Computation of Optimal Stabilizing PID Controller for Interval Plants Using PSO

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Abstract

This paper presents a method for calculating optimum values of stabilizing proportional-integral-derivative(PID) controller for interval plants using particle swarm optimization(PSO) technique. This method is based on plotting the stability boundary locus in the (K_p,K_i)-plane for fixed K_d and then compute the stability region of PID controllers. Once the stability region of K_p,K_i locus is obtained, the best possible PID controller parameters are computed using PSO technique. The proposed method has got advantages like, It does not require sweeping over the parameters and also does not need a linear programming to solve set of inequalities over existing methods. To show the efficacy of the proposed method, a typical unstable fixed coefficient and interval plant is considered and simulated in MATLAB.The simulation results obtained by the proposed method are successfully verified.

Keywords :**stabilizing controllers,Kharitonov polynomials,intervalplants,PSOalgorithm**

I. INTRODUCTION

Now a days 95% of contemporary practical industrial applications use PID control algorithms[2] and thus the appropriate PID control design is still topical especially for systems under some nonlinearities[5-7]. Controller is a device possibly in the form of a chip, an alogue electronics or computer, which monitors and physically alters the operating conditions of dynamic systems.PID controller is a generic   control loop feedback mechanism widely used in industrial control systems. A PID controller attempts to correct the error between a measured process variable and a desired set point by calculating and then outputting a corrective action that can adjust the process accordingly and rapidly, to keep the error minimum. The linear programming characterization of stabilizing PID controllers obtained in[3].This characterization has computationally efficient and revealed important properties of PI&PID controllers. This method shown for a fixed proportional gain, the set of stabilizing integral &derivative gains lies in a convex set. The computation time of this method increases in an exponential manner with order of system considered. This method needs sweeping over proportional gain to find stabilizing values which is a disadvantage of this method. Computation of stabilizing PID controllers with Tan's method published in[5].This technique has been extended for robust stabilization of interval plants through combination of kharitonov theorem[5].All those methods are finding only the possible stabilizing areas value in that stabilizing areas. This method is very important since it can be applicable for both stable and unstable systems. More   recent work on systems with uncertain parameters has been based on kharitonov's result[5,7] on the stability of interval polynomials. Kharitonov showed that for the interval polynomial

$$P(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots + a_n s^n$$

Where $a_i \in [\underline{a_i}, \overline{a_i}]$, i=1,2,….,n,the stability of the set should be found by applying the routh criterion to the following four polynomials

$$P_1(s) = \underline{a_0} + \underline{a_1} s + \overline{a_2} s^2 + \overline{a_3} s^3 + \cdots$$

$$P_2(s) = \underline{a_0} + \overline{a_1} s + \overline{a_2} s^2 + \underline{a_3} s^3 + \cdots$$

$$P_3(s) = \overline{a_0} + \underline{a_1} s + \underline{a_2} s^2 + \overline{a_3} s^3 + \cdots$$

$$P_4(s) = \overline{a_0} + \overline{a_1}s + \underline{a_2}s^2 + \underline{a_3}s^3 + \cdots$$

In this paper, a method for computation of stabilizing PID controllers for interval plant using PSO techniques is proposed. The proposed method has got advantage of computing optimum values of controller parameters.

II. PARTICLE SWARM OPTIMIZATION

James Kennedy an American social psychologist along with Russell C.Eberhart innovated a new evolutionary computational technique termed as PSO in 1995.The approach is based on the swarm behavior such as birds finding food by flocking. The basic variant of the PSO algorithm works by having a population of candidate solution[10-11].The movements of the particles are guided by their own best known position in the search-space as well as the entire swarm's best known position[10].when improved positions are being discovered these will then come to guide the movements of the swarm. The process is repeated and by doing so it is hoped, but not guaranteed, that a satisfactory solution will eventually be discovered. The initial position of the particle is taken as the best position for the start and then the velocity of the particle is updated based on the experience of other particles of the swarming population.

