

Computation Of Natural Frequencies Of Multi Degree Of Freedom System

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Abstract

Wear and tear of machinery manifests into vibrations or changing the pattern of vibrations. Analysis of vibrations for frequency response and time response has become indispensable for major process machinery in trouble shooting. Computation of natural frequencies for an n-degree freedom systems and relative amplitudes of vibrating masses help the designer in choosing the parameters of the system and operating frequencies for the safer operations. Classical mathematical methods are not enough for real time computation of frequency response as they consume more time and effort. In this project we have attempted to analyze a multi-degree freedom system for natural frequencies and thereby pure mode shapes. The pure mode shapes can be later superimposed to get the actual displacement pattern of the system. We have developed a multi-degree freedom system by developing a program in *Mat Lab* platform.

1. Introduction

Vibrations occur in many aspects of our life. For example in human body, there are low frequency oscillations of lungs and the heart, high frequency oscillations of ear, oscillations of the larynx as one speaks, and oscillations induced by rhythmical body Motions such as walking, jumping, and dancing. Many manmade systems also experience or produce vibrations. For example, any unbalance in machines with rotating parts such as fans, ventilators, centrifugal pumps, rotary presses, and turbines can cause vibrations. Building and structures can experience vibrations due to operating machinery, passing vehicular, air and railway traffic; or natural phenomena such as earthquakes and winds. Pedestrian bridges and floors in buildings also experience vibrations due to human moment on them.

From the above examples the definition of vibrations can be drawn as "**Fluctuations of mechanical or structural systems about an equilibrium position**". Vibrations are initiated when an inertia element is displaced from its equilibrium Position due to an energy imparted to the system by an external source.

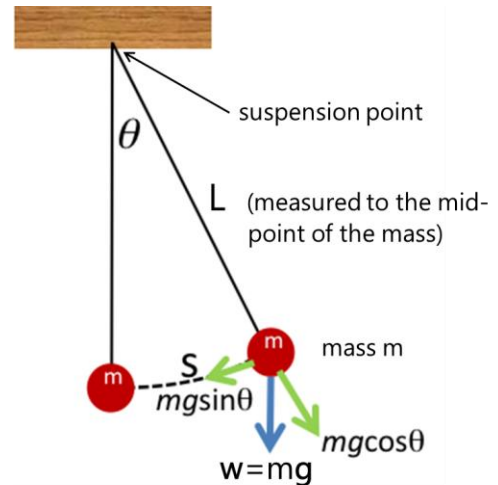


Fig.1 Simple pendulum

2. Modelling of vibration systems

All physical systems are inherently non linear. Exact mathematical modelling of a physical system leads to non-linear differential equations, which often have no analytical solutions. So the mathematical model is only an approximation to the true physical system. The elements that comprise a vibratory system model are:

1. System elements
2. Inertia elements
3. Dissipation elements

External applied forces and moments and external disturbances from prescribed initial displacements and/or initial velocities.

A linear spring obeys a force –displacement law of

$$F=K*X$$

Where K is called the spring stiffness and has dimensions of force/ length

3. HARMONIC EXCITATION OF ONE – DEGREE- OF- FREEDOM SYSTEMS:

FREE VIBRATIONS:

Free vibrations are oscillations about a system's equilibrium position that occurs in the absence of an external excitation. Free vibrations are a result of a kinetic energy imparted to the system or a displacement from the equilibrium position that leads to a difference in potential energy from the system's equilibrium position.

FORCED VIBRATIONS:

Forced vibrations of a one degree freedom system occur when work is being done on system while the vibrations occur. Examples of forced excitation include the ground motion during an earthquake or the motion caused by an unbalanced reciprocating component. Consider a spring mass system excited by a sinusoidal forcing function $F_0 \sin \omega t$ as shown in figure. At any instant, when the mass is displaced from the mean position through a distance x in the downward direction, the external forces acting on the system are

- kx , in the upward direction.
- $F_0 \sin \omega t$, in the downward direction

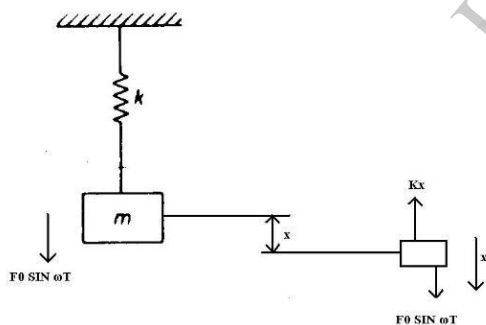


Fig 2. Spring mass system

$$m \ddot{x} = -kx + F_0 \sin(\omega t)$$

This is a linear, non-homogeneous, second order differential equation. The solution of this equation consists of two parts, complementary function and particular integral. The complementary function is obtained from the differential equation.

$$m \ddot{x} + kx = 0$$

4. HARMONIC EXCITATION OF MULTI-DEGREE OF FREEDOM SYSTEMS

The procedure for analyzing multi-degree of freedom system is only an extension of the method used for analyzing single degree of freedom system. However, the use of influence coefficients and writing the equations in matrix form made the things sufficiently simpler. Besides, the use of equations in matrix forms facilitates the application of computer methods for their solution. Two methods are introduced to drive the differential equations governing the motion of multi-degree of freedom systems.

