

# Complexity Reduction of Maximum Likelihood Detector using closed form approximation in Cooperative Networks

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**Abstract**— The maximum likelihood detection in differential AF and DF cooperative networks using M-ary differential phase shift keying with Rayleigh fading, has practical complexity. To reduce the complexity of the maximum likelihood detector, two algorithms were analysed. The first algorithm eliminates number of symbols in ML search. In high signal-to-noise ratios, this algorithm determines a single symbol, means without calculating the likelihood function, maximum likelihood estimate of the information symbol, can be obtained. For low to medium signal-to-noise ratios multiple symbols were determined, thus the closed-form approximate expression was derived, which is very accurate throughout the whole signal to noise ratio range. Closed form approximation requires only five sample evaluation per symbol. Combining these algorithms closed-form approximate maximum likelihood detector was obtained. Finally, the obtained BER performance of closed form ML detector is compared with bit error performance of diversity combiner.

**Index terms**— Maximum Likelihood Detector (ML), Amplify and Forward (AF), Decode and forward (DF), M-Differential Phase shift keying (M-DPSK).

## I.INTRODUCTION

In cooperative networks the function of relay is to retransmit received signal from source to destination. Here the information symbol is modulated by differential phase shift keying and corresponding symbol is detected using maximum likelihood detector at the receiver. Objective of the paper is to reduce complexity of maximum likelihood detector to estimate the single information symbol. As the instantaneous channel state information is not available at the receiver M-differential phase shift keying modulation is used. To reduce complexity, two algorithms were analysed and denoted as closed form approximate ML detector. In high SNR, the algorithm detects single information symbol without calculating likelihood function. For low to medium SNR, multiple symbols were detected hence, calculating likelihood function becomes inevitable. Thus closed form approximation was made for calculating the likelihood function.

## II.RELATED WORKS

In the existing system, non coherent modulations were used in amplify and forward networks. In [2] maximum likely ctois used for non-coherent frequency shift keying in Rayleigh fading. However ML detection in [2] has no closed form solutions are very complex for implementation. In [3] suboptimum detectors is used, it has lower complexity but some degradation in performance. In [4] simple diversity combiner is used but diversity combiner suffers from strict sub optimality as it does not exploits non Gaussianity of overall noise at destination. This may lead to significant performance loss compared with optimum (ML) detector.

## III.SYSTEM MODEL

Cooperative networks consist of source S, relay R, and destination D shown in Fig 1. Let  $h_{sd}$  denote channel coefficient of direct link between S and D,  $h_{sr}$  denote channel coefficient of first hop from S and R,  $h_{rd}$  denote channel coefficient of second hop from R and D.

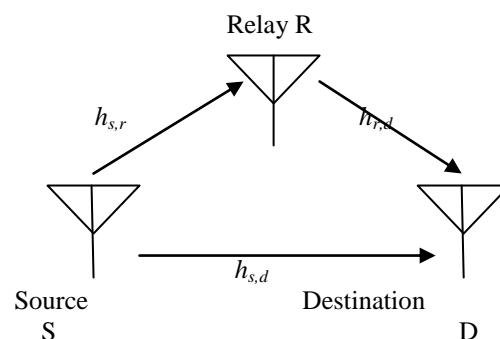


Fig1: General cooperative network

Channel coefficients are independent circularly symmetric complex Gaussian random variables with zero mean and variances  $\Omega_{sd}$ ,  $\Omega_{sr}$  and  $\Omega_{rd}$  respectively. Here M-DPSK modulation is used hence each information symbol is drawn from

M-phase constellation set  $C := \{e^{j2\pi n/M} : n=0, \dots, M-1\}$  and is differentially encoded into two information bearing symbols by  $x(k) = x(k-1)c(k)$ ,  $k=1, 2, \dots$  is symbol index. Transmission divided into two phases. In phase 1, S broadcasts  $x(k)$  with power  $E_s$ , while R and D listen. The received signals at the relay and the destination are, respectively, given by

$$y_{sr}(k) = \sqrt{E_s}h_{sr}x(k) + u_{sr}(k) \quad (1)$$

$$y_{sd}(k) = \sqrt{E_s}h_{sd}x(k) + u_{sd}(k) \quad (2)$$

In phase 2, R scales its received signal by a factor  $s_r(k)$  and forwards to destination. Then received signal at phase 2 is given by,

$$y_{rd}(k) = \sqrt{E_s}h_{rd}h_{sr}s_r(k)x(k) + h_{rd}s_r(k)u_{sr}(k) + u_{rd}(k) \quad (3)$$

