Completely $B^{\#}$ Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

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***Abstract* — In this chapter we have introduced two types of $b^{\#}$ continuous mappings namely intuitionistic fuzzy completely $b^{\#}$ continuous mappings and intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings. Also we have provided some interesting results based on these continuous mappings.**

***Keywords* — Intuitionistic fuzzy sets, intuitionistic fuzzy topology, intuitionistic fuzzy completely $b^{\#}$ continuous mapping.**

I INTRODUCTION

Intuitionistic fuzzy set is introduced by Atanassov in 1986. Using the notion of intuitionistic fuzzy sets, Coker [1997] has constructed the basic concepts of intuitionistic fuzzy topological spaces. The concept of $b^{\#}$ closed sets and $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces are introduced by Gomathi and Jayanthi (2018). In this paper we have introduced intuitionistic fuzzy completely $b^{\#}$ continuous mappings and intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings. Also we have provided some interesting results based on these continuous mappings.

II PRELIMINARIES

Definition 2.1: [Atanassov 1986] An intuitionistic fuzzy set(IFS) A is an object having the form A= $\{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$,where the functions $\mu_A: X \to [0, 1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively , and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X) , the set of all intuitionistic fuzzy sets in X. An IFS A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$.

Definition 2.2: [Atanassov 1986] Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$ and $B = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$. Then the following properties hold:

i. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
ii. A=B if and only if $A \subseteq B$ and $A \supseteq B$,
iii. $A^c = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$,
iv. $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle: x \in X\}$,
v. $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x) , \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The IFSs $0_{\sim} = \langle x, 0, 1\rangle$ and $1_{\sim} = \langle x, 1, 0\rangle$ are respectively the empty set and whole set of X.

Definition 2.3: [Coker, 1997] An intuitionistic fuzzy topology (IFT) on X is a family τ of IFSs in X satisfying the following axioms:

i. $0_{\sim}, 1_{\sim} \in \tau$
ii. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
iii. $\cup G_i \in \tau$ for any $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called the intuitionistic fuzzy topological space (IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X. Then the complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4: [Coker, 1997] Let (X,τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$int(A) = \cup \{G/G \text{ is an IFOS in X and } G \subseteq A\},$$

$$cl(A) = \cap \{K/K \text{ is an IFCS in X and } A \subseteq K\}.$$

Definition 2.5: [Gurcay, Coker and Hayder, 1997] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an

i. intuitionistic fuzzy semi closed set if $int(cl(A)) \subseteq A$
ii. intuitionistic fuzzy pre closed set if $cl(int(A)) \subseteq A$
iii. intuitionistic fuzzy regular closed set if $cl(int(A)) = A$
iv. intuitionistic fuzzy α closed set if $cl(int(cl(A))) \subseteq A$
v. intuitionistic fuzzy β closed set if $int(cl(int(A))) \subseteq A$

Definition 2.6: [Hanafy, 2009] An IFS $A=\langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an intuitionistic fuzzy γ closed set if $int(cl(A)) \cap cl(int(A)) \subseteq A$.

Definition 2.7: [Gomathi and Jayanthi, 2018] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an intuitionistic fuzzy $b^{\#}$ closed set (IF$b^{\#}$CS) if $int(cl(A)) \cap cl(int(A)) = A$.

Definition 2.8: [Coker, 1997] Let X and Y be two non empty sets and $f: X \to Y$ be a mapping. If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle / y \in Y\}$ is an IFS in Y, then the preimage of B under f is denoted and defined by $f^{-1}(B)= \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) / x \in X \rangle \}$, where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ for every $x \in X$.

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Definition 2.9: [Gurcay, Coker and Hayder, 1997] Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f said to be an intuitionistic fuzzy continuous mapping if $f^{-1}(V)$ is an IFCS in (X,τ) for every IFCS V of (Y,σ).