1. The free body diagram method
2. The equivalent systems method

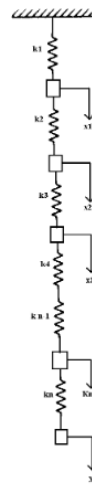


Fig 3. N DOF system

5. METHODOLOGY:

NUMERICAL METHODS:

The actual solution of the determinants of higher order becomes more and more difficult with increasing number of degrees –of–freedom as the exact analysis is associated with laborious calculations. Even with high-speed digital computers that can solve equations of many degree-of-freedom that result beyond the first few normal modes are often unreliable and meaningless. So, Numerical methods are used to solve these problems.

- RAYLEIGH METHOD
- RAYLEIGH-RITZ METHOD
- HOLZER'S METHOD
- METHOD OF MATRIX ITERATION

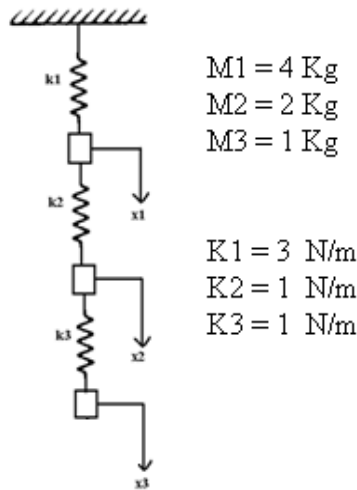


Fig 4. Different masses in n-DOF spring system

By using differential equations of motions for the three masses in terms

$$\begin{aligned}
 -x_1 &= \partial_{11}m_1 x_1 + \partial_{12}m_2 x_2 + \partial_{13}m_3 x_3 \\
 -x_2 &= \partial_{21}m_1 x_1 + \partial_{22}m_2 x_2 + \partial_{23}m_3 x_3 \\
 -x_3 &= \partial_{31}m_1 x_1 + \partial_{32}m_2 x_2 + \partial_{33}m_3 x_3
 \end{aligned}$$

Replacing $-x$ by $-\omega^2 x$ we have

$$\begin{aligned}
 x_1 &= \partial_{11}m_1 \omega^2 x_1 + \partial_{12}m_2 \omega^2 x_2 + \partial_{13}m_3 \omega^2 x_3 \\
 x_2 &= \partial_{21}m_1 \omega^2 x_1 + \partial_{22}m_2 \omega^2 x_2 + \partial_{23}m_3 \omega^2 x_3 \\
 x_3 &= \partial_{31}m_1 \omega^2 x_1 + \partial_{32}m_2 \omega^2 x_2 + \partial_{33}m_3 \omega^2 x_3
 \end{aligned}$$

The modes have been repeated with sufficient accuracy in the 4th iterative

$$\begin{aligned}
 \omega^2 m / 3k & \times 14.28 = 1 \\
 \omega & = 0.458 \sqrt{(k/m)}
 \end{aligned}$$

Natural frequencies are = 1, 3.16, 4

6. MATLAB

MATLAB (short for Matrix Laboratory) is a special-purpose computer program optimized to perform engineering and scientific calculations. It is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses include:

- Math and computation
- Algorithm development
- Modelling, simulation and prototyping

- Data analysis, exploration and visualization
- Scientific and engineering graphics
- Application development, including Graphical User Interface (GUI) building

7. Test Cases:

```

% DOF = input ('enter the number of degrees of
freedom: ');
% MASS = input ('enter masses');
% STFNS_VALS = input ('enter stiffness values of
springs: ');
% AMPLITUDE = input ('Enter Random values for
amplitude: ');

```

%creation of global stiffness matrix

```

DOF = 3
STFNS_VALS = [3 1 1 0]
MASS = [4 2 1]
AMPLITUDE = [1; 2; 3]

```

```

STFNS_MTRX = zeros (DOF, DOF);
MASS_MTRX = zeros (DOF, DOF);
AMPLITUDE_MTRX = zeros (DOF, 1);

```

```

fprintf ('the global stiffness matrix is:\n');

```

```

for i = 1: DOF
    for j = 1:DOF
        if i == j
            STFNS_MTRX(i,j) = (STFNS_VALS(i) +
STFNS_VALS(i+1));
        elseif j==i+1
            STFNS_MTRX(i,j) = -STFNS_VALS(j);
        elseif i==j+1
            STFNS_MTRX (i,j) = -STFNS_VALS(i);
        else STFNS_MTRX(i,j)=0;
        end
        fprintf ('%d ',STFNS_MTRX(i,j));
    end
    fprintf ('\n');
end