The updated velocity and the distances are

$$V_{i,m}^{t+1} = w * V_{i,m}^t + c1 * rand() * \left(Pbest_{i,m} - X_{i,m}^t\right) + c2 * rand() * \left(Gbest_m - X_{i,m}^t\right)$$

$$X_{i,m}^{(t+1)} = X_{i,m}^{(t)} + V_{i,m}^{(t+1)}$$

CONCEPT OF FITTNESS FUNCTION

For our case of design, we tune all the three parameters of PID such that it gives the best output results or in other words we have to optimize all the parameters of the PID for best results. The performance of the point or the combination of PID parameters is determined by a fitness function or the cost function[10] .This fitness function consists of several component functions which are the performance index of the design. The point in the search space is the best point for which the fitness function attains an optimal value[1]. The fitness function is a function of steady state error, peak overshoot, risetime and settling time. For this design the best point is the point where the fitness function as the minimal value.

The choosen fitness function is

$$F = (1 - \exp(-\beta))(M_P + E_{SS}) + (\exp(-\beta))(T_s - T_r)$$

Where F:Fitness function

M_P:Peak Overshoot

T_s:Settling Time

T_r:Rise Time

β:Scaling Factor

E_{SS}=steady state error

For our case of design we have taken the scaling factor β=1.

III. COMPUTATION OF STABILIZING PID CONTROLLER PARAMETERS FOR FIXED COEFFICIENT PLANT

Consider single-input single-output(SISO) Control system .where

$$G(s) = \frac{N(s)}{D(s)} \qquad (1)$$

is the plant to be controlled and C(s) is a PID controller of the form

$$C(s) = K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s} \qquad (2)$$

The problem is to compute the parameters of the PID controller of eq(2) which stabilize the system of Figure 1.
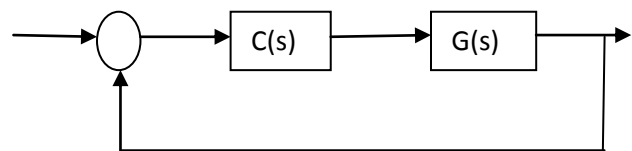

Figure 1 :A SISO control system

Decomposing the numerator and the denominator polynomials of eq(1) into their even and odd parts, and substituting s=jω ,gives

$$G(j\omega) = \frac{N_e(-\omega^2) + j\omega\, N_o(-\omega^2)}{D_e(-\omega^2) + j\omega\, D_o(-\omega^2)} \qquad (3)$$

The closed loop transfer function of the system represented in figure1 is given by

$$T(s) = \frac{G(s)C(s)}{1+G(s)c(S)}$$

The closed loop characteristic polynomial of the system can be written as

$\Delta(s) = 1 + G(s)C(s) =$
$$\frac{-\omega^2 K_p N_o(-\omega^2) + K_i N_e(-\omega^2) - \omega^2 K_d N_e(-\omega^2) + j[K_p \omega N_e(-\omega^2) + K_i \omega N_o(-\omega^2) - \omega^3 K_d N_o(-\omega^2)]}{-\omega^2 D_0(-\omega^2) - \omega^2 K_p N_o(-\omega^2) + K_i N_e(-\omega^2) - \omega^3 K_d N_e(-\omega^2) + j[\omega D_e(-\omega^2) + K_p \omega N_e(-\omega^2) + K_i \omega N_o(-\omega^2) - \omega^3 K_d N_o(-\omega^2)]}$$

Take the denominator of the transfer function and separating real and imaginary parts.

$$-\omega^2 D_0(-\omega^2) - \omega^2 K_p N_o(-\omega^2) + K_i N_e(-\omega^2) - \omega^3 K_d N_e(-\omega^2)$$
$$+ j[\omega D_e(-\omega^2) + K_p \omega N_e(-\omega^2) + K_i \omega N_o(-\omega^2) - \omega^3 K_d N_o(-\omega^2)]$$

Then, equating the real and imaginary parts of denominator polynomials to zero, obtain

REAL:

$$-\omega^2 K_p N_o(-\omega^2) + K_i N_e(-\omega^2) - \omega^3 K_d N_e(-\omega^2) = \omega^2 D_o(-\omega^2) \quad (4)$$