Definition 2.10: [Joung Kon Jeon, 2005] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f said to be an

i. intuitionistic fuzzy semi continuous mapping if $f^{-1}(V)$ is an IFSCS in (X, τ) for every IFCS V of (Y,σ).
ii. intuitionistic fuzzy α continuous mapping if $f^{-1}(V)$ is an IFαCS in (X, τ) for every IFCS V of (Y, σ).
iii. intuitionistic fuzzy pre continuous mapping if $f^{-1}(V)$ is an IFPCS in (X, τ) for every IFCS V of (Y, σ).
iv. intuitionistic fuzzy β continuous mapping if $f^{-1}(V)$ is an IFβCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.11: [Gomathi and Jayanthi, 2018] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

i) intuitionistic fuzzy $b^{\#}$ continuous mapping if f^{-1} (V) is an $IFb^{\#}CS$ in (X, τ) for every IFCS V of (Y, σ).
ii) intuitionistic fuzzy contra $b^{\#}$ continuous mapping if f^{-1} (V) is an $IFb^{\#}CS$ in (X, τ) for every IFOS V of (Y, σ).
iii) intuitionistic fuzzy $b^{\#}$ irresolute mapping if f^{-1} (V) is an $IFb^{\#}CS$ in (X, τ) for every $IFb^{\#}CS$ V of (Y, σ).

Definition 2.12: [Hanafy and El-Arish, 2003] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy completely continuous mapping if $f^{-1}(V)$ is an IFROS in (X, τ) for every IFOS V of (Y, σ).

Definition 2.13: [Coker and Demirci, 1995] Intuitionistic fuzzy point (IFP), written as $p_{(\alpha, \beta)}$, is defined to be an IFS of X given by $p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha,\beta) & if\ x=p \\ (0,1) & otherwise \end{cases}$. An IFP $p_{(\alpha, \beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.14: [Thakur and Rekha Chaturvedi, 2008] Two IFSs A and B are said to be q-coincident ($A\ _q\ B$) if and only if there exist an element x ∈ X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.15: [Seok Jong Lee and Eun Pyo Lee, 2000] Let $p_{(\alpha, \beta)}$ be an IFP in (X, τ). An IFS A of X is called an intuitionistic fuzzy neighbourhood of $p_{(\alpha, \beta)}$ if there exist an IFOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

Definition 2.16: [Dhivya and Jayanthi, 2019] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping if $f^{-1}(V)$ is an $IFb^{\#}CS$ in (X, τ) for every IFRCS V of (Y, σ).

III COMPLETELY $b^{\#}$ CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

In this chapter we have introduced and investigated intuitionistic fuzzy completely $b^{\#}$ continuous mappings and intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings. We have provided many interesting results using these continuous mappings.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy completely $b^{\#}$ continuous mapping if $f^{-1}(V)$ is an IFRCS in (X, τ) for every $IFb^{\#}CS$ V of (Y, σ).

Example 3.2: Let X = {a, b}, Y = {u, v}. Then τ = {$0_\sim$, G_1, G_2 $1_\sim$} and σ = {$0_\sim$, G_3, G_4 $1_\sim$} are IFS on X and Y respectively, where, G_1 = ⟨x, (0.2_a, 0.3_b), (0.4_a, 0.5_b)⟩, G_2 = ⟨x, (0.4_a, 0.5_b), (0.2_a, 0.3_b)⟩, G_3 = ⟨y, (0.2_u, 0.3_v), (0.4_u, 0.5_v)⟩ and G_4 = ⟨y, (0.4_u, 0.5_v), (0.2_u, 0.3_v)⟩. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

Proposition 3.3: A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping if and only if the inverse image of each $IFb^{\#}OS$ in Y is an IFROS in X.

Proof: Obviously.

Proposition 3.4: If f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping where Y is an $IFT_{b^{\#}}$ space[4], then for each IFP $p_{(\alpha, \beta)} \in X$ and for every intuitionistic fuzzy neighbourhood A of $f(p_{(\alpha, \beta)})$, there exists an IFROS B of X such that $p_{(\alpha, \beta)} \in B$ and $f(B) \subseteq A$.

