modeling of Plate with center crack

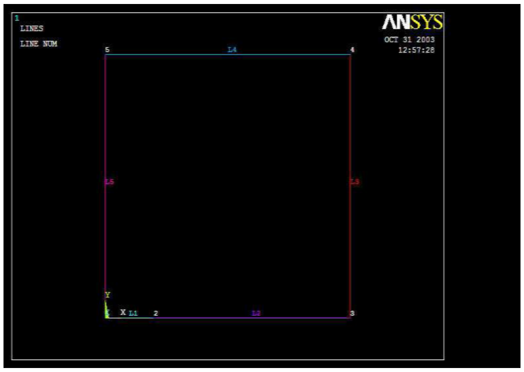
Creating Key points

Table 2: Keypoints Data

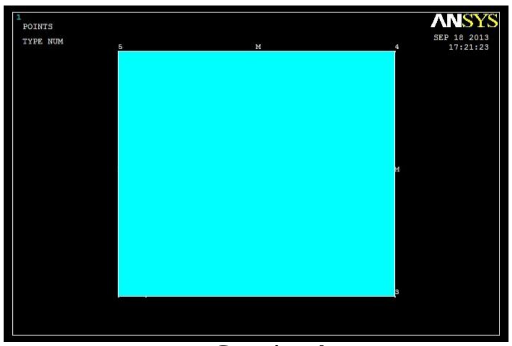
Key points #	**X**	**Y**
1	**0**	**0**
2	**0.01**	**0**
3	**0.1**	**0**
4	**0.1**	**0.1**
5	**0**	**0.1**



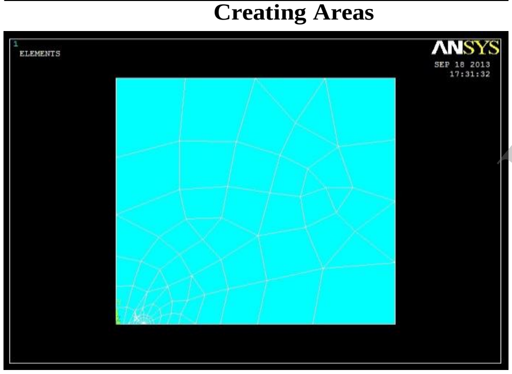
Material Properties



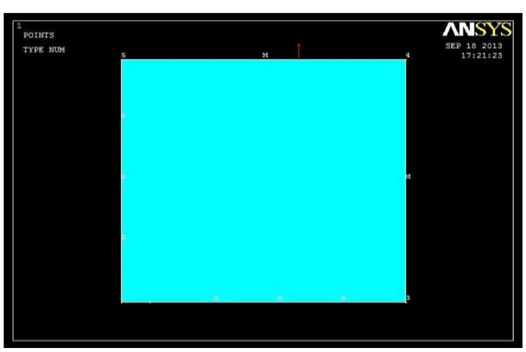
Creating Lines



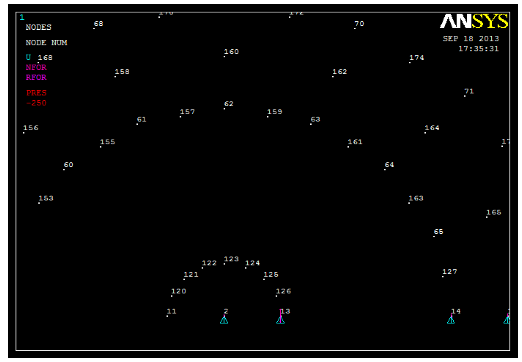
Creating Areas



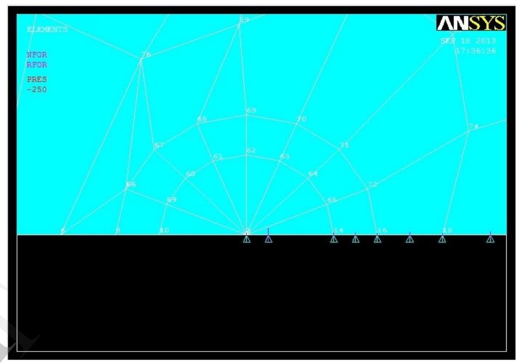
Meshing



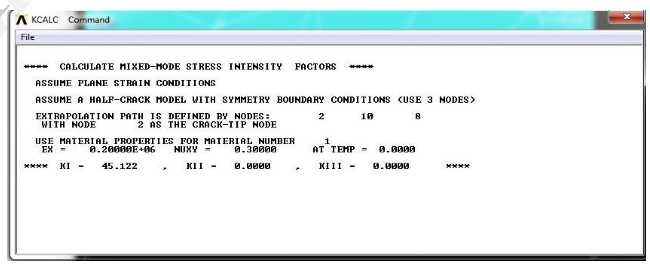
Applying Loads



. Create crack tip



Create local coordinate system



Stress intensity factor using KCALC

The window shows that the SIF's at the crack tip

$K_1 = 45.122$; $K_2 = 0$; $K_3 = 0$;.

We have calculated stress intensity factor values for different crack lengths using ANSYS software by using Displacement Extrapolation Technique. We have also calculated the stress intensity factor values by the literature formulae. And we have compared ANSYS values and literature values and find the difference between those values.The below table gives the theoretical value of stress intensity factor and ANSYS values using displacement extrapolation technique at varying crack lengths of 10mm to 52mm for different pressures of 250,275 and 300N/mm^2 ,and the difference between the two values at a pressures.