IMAGINARY:

$$K_p N_e(-\omega^2) + K_i N_o(-\omega^2) + K_d \omega^2 N_o(-\omega^2) = -D_e(-\omega^2) \quad (5)$$

Let

$$P_1 = -\omega^2 N_o(-\omega^2)$$

$$P_2 = N_e(-\omega^2)$$

$$P_3 = N_e(-\omega^2)$$

$$P_4 = N_o(-\omega^2)$$

$$P_5 = \omega^2 D_o(-\omega^2) + K_d \omega^3 N_e(-\omega^2)$$

$$P_6 = -D_e(-\omega^2) + K_d \omega^2 N_o(-\omega^2) \quad (6)$$

For the fixed value of K_d in eq (6) the above eq(4) & eq(5) can be modified as

$$K_p P_1 + K_i P_2 = P_5$$

$$K_p P_3 + K_i P_4 = P_6$$

Solving the above two equations, we get

$$K_p = \frac{P_5 P_4 - P_6 P_2}{P_1 P_4 - P_2 P_3} \quad (7)$$

And

$$K_i = \frac{P_6 P_1 - P_5 P_3}{P_1 P_4 - P_2 P_3} \quad (8)$$

Solving the two equations (7)&(8) simultaneously, the stability boundary locus in (K_p, K_i) plane for fixed K_d can be obtained. The stability boundary locus divides the parameter plane $((K_p, K_i)$ -plane) into stable and unstable regions. Choosing a test point with in each region, the stable region which contains the values of stabilizing K_p and K_i parameters can be determined. In the stable region, we determine the optimum values using PSO so as to minimize the fitness function given in section II.

IV. COMPUTATION OF STABILIZING PID CONTROLLER FOR INTERVAL PLANT

In [6],a constant gain controller which stabilizes an interval plant is proposed. This method is based on Hermite-Biehler theorem.In this section, instead of using Hermite-Biehler theorem, we have used Kharitonov's theorem[5]. The stability boundary locus is used to find all the stabilizing values of PID controllers. In that stability region the given interval plant is Hurwitz stable.

Consider a unity feedback system with a PID controller and an interval plant

$$G(s) = \frac{N(s)}{D(s)} = \frac{q_m s^m + q_{m-1} s^{m-1} \ldots + q_0}{p_n s^n + p_{n-1} s^{n-1} \ldots + p_0} \quad (9)$$

Where $q_i \in [\underline{q_i}, \overline{q_i}]$, i=0,1,2,……..,m and $p_i \in [\underline{p_j}, \overline{p_j}]$, j=0,1,2,……,n.let the kharitonov polynomials associated with N(s) and D(s) be respectively.

$$N_1(s) = \underline{q_0} + \underline{q_1}s + \overline{q_2}s^2 + \overline{q_3}s^3 + \ldots$$

$$N_2(s) = \underline{q_0} + \overline{q_1}s + \overline{q_2}s^2 + \underline{q_3}s^3 + \ldots$$

$$N_3(s) = \overline{q_0} + \underline{q_1}s + \underline{q_2}s^2 + \overline{q_3}s^3 + \ldots$$

$$N_4(s) = q_0 + q_1 s + q_2 s^2 + q_3 s^3 + \ldots$$

And

$$D_1(s) = p_0 + p_1 s + p_2 s^2 + p_3 s^3 + \ldots$$

$$D_2(s) = p_0 + p_1 s + p_2 s^2 + p_3 s^3 + \ldots$$

$$D_3(s) = p_0 + p_1s + p_2s^2 + p_3s^3 + \ldots$$

$$D_4(s) = p_0 + p_1s + p_2s^2 + p_3s^3 + \ldots$$

By taking all combinations of the $N_i(s)$ and $D_j(s)$ for i,j=1,2,3,4,..the following sixteen kharitonov plants family can be obtained.

$$G_{ij}(s) = \frac{N_i(s)}{D_j(s)}$$

Where i,j=1,2,3,4.