Proof: Let $p_{(\alpha, \beta)}$ be an IFP of X and let A be an intuitionistic fuzzy neighbourhood of $f(p_{(\alpha, \beta)})$ such that $f(p_{(\alpha, \beta)}) \in C \subseteq A$, where C is an IFOS in X. Since every IFOS is an $IFb^{\#}OS$ in an $IFT_{b^{\#}}$ space, C is an $IFb^{\#}OS$ in Y as Y is an $IFT_{b^{\#}}$ space. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p_{(\alpha, \beta)} \in f^{-1}(C)$. Put $B = f^{-1}(C)$. Therefore $p_{(\alpha, \beta)} \in B = f^{-1}(C) \subseteq f^{-1}(A)$.Thus $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. That is $f(B) \subseteq A$.

Proposition 3.5: A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping then $cl(int(f^{-1}(cl(B)))) \supseteq f^{-1}(B)$ for every IFS B in Y where Y is an $IFT_{b^{\#}}$ space.

Proof: Let B ⊆ Y be an IFS. Then cl(B) is an IFCS in Y and hence an $IFb^{\#}CS$ in Y as Y is an $IFT_{b^{\#}}$ space. By hypothesis, $f^{-1}(cl(B))$ is an IFRCS in X. Hence $cl(int(f^{-1}(cl(B)))) = f^{-1}(cl(B)) \supseteq f^{-1}(B)$.

Proposition 3.6: Let f: (X, τ) → (Y, σ) be an mapping. Then the following are equivalent:

i. f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping
ii. $f^{-1}(V)$ is an IFROS in X for every $IFb^{\#}OS$ V in Y
iii. for every IFP $p_{(\alpha,\ \beta)} \in X$ and for every $IFb^{\#}OS$ B in Y such that $f(p_{(\alpha,\ \beta)}) \in B$ there exists an IFROS in X such that $p_{(\alpha,\ \beta)} \in A$ and $f(A) \subseteq B$

Proof: (i)$\Rightarrow$(ii): Let V be an $IFb^{\#}OS$ in Y. Then V^c is an $IFb^{\#}CS$ in Y. Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(V^c)$ is an IFRCS in X. Since $f^{-1}(V^c) = (f^{-1}(V))^c$, $f^{-1}(V)$ is an IFROS in X.

(ii)$\Rightarrow$(iii): Let $p_{(\alpha,\ \beta)} \in X$ and B $\subseteq$Y such that $f(p_{(\alpha,\ \beta)}) \in$ B. This implies $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since B is an $IFb^{\#}OS$ in Y, by hypothesis $f^{-1}(B)$ is an IFROS in X. Let A = $f^{-1}(B)$. Then $p_{(\alpha,\ \beta)} \in f^{-1}(f(p_{(\alpha,\ \beta)})) \in f^{-1}(B)$=A. Therefore $p_{(\alpha,\ \beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.This implies $f(A) \subseteq B$.

(iii)$\Rightarrow$(ii): Let B $\subseteq$ Y be an $IFb^{\#}OS$. Let $p_{(\alpha,\ \beta)} \in X$ and $f(p_{(\alpha,\ \beta)}) \in$ B. By hypothesis, there exists an IFROS C in X such that $p_{(\alpha,\ \beta)} \in C$ and f(C) $\subseteq$ B. This implies C $\subseteq$ $f^{-1}(f(C)) \subseteq f^{-1}(B)$. Therefore $p_{(\alpha,\ \beta)} \in C \subseteq f^{-1}(B)$. That is $f^{-1}(B) = \bigcup_{p(\alpha,\beta)\in f^{-1}(B)} p_{(\alpha,\beta)} \subseteq \bigcup_{p(\alpha,\beta)\in f^{-1}(B)} C \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = \bigcup_{p(\alpha,\beta)\in f^{-1}(B)} C$. Since the union IFROSs is an IFROS, $f^{-1}(B)$ is an IFROS in X. Hence f is intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