#### A. Amplify and forward scheme:

For phase 2 relay which amplifies the received signal by a factor  $s_r(k)$

$$s_r(k) = \sqrt{(E_r)/(E_s\Omega sr + \sigma^2)} \quad (4)$$

#### B. Decode and forward scheme:

Here the relay first decodes the received signals,

$$\tilde{d}(k) = \text{sign}(R\{y_{sr}^*(k-1)y_{sr}^*(k)\}) \quad (5)$$

The decoded signal is re-encoded as follows

$$s_r(k) = s_r(k-1)\tilde{d}(k) \quad (6)$$

$u_{sr}(k)$ ,  $u_{sd}(k)$ ,  $u_{rd}(k)$  are noise components and are independent CSCG random variable with variance  $\sigma^2$ . The average received SNR of S-to-R, S-to-D, and R-to-D are expressed as  $\bar{\gamma}_{sr} = E_s\Omega sr/\sigma^2$ ,  $\bar{\gamma}_{sd} = E_s\Omega sd/\sigma^2$ ,  $\bar{\gamma}_{rd} = E_r\Omega rd/\sigma^2$ .

### IV. ML DETECTOR FOR M-DPSK

In DPSK signalling, information  $c(k)$  is carried by the phase difference of two consecutive signals, hence the receiving signals has been considered as vector.

$$\psi \text{ is equal to } \begin{cases} \left\{ e^{\frac{j2\pi n}{M}} : n = n_1, \dots, n_2, \right. \\ \left. \left\{ e^{\frac{j2\pi n}{M}} : n = n_1, \dots, n_2, n_2, \dots, M-1, \text{ if } n_2 - n_1 > \frac{M}{2}, \text{ or } n_2 - n_1 = \frac{M}{2} \right. \right. \\ \left. \left. \text{ and } \varphi_1, \varphi_2 \in \left[ \frac{2n_1\pi}{M}, \frac{2n_2\pi}{M} \right] \right. \right. \\ \left. \left. \text{ otherwise } \right. \right. \\ C, \end{cases} \quad (13)$$

$$\begin{aligned} \varphi_2 := & 4\left\{ \frac{\bar{\gamma}_{sd}}{1+2\bar{\gamma}_{sd}} y_{sd}(k) y_{sd}^*(k-1) \right. \\ & \left. + \frac{1+\bar{\gamma}_{sr}-\sqrt{1+2\bar{\gamma}_{sr}}}{2\bar{\gamma}_{sr}} y_{rd}(k) y_{rd}^*(k-1) \right\} \quad (8) \end{aligned}$$

$$m1 := \text{round}\left(\frac{M}{2\pi}\varphi_1\right) \bmod M \quad (9)$$

$$m2 := \text{round}\left(\frac{M}{2\pi}\varphi_2\right) \bmod M \quad (10)$$

$$n1 := \min(m1, m2) \quad (11)$$

$$n2 := \max(m1, m2) \quad (12)$$

### V. ML DETECTOR APPROXIMATE METHOD

For low to medium SNR number of symbols estimated was more than one, so likelihood function calculation becomes inevitable. So the analysed simple closed form approximation was given below.

#### A. Closed-Form Approximation

Using five sample Gaussian Legendre quadrature the term  $I(\epsilon_1, \epsilon_2, \beta_1, \beta_2)$  can be approximated as follows:

$$I(\epsilon_1, \epsilon_2, \beta_1, \beta_2) \approx I_{approx} := \sum_{i=1}^5 \omega_i L(z_i) \quad (14)$$

$$L(z) = \frac{\exp\left\{-\left(\frac{\beta_1}{1-\epsilon_1 \ln \frac{1+z}{2}} + \frac{\beta_2}{1-\epsilon_2 \ln \frac{1+z}{2}}\right)\right\}}{2\left(1-\epsilon_1 \ln \frac{1+z}{2}\right)\left(1-\epsilon_2 \ln \frac{1+z}{2}\right)} \quad (15)$$