Define the set S(C(s)G(s)) which contains all the values of the parameters of the controller C(s) which stabilize G(s).The closed loop response calculated all the sixteen plants. The procedure in section II is repeated for all the sixteen plants and obtain (Kp&Ki)- locus. From the obtained locii, we choose the common region for Kp&Ki. From this common region we take the optimum values using PSO so as to minimize the fitness function given in section II.

NUMERICAL EXAMPLE:

Consider a control system of figure 1 with an interval transfer function

$$G(s) = \frac{K}{a_2s^3 + a_1s^2 + a_0s} \qquad (10)$$

Where K∈ [190,210],a_2 ∈[1,1],

a_1 ∈[9.245,10.749]and a_0 ∈[20.5077,21.0084]. The objective function is to calculate all the parameters of a PID controller which stabilizes G(s).The eq(10) has eight Kharitonov plants since there are only 2 Kharitonov polynomials in numerator.

The kharitonov polynomials associated with N(s)&D(s) be respectively are

Numerator Kharitonov polynomials are :

$N_1(s)$=190

$N_2(s)$=210 (11)

Denominator Kharitonov polynomials are:

$$D_1(s) = 20.5077s + 10.749s^2 + s^3$$

$$D_2(s) = 21.0084s + 10.749s^2 + s^3$$

$$D_3(s) = 20.5077s + 9.245s^2 + s^3$$

$$D_4(s) = 21.00084s + 9.245s^2 + s^3 \qquad (12)$$

According to the khartinov theorem [5],the interval plant given in (10) is robustly stable if and only if all of its 4 kharitonov denominator polynomials (12) are Hurwitz stable Therefore,it can be easily shown that the $D_i(s)$ (for i=1,2,3,4) are not Hurwitz, there by the interval plant G(s) is unstable.

Therefore, it is required to compute a PID controller which stabilizies the interval plant. To compute stabilizing PID controller, let us consider first kharitonov plant $G_{11}(s)$ of an interval plant G(s).

$$G_{11}(s) = \frac{190}{s^3 + 10.749s^2 + 20.5077s} \qquad (13)$$

It has been easily shown that the first Kharitonov plant G_{11} is unstable since it has one root in the right half of the S-plane.

Let the stabilizing PID controller defined in eq(2).

$$C(s) = K_p + \frac{K_i}{s} + K_ds = \frac{K_ds^2 + K_ps + K_i}{s}$$

The closed loop transfer function of the system $G_{11}(s)$ with controller is given by

$$T_1(s) = \frac{G_{11}(s)C(s)}{1 + G_{11}(s)c(S)}$$

The characteristic polynomial of the eq(13) is given by

$$\Delta_1(s) = 1 + G_{11}(s)C(s) =$$

$$s^4 + 10.749s^3 + 20.5077s^2 + 190(k_ds^2 + k_ps + k_i) \quad (14)$$

Substitute s= jω in above equation and using the eq (3) to (5) we obtain expressions K_p, K_i for fixed K_d.

$$K_p = 0.0565\omega^2 \qquad (15)$$

$$K_i = 0.83\omega^2 - 0.00526\omega^4 \qquad (16)$$

The aim is to compute all the stabilizing values of K_p, K_i which make the characteristic polynomial of eq.(14) Hurwitz stable. For a range of frequencies ∈(0,4.528), the stability boundary locus in K_p, K_i plane is drawn using the equations(15)&(16).The range of frequencies ω can be calculated by solving the equation Im[G(s)]=0.The stability boundary locus in K_p, K_i plane is shown in Figure2. Same procedure is applied for all the 8 kharitonov plants. The stability region for all the 8 kharitonov plants can be seen in Figure3.we take the common stability region from Figure3,that region be seen in Figure4.It can be easily seen that the stability boundary locus can be divided into two regions, namely stable and unstable regions. Choose a

test point in order to find the stability region in K_p, K_i plane. For this test point, determine the $\Delta(s)$ and find the roots. If the $\Delta(s)$ is found to be Hurwitz stable for the test point, there by this region is a stable region. In this stable region, we determine the optimum values of PID parameters using PSO so as to minimize the fitness function $F = (1 - \exp(-\beta))(M_P + E_{SS}) + (\exp(-\beta))(T_s - T_r)$.