Proposition 3.7: A mapping f : X → Y is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping then the following are equivalent:

i. For any $IFb^{\#}OS$ A in Y and for any IFP $p_{(\alpha,\ \beta)} \in X$, if $f(p_{(\alpha,\ \beta)})_q A$, then $p_{(\alpha,\ \beta)q}\, int(f^{-1}(A))$.
ii. For any $IFb^{\#}OS$ A in Y and for any $p_{(\alpha,\ \beta)} \in X$, if $f(p_{(\alpha,\ \beta)})_q A$, then there exists an IFOS B such that $p_{(\alpha,\ \beta)q}$ B and f(B) $\subseteq$ A.

Proof: (i)$\Rightarrow$(ii): Let A $\subseteq$ Y be an $IFb^{\#}OS$ and let $p_{(\alpha,\ \beta)} \in X$. Let $f(p_{(\alpha,\ \beta)})_q$ A. Then $p_{(\alpha,\ \beta)q} f^{-1}(A)$ (i) implies that $p_{(\alpha,\ \beta)q}$ int(f^{-1}(A)) where int(f^{-1}(A)) is an IFOS in X. Let B = int(f^{-1}(A)). Since int(f^{-1}(A)) $\subseteq f^{-1}$(A), B $\subseteq f^{-1}$(A). Then f(B) $\subseteq$ f(f^{-1}(A)) $\subseteq$ A.

(ii)$\Rightarrow$(i): Let A $\subseteq$ Y be an $IFb^{\#}OS$ and let $p_{(\alpha,\ \beta)} \in X$. Suppose $f(p_{(\alpha,\ \beta)})_q$ A, then by (ii) there exists an IFOS B in X such that $p_{(\alpha,\ \beta)q}$ B and f(B) $\subseteq$ A. Now B $\subseteq$ f^{-1}(f(B)) $\subseteq$ f^{-1}(A). That is B = int(B) $\subseteq$ int(f^{-1}(A)). Therefore $p_{(\alpha,\ \beta)q}$B implies $p_{(\alpha,\ \beta)q}$ int(f^{-1}(A)).

Proposition 3.8: Let f_1: (X, τ) → (Y, σ) and f_2 : (X, τ) → (Y, σ) be any two intuitionistic fuzzy completely $b^{\#}$ continuous mappings. Then the mapping (f_1,f_2) : (X, τ) → $(Y \times Y, \sigma \times \sigma)$ is also an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

Proof: Let $A \times B$ be an $IFb^{\#}CS$ of $Y \times Y$. Then $(f_1, f_2)^{-1}(A \times B)(x)=(A \times B)(f_1(x),f_2(x))=$
$\langle x, \min(\mu_A(f_1(x)), \mu_B(f_2(x))), \max(\nu_A(f_1(x)), \nu_B(f_2(x)))\rangle =$
$\langle x, \min(f_1^{-1}(\mu_A)(x), f_2^{-1}(\mu_B)(x)), \max(f_1^{-1}(\nu_A)(x), f_2^{-1}(\nu_B)(x)\rangle =$
$f_1^{-1}(A) \cap f_2^{-1}(B)(x)$. Since f_1 and f_2 are an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, f^{-1}(A) and f^{-1}(B) are IFROSs in X. Since the intersection of two IFROSs is an IFROS, $f_1^{-1}(A) \cap f_2^{-1}(B)$ is an IFROS in X. Hence (f_1,f_2) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

Proposition 3.9: Let f : X → Y and g : Y → Z be any two mappings. If f and g are intuitionistic fuzzy completely $b^{\#}$ continuous mapping, then g ∘ f is also an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an $IFT_{b^{\#}}$ space.