#### B. Closed-Form Approximate condition for symbol estimation

For  $|\Psi| = 1$ , the ML estimate of  $c(k)$  is immediately obtained from the singleton  $\Psi$  without solving the LF integral. For  $|\Psi| > 1$ , the LF integral is approximately calculated. Therefore, closed-form approximate ML detectors are as follows:

$$\begin{cases} \text{If } |\Psi| = 1, \hat{c}(k) = \text{the single element in } \Psi \\ \text{otherwise, } \hat{c}(k) \approx \arg \max_{c(k) \in \Psi} \{ \epsilon_0 \beta_0 + \ln I_{approx} \} \end{cases} \quad (16)$$

The parameters involved in this above equation are defined as,

$$\begin{aligned} \epsilon_1 &:= \frac{\bar{\gamma}_{rd}}{1+\bar{\gamma}_{sr}} \quad (18) \\ \epsilon_2 &:= \frac{(1+2\bar{\gamma}_{sr})}{1+\bar{\gamma}_{sr}} \bar{\gamma}_{sr} \quad (19) \end{aligned}$$

$$\varphi_1 := 4\{y_{sd}(k)y_{sd}^*(k-1)\} \quad (7)$$

$$\beta_0 := \frac{R\{y_{sd}(k-1)y_{sd}^*(k)c(k)\}}{\sigma^2} \quad (20)$$

$$\beta_1 := \frac{|y_{rd}(k)-y_{rd}(k-1)c(k)|^2}{2\sigma^2} \quad (21)$$

$$\beta_2 := \frac{|y_{rd}(k)+y_{rd}(k-1)c(k)|^2}{2\sigma^2} \quad (22)$$

Thus the estimated value  $\hat{c}(k)$  should be within constellation  $\psi$ .

### C. Complexity Comparison

The diversity combiner equation used to estimate the information symbol is as follows:

$$\hat{c}(k) = \arg \max_{c(k) \in \psi} R\left\{ \left[ \frac{\bar{y}_{sr} + 1}{\bar{y}_{sr} + \bar{y}_{rd} + 1} y_{rd}(k) y_{rd}^*(k-1) + y_{sd}(k) y_{sd}^*(k-1) \right] c(k) \right\} \quad (23)$$

Here for estimating the single information symbol, the detector needs to check the entire constellation set.

## VI. SIMULATION RESULTS

The distance between source and destination is normalized to unity and denoted by  $d$ ,  $0 \leq d \leq 1$ . The total signal transmission power in the network is  $E = E_s + E_r$ . For various SNR, average bit error rate value can be evaluated and the inference values are tabulated for both amplify and forward networks, as well as decode and forward networks by using ML detector and diversity combiner.