These values of K_p=0.66, K_i=0.45 and K_d=0.73 are obtained. Substituting these values in $\Delta(s)$,then $\Delta(s)$ can be written as $\Delta(s) = s^4 + 10.74s^3 + 158.5s^2 + 125.4s + 85.5$ .Therefore the $\Delta(s)$ has all of its roots lie in left half of the S-plane,which represents stable polynomial. There by the obtained PID controller stabilizies the interval plant.The step response of 8 kharitonov plants with PID controller is shown in Figure 5.It has been observed from the Figure5, that the designed PID controller not only stabilizes the first kharitonov unstable plant $G_{11}(s)$ but also all other possible kharitonov plants .Hence ,the designed PID controller robustly stabilizes the interval plant.
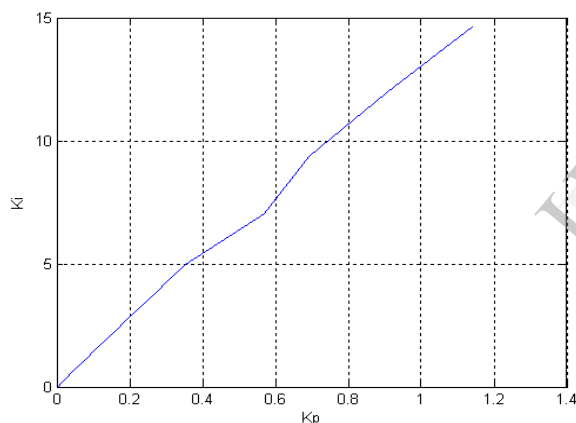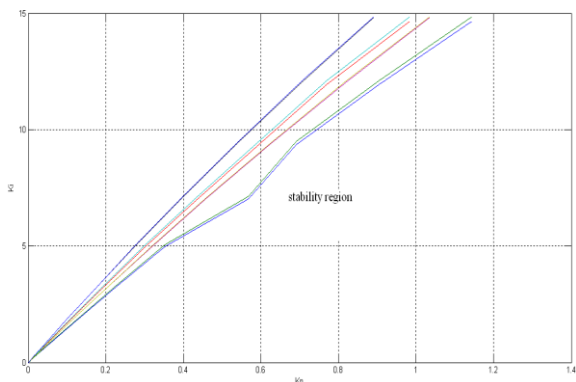

Figure4. Common stability region for 8 interval plants

Figure2.Stability region for $G_{11}(s)$
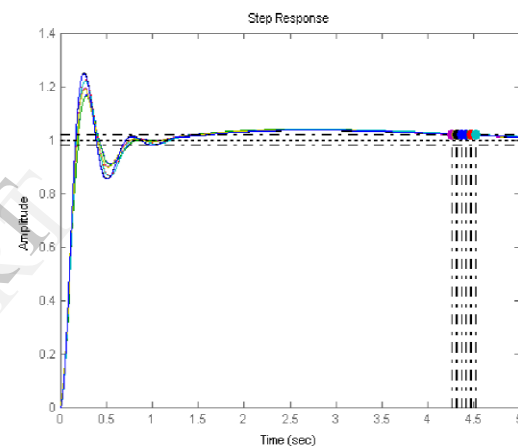
Figure5.Step response of 8 Kharitionv plants with optimum PID controller parameters

Figure3.Stability region for 8 Kharitonov plants

V. CONCLUSION

In this paper, a new approach is proposed for the computation of stabilizing PID controller for interval plants. The proposed method is based on plotting stability boundary locus in K_p, K_i plane for fixed K_d.After obtaining the stability region, the optimum PID controller parameters are determined by using PSO technique, which minimizes an objective function. To show the efficacy of the proposed method , a typical numerical example is considered from literature and simulated through MATLAB. The proposed method not only stabilizes robustly but also give the optimum PID parameters.

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