```

```

end
SPRSE_STFNS_MTRX = sparse (STFNS_MTRX);
MASS_MTRX = diag(MASS)
SPRSE_MASS_MTRX = sparse (MASS_MTRX);
INV_MASS_MTRX = inv (SPRSE_MASS_MTRX);
MASS_X_STFNS = (INV_MASS_MTRX *
STFNS_MTRX);

```

```

cond = 1;
while (cond ~= 0)

```

```

AMPLITUDE_MTRX = (MASS_X_STFNS *
AMPLITUDE);
AMPLITUDE_MTRX =
(AMPLITUDE_MTRX/AMPLITUDE_MTRX(1));
AMPLITUDE_MTRX = num2str
(AMPLITUDE_MTRX,3);
AMPLITUDE          = num2str
(AMPLITUDE,3);
AMPLITUDE_MTRX = str2num
(AMPLITUDE_MTRX);
AMPLITUDE          = str2num (AMPLITUDE);
cond              = AMPLITUDE -
AMPLITUDE_MTRX;
cond              = sum (cond(:));

```

```

AMPLITUDE      = AMPLITUDE_MTRX;

```

```

end

```

```

fprintf ('the final amplitude of the system are:\n');
fprintf ('%g \n',AMPLITUDE_MTRX);

```

```

LAMDA  = eig(MASS_X_STFNS);
fprintf ('the natural frequencies of the system in rad/sec
are:');

```

```

OMEGA  = sqrt(LAMDA)

```

```

for i = 1:DOF

```

```

    fprintf('the displacement when vibration with w %d
natural frequency is \n',i );
    TIME  = linspace (0, 20, 123);
    WT    = (OMEGA (i, 1)* TIME);
    DISP  = AMPLITUDE_MTRX (i,1)*sin(WT);
    fprintf ('%g \n', DISP);

```

```

    figure, plot (TIME, DISP );
    title('Displacement Vs Time')
    xlabel ('Time (sec)')
    set (gca, 'XGrid', 'on', 'YGrid', 'on')
    ylabel ('Displacement (mts)')

```

```

end

```

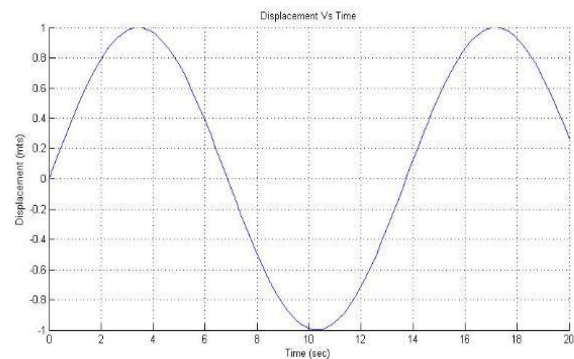
8. Results:

The final amplitude of the system is:

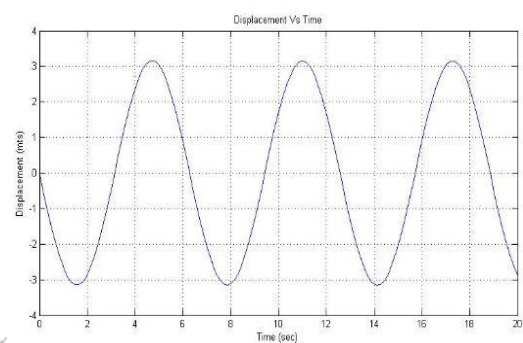
	1
	-3.15
	3.98

The natural frequencies of the system in rad/sec are:

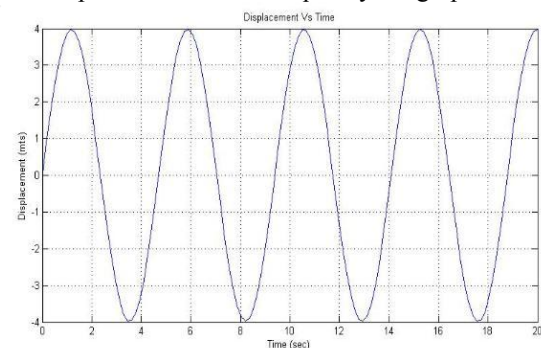
OMEGA =	0.4576
	1.0000
	1.3381



Graph 1.The Natural frequency ω_1 graph



Graph 2.The Natural frequency ω_2 graph



Graph 3.The Natural frequency ω_3 graph

9. Conclusion

Computer of natural frequencies and relative amplitudes has become indispensable in the design of multi degree freedom vibration systems. The code we have developed in Matlab helps a perspective designer in choosing the operating speed of the system. The program also plots the amplitudes of the vibrating bodies, with the help of which the designer can tryout different system parameters in real time to arrive at optimum specifications of the system. Thus optimization of the parameters can be achieved without the application of tools like artificial neural networks

10. References

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- [4] R.J. Astley, G. Gabard, Computational Aero-Acoustics (CAA) for Aircraft Noise Prediction, Elsevier Ltd Pages 4081-4082

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