### A. Simulation Graph:

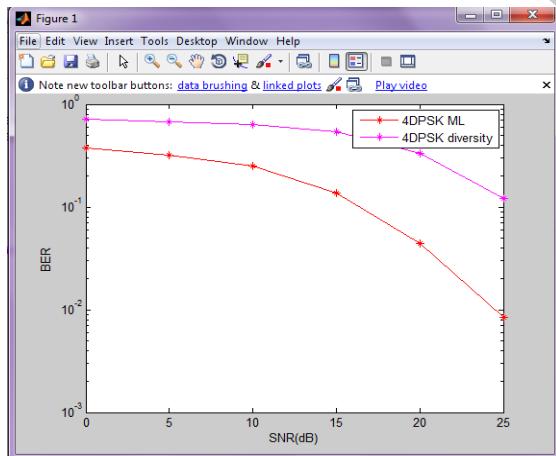


Fig 2:4DPSK for Amplify and Forward network

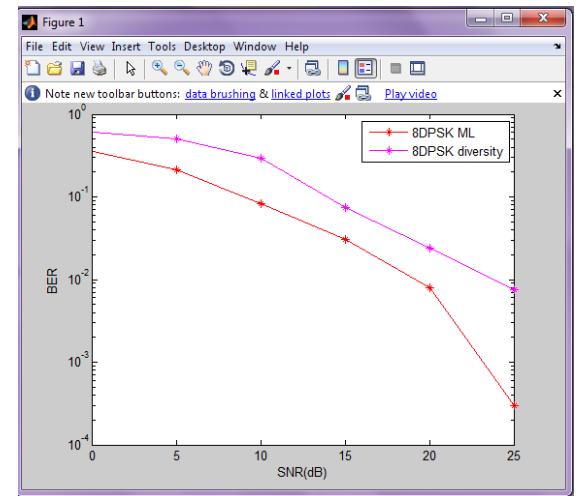


Fig 3:8DPSK for Amplify and Forward network

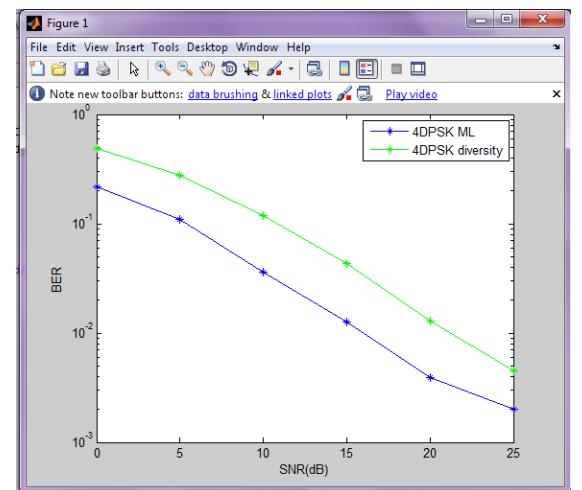


Fig 4:4DPSK for Decode and Forward network

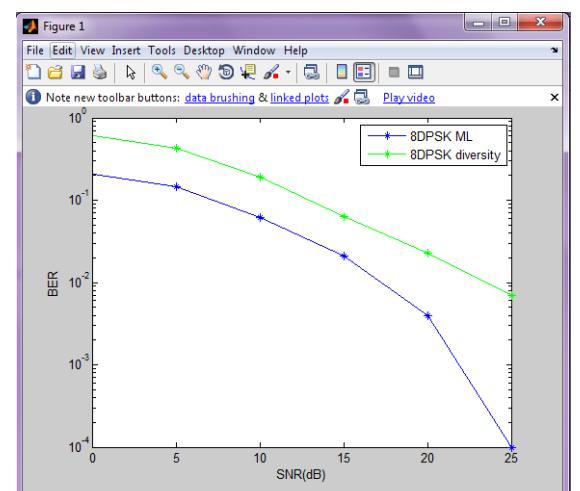


Fig 5:8DPSK for Decode and Forward network

## B. Inferences: Average BER

SNR	AMPLIFY AND FORWARD			
	ML DETECTOR		DIVERSITY NER	
	4D	8D	4D	8D SK
0	0.3	0.3	0.71	0.6
5	0.3	0.2	0.67	0.4
10	0.2	0.0	0.63	0.2
15	0.1	0.0	0.53	0.0
20	0.0	0.0	0.33	0.0
25	0.0	0.0	0.12	0.0

Table 1: Average BER for Amplify and Forward

SNR	DECODE AND FORWARD			
	ML DETECTOR		DIVERSITY NER	
	4D	8D	4D	8D SK
0	0.2	0.2	0.49	0.6
5	0.1	0.1	0.27	0.4
10	0.0	0.0	0.11	0.1
15	0.0	0.0	0.04	0.0
20	0.0	0.0	0.01	0.0
25	0.0	0.0	0.00	0.0

Table 2: Average BER for Decode and Forward

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## VII CONCLUSION

In this paper, closed form approximation for maximum likelihood detector was derived for Amplify and Forward, decode and forward cooperative networks employing  $M$ -Differential phase shift keying in Rayleigh fading. The algorithm was analysed, which can substantially reduce the complexity of ML detection. Specifically, the first algorithm substantially narrows down the ML search while causing no loss of optimality. The second algorithm provides a very accurate closed-form approximation for the LF. Combining these algorithms, a closed-form approximate ML detector was obtained. In future closed form approximation can be done for amplify and forward networks using multiple relays.