Proof: Let B be an $IFb^{\#}CS$ in Z. Since g is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, g^{-1}(B) is an IFRCS in Y. Since every IFRCS is an IFCS, g^{-1}(B) is an IFCS in Y. As Y is an $IFT_{b^{\#}}$ space, g^{-1}(B) is an $IFb^{\#}CS$ in Y. Now as f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is an IFRCS in X. Hence g ∘ f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

Proposition 3.10: Let f : X → Y and g : Y → Z be any two mappings. If f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping and g is an intuitionistic fuzzy $b^{\#}$ irresolute mapping then g ∘ f is also an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

Proof: Let B be an $IFb^{\#}CS$ in Z. Since g is an intuitionistic fuzzy $b^{\#}$ irresolute mapping, g^{-1}(B) is an $IFb^{\#}CS$ in Y. Also, since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFRCS in X. Since $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, g ∘ f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

Proposition 3.11: Let f : X → Y and g : Y → Z be any two mappings. If f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping and g is an intuitionistic fuzzy $b^{\#}$ continuous mapping then g ∘ f is also an intuitionistic fuzzy completely continuous mapping.

Proof: Let B be an IFCS in Z. Since g is an intuitionistic fuzzy $b^{\#}$ continuous mapping, g^{-1}(B) is an $IFb^{\#}CS$ in Y. Also, since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFRCS in X. Since $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, (g ∘ f) is an intuitionistic fuzzy completely continuous mapping.

Proposition 3.12: Let f : X → Y and g : Y → Z be any two mappings. If f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping and g is an intuitionistic fuzzy $b^{\#}$ continuous mapping then g ∘ f is also an intuitionistic fuzzy completely continuous mapping.

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Proof: Let B be an IFCS in Z. Since g is an intuitionistic fuzzy $b^{\#}$ continuous mapping, $g^{-1}(B)$ is an $IFb^{\#}CS$ in Y. Also, since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFRCS in X. Since $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, $g \circ f$ is an intuitionistic fuzzy completely continuous mapping.

Proposition 3.13: Let f : X → Y and g : Y → Z be any two mappings. If f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping and g is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping then $g \circ f$ is also an intuitionistic fuzzy $b^{\#}$ irresolute mapping.

Proof: Let B be an $IFb^{\#}CS$ in Z. Since g is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $g^{-1}(B)$ is an IFRCS in Y. Also, since f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an $IFb^{\#}CS$ in X. Since $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, $g \circ f$ is an intuitionistic fuzzy $b^{\#}$ irresolute mapping.

Definition 3.14: A mapping f: $(X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping if $f^{-1}(V)$ is an intuitionistic fuzzy clopen set in (X, σ) for every $IFb^{\#}CS$ V of (Y, σ).

Example 3.15: Let X = {a, b}, Y = {u, v}. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ are IFS on X and Y respectively, where, $G_1 = \langle x, (0.2_a, 0.3_b), (0.4_a, 0.5_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.5_b), (0.2_a, 0.3_b)\rangle$, $G_3 = \langle y, (0.2_u, 0.3_v), (0.4_u, 0.5_v)\rangle$ and $G_4 = \langle y, (0.4_u, 0.5_v), (0.2_u, 0.3_v)\rangle$.Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping.

Proposition 3.16: A mapping f : $(X,\tau) \to (Y,\sigma)$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping if and only if the inverse image of each $IFb^{\#}OS$ in Y is an intuitionistic fuzzy clopen in X.

Proof: Straight forward.

Proposition 3.17: A mapping f : $(X,\tau) \to (Y,\sigma)$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy continuous mapping where Y is an $IFT_{b^{\#}}$ space.

Proof: Let B be an IFCS in Y. Since every IFCS is an $IFb^{\#}CS$ in an $IFT_{b^{\#}}$ space, B is an $IFb^{\#}CS$ in Y, as Y is an $IFT_{b^{\#}}$ space. Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Thus $f^{-1}(B)$ is an IFCS in X. Hence f is an intuitionistic fuzzy continuous mapping.

Proposition 3.18: A mapping f:$(X,\tau) \to (Y,\sigma)$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, where X and Y are $IFT_{b^{\#}}$ spaces.