S.No	Crack length a mm	Theoretical value K $N\backslash mm^{(-3/2)}$	DET K value $N\backslash mm^{(-3/2)}$	Difference between them
1	10	44.300	45.10	0.8
2	12	48.523	50.76	2.27
3	14	52.381	53.21	0.8
4	16	56.016	57.59	1.5
5	18	59.405	61.87	2.4
6	20	62.609	63.63	1.03
7	24	68.562	69.32	0.76
8	28	74.030	76.43	2.4
9	30	76.615	78.65	2.0
10	34	81.537	82.22	0.6

Stress Intensity Factor values at 250N/mm^2

.No	Crack length a Mm	Theoretical value K $N\backslash mm^{(-3/2)}$	DET K value $N\backslash mm^{(-3/2)}$	Difference between them
1	10	48.730	49.558	0.79
2	12	53.376	54.29	0.91
3	14	56.247	58.93	2.6
4	16	61.618	63.54	1.9
5	18	64.732	68.03	3.2
6	20	68.870	70.31	1.4
7	24	75.416	76.74	1.3
8	28	81.433	81.12	0.3
9	30	84.236	85.64	1.4
10	32	87.025	89.91	2.8

Stress Intensity Factor values at 275N/mm^2

5. Residual strength curve:

In general, the construction of a residual strength diagram involves three steps:

- The development of the relationship between the applied stress σ, the crack length parameter a, and the applied stress-intensity factor K for the given structural configuration.
- The selection of an appropriate failure criterion based for the expected material behavior at the crack tip.
- The fracture strength (σf) values for critical crack sizes (ac) are obtained utilizing the results of the first two steps and residual strength diagram (σf vs. ac) for the given structural configuration is plotted.

To understand these three steps for constructing a residual strength diagram, in the following steps the description of the above steps is given.

Step 1: Define the stress- intensity factor relationship, the SIF for a wide centre crack plate is given as

$$K = \sigma\sqrt{\pi a}$$

Step 2: Define the failure criterion. For this plate it is abrupt failure occurs and the condition that defines the fracture is

$$K = K_{cr} = K_c = 90 ksi\sqrt{in}$$

Step 3: Utilize the results of the first two steps to derive a relationship between fracture strength σ_f and critical crack size (a_c), the σ_f vs. a_c relationship is given by

$$\sigma_f\sqrt{a_c} = 90/\sqrt{\pi}$$

For a half crack size (a_c) of 20mm the fracture strength σ_f is about a constant value. Other σ_f vs. a_c values can be similar obtained. Once a sufficient number of values are available, the residual strength diagram can be developed, or one could also attack the problem in the graphic manner that is explained using the following steps.

Step 1: Construct a plot of K vs a by using the equation in step 1 for various values of stress and crack lengths.

Step 2: Superimpose the horizontal line K= Kcr= 90 ksi$\sqrt{in}$ on the diagram. This line represents the critical stress intensity, i.e., fracture toughness, for this material and is independent of crack length.

Step 3: Complete the residual strength diagram, utilize the intersection line with curves where the failure criterion is satisfied, i.e., where Kcr=$\sigma_f\sqrt{\pi a_c}$. The values of the respective stresses and the crack sizes these points are termed to be the failure stresses and the critical crack sizes for the given structures, i.e., model. The residual strength diagram is finally constructed by plotting the σ_f vs. a_c curve.

Based on the following steps critical crack length has been calculated for the following loads 250 N/mm^2 , 275 N/mm^2 , 300 N/mm^2 by the reference of residual strength diagram, the following diagram shows the residual strength.

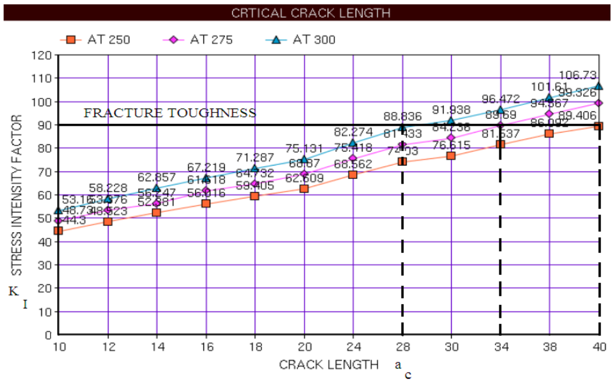


Fig.5:Determination of Critical Crack Length

S.NO	Load N/mm2	Critical Crack Length mm
1	250	40
2	275	34
3	300	28

Critical Crack Lengths

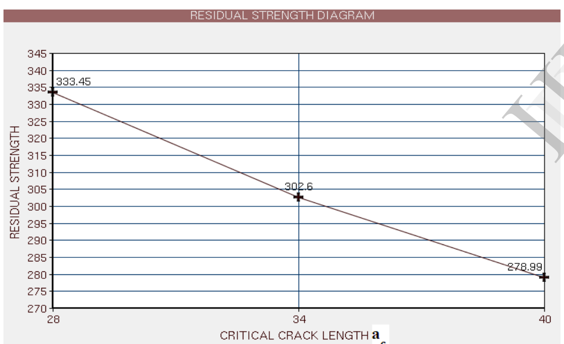


Fig.6:Residual Strength Diagram

6. Conclusion

In this paper we used Displacement Extrapolation Technique for the calculation of SIF KI in 2D structure of ASTM A36 Steel and been compared with theoretically calculated SIF. To know the Structural Integrity, the critical crack length is the important parameter, calculated critical crack length for loads 250N/mm^2, 275N/mm^2, 300N/mm^2 are 40mm, 34mm, 28mm. From this study, it has been observed that SIF value increases with increase in crack length and the component failed when the SIF reaches its critical value i.e. fracture toughness Using the residual strength diagram the propagation of the crack for different loads are studied.

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