Proof: Let B be an IFRCS in Y. Since every IFRCS is an IFCS, B is an IFCS in Y. Since Y is an $IFT_{b^{\#}}$ space, B is an $IFb^{\#}CS$ in Y. Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Thus $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an $IFb^{\#}CS$ in an $IFT_{b^{\#}}$ space, $f^{-1}(B)$ is an $IFb^{\#}CS$ in X, as X is an $IFT_{b^{\#}}$ space. Hence f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

Proposition 3.19: A mapping f:$(X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping and then f is an intuitionistic fuzzy $b^{\#}$ continuous mapping where X and Y are $IFT_{b^{\#}}$ spaces.

Proof: Let B be an IFCS in Y. Since every IFCS is an $IFb^{\#}CS$ in an $IFT_{b^{\#}}$ space, B is an $IFb^{\#}CS$ in Y, as Y is an $IFT_{b^{\#}}$ space. Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Thus $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an $IFb^{\#}CS$ in an $IFT_{b^{\#}}$ space, $f^{-1}(B)$ is an $IFb^{\#}CS$ in X, as X is an $IFT_{b^{\#}}$ space. Hence f is an intuitionistic fuzzy $b^{\#}$ continuous mapping.

Proposition 3.20: A mapping f : $(X,\tau) \to (Y,\sigma)$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy semi continuous mapping, where Y is an $IFT_{b^{\#}}$ space.

Proof: Let B be an IFCS in Y. Since every IFCS is an $IFb^{\#}CS$ in an $IFT_{b^{\#}}$ space, B is an $IFb^{\#}CS$ in Y as Y is an $IFT_{b^{\#}}$ space. Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Thus $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an IFSCS, $f^{-1}(B)$ is an IFSCS in X. Hence f is an intuitionistic fuzzy semi continuous mapping.

Proposition 3.21: A mapping f : $(X, \tau) \to (Y,\sigma)$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy α continuous mapping, where Y is an $IFT_{b^{\#}}$ space.

Proof: Let B be an IFCS in Y. Since every IFCS is an $IFb^{\#}CS$ in an $IFT_{b^{\#}}$ space, B is an $IFb^{\#}CS$ in Y, as Y is an $IFT_{b^{\#}}$ space. Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Thus $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an $IF\alpha CS$, $f^{-1}(B)$ is an $IF\alpha CS$ in X. Hence f is an intuitionistic fuzzy α continuous mapping.

Proposition 3.22: A mapping f : $(X,\tau) \to (Y,\sigma)$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping then f is

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an intuitionistic fuzzy pre continuous mapping, where Y is an IFT $_{b^\#}$ space.

Proof: Let B be an IFCS in Y. Since every IFCS is an IFb$^\#$CS in an IFT $_{b^\#}$ space. B is an IFb$^\#$CS in Y as Y is an IFT $_{b^\#}$ space. Since f is an intuitionistic fuzzy perfectly b$^\#$ continuous mapping, f^{-1}(B) is an intuitionistic fuzzy clopen set in X. Thus f^{-1}(B) is an IFCS in X. Since every IFCS is an IFPCS, f^{-1}(B) is an IFPCS in X. Hence f is an intuitionistic fuzzy pre continuous mapping.

Proposition 3.23: Let f : X → Y and g : Y → Z be any two intuitionistic fuzzy perfectly b$^\#$ continuous mappings where Y is an IFT $_{b^\#}$ space. Then their composition g ∘ f : X → Z is an intuitionistic fuzzy perfectly b$^\#$ continuous mapping.

Proof: Let A be an IFb$^\#$CS in Z. Then by hypothesis, g^{-1}(A) is an intuitionistic fuzzy clopen set in Y. Since Y is an IFT $_{b^\#}$ space, g^{-1}(A) is an IFb$^\#$CS in Y. Again by hypothesis, $f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy clopen set in X. Since $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$, $(g \circ f)^{-1}(A)$ is an intuitionistic fuzzy clopen set in X. Hence g ∘ f is an intuitionistic fuzzy perfectly b$^\#$ continuous mapping.

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