Comparison of SLM and PTS method for PAPR reduction in OFDM

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Abstract— Orthogonal frequency division multiplexing (OFDM) also referred to as multicarrier communication systems, have become a key technology in current and for future communication systems. Due to OFDM's immunity to many channel imperfections, it is the ideal modulation scheme for many applications which transmit signals in hostile environments. A major drawback of OFDM is the high peak-toaverage-power ratio (PAPR) problem, which can lead to low power efficiency and nonlinear distortion at the transmitter power amplifier. Selected mapping (SLM) and Partial transmit sequences (PTS) are powerful and distortion less peak power reduction schemes for OFDM. In SLM the transmitter selects one favourable transmit signal from a set of sufficiently different signals which all represent the same information, while in PTS the transmitter constructs its transmit signal with low PAPR by coordinated addition of appropriately phase rotated signal parts. In this paper the technique of SLM is simulated for OFDM and it is found that using this technique with 8 alternative sub carrier vectors, PAPR is reduced to approximately 4 dB whereas using PTS with 8 sub block partitions, PAPR is reduced to approximately 5 dB.

Keywords— Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), selected mapping, partial transmit sequences.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing has many advantages including resistance to multipath fading and high data rates. It has therefore been chosen as the modulation standard for IEEE802.11a/g WLAN, Worldwide Interoperability for Microwave Acess (WiMax), Digital Video Broadcasting (DVB). However a major drawback of OFDM is the manner in which the phases can align in the frequency domain causing high peaks to result in the time domain [1]. High peak values cause saturation of the power amplifier and both in-band and out-of-band distortion when limiting effects occurs. To prevent such phenomena amplifiers are normally "backed off" by approximately the PAPR. This however severely impacts power amplifier efficiency, making it preferable to reduce the PAPR of the signal before it enters the power amplifier [1]. For PAPR reduction number of techniques have been developed e.g. Clipping, Coding, Selected Mapping, Partial Transmit Sequences, Tone Injection, and Tone Reservation. Selected mapping (SLM) & Partial

transmit sequences (PTS) are powerful and distortion less peak power reduction schemes for OFDM. In SLM the transmitter selects one favourable transmit signal from a set of sufficiently different signals which all represent the same information, while in PTS the transmitter constructs its transmit signal with low PAPR by coordinated addition of appropriately phase rotated signal parts[2].

II. PAPR IN OFDM

OFDM signals have a higher Peak-to-Average Power Ratio (PAPR) than single carrier signals. The reason for this is that in the time domain, a multicarrier signal is the sum of many narrowband signals. At some time instances, this sum is large, at other times it is small, which mean that the peak value of the signal is substantially larger than the average value. This high PAPR is one of the most important implementation challenges that face OFDM because it reduces the efficiency and hence increases the cost of the RF power amplifier.

Let $A = [A_0A_1...,A_{N-1}]$ denote an input symbol sequence in the frequency domain, where A_K represents the complex data of the kth subcarrier and N the number of sub carriers of OFDM signal. Let T be a period of input symbol and NT a period of OFDM signal. The OFDM signal is generated by summing all the N modulated subcarriers each of which is separated by 1/T.

Then the complex OFDM signal in time domain is expressed as

$$a_{t} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_{n} e^{i2\pi nt/NT}, 0 \le t \le NT$$
(1)

Where, t is continuous time index. The OFDM signal sampled at Nyquist rate can be written as

$$a_{k} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_{n} e^{j2\pi k/N}, k = 0, 1, \dots, N-1$$
(2)

Which can also be in a vector form, called an OFDM signal sequence, as $a = [a_0a_1...a_{N-1}]$. In fact, *a* correspond to inverse fast Fourier transform (IFFT) of A. The PAPR of OFDM signal sequence *a* is defined as the ratio between

maximum instantaneous power and its average power, which can be written as,

$$PAPR(a) = \frac{\max_{0 \le k \le N-1} |a_k|^2}{E\left[\left|a_k\right|^2\right]}$$
(3)

Where E[.] denotes the expectation operator [3][6].

Statistically it is possible to characterize the PAPR distribution (probability that PAPR exceeds given threshold χ_0) using its cumulative distribution function (CDF) or complementary cumulative distribution function (CCDF). For the case of OFDM, the following expression for the CCDF holds,

$$P_{r} \{ PAPR > \chi_{0} \} = 1 - (1 - \exp(-\chi_{0}))^{N}$$

$$CCDF_{PAPR} = 1 - CDF_{PAPR}$$
(4)

III. SELECTED MAPPING

In this most general approach it is assumed that U statistically independent alternative transmits sequences $a_{\mu}^{(u)}$ represent the same information. Then, that sequence $\tilde{a}_{\mu} = a_{\mu}^{(\tilde{u}_{\mu})}$ with the lowest PAPR, denoted as $\tilde{\chi}_{\mu}$, is selected for transmission. The probability that $\tilde{\chi}_{\mu}$ exceeds χ_{0} is approximated by,



Fig. 1 PAPR reduction in SLM-OFDM [2]

Because of the selected assignment of binary data to the transmit signal, this principle is called selected mapping. A set of U markedly different, distinct, pseudo-random but fixed vectors,

$$P_{v}^{(u)} = \left[P_{0}^{(u)}, \dots, P_{D-1}^{(u)}\right]$$
$$P_{v}^{(u)} = e^{+j\varphi_{v}^{(u)}}, \varphi_{v}^{(u)} \in \left[0, 2\pi\right), 0 \le v < N, 1 \le u \le U, \qquad (6)$$

must be defined. The subcarrier vector A_{μ} is multiplied subcarrier wise with each one of the U vectors $P^{(u)}$, resulting in a set of U different subcarrier vectors $A_{\mu}^{(u)}$ with components,

$$A_{\mu,\nu}^{(u)} = A_{\mu,\nu} \cdot P_{\nu}^{(u)}, 0 \le \nu < N, 1 \le u \le U \quad (7)$$

Then, all U alternative subcarrier vectors are transformed into time domain to get $a_{\mu}^{(u)} = IDFT \{A_{\mu}^{(u)}\}$ and finally that transmit sequence $\tilde{a}_{\mu} = a_{\mu}^{(\tilde{u}_{\mu})}$ with the lowest PAPR $\tilde{\chi}_{\mu}$ is chosen. The SLM-OFDM transmitter is depicted in Fig. 1 where it is visualized that one of the alternative subcarrier vectors can be the unchanged original one. Optionally, differentially encoded modulation may be applied before the IDFT and right after generating the alternative OFDM symbols. At the receiver, differential demodulation has to be implemented right after the DFT [2][5].

IV. PARTIAL TRANSMIT SEQUENCES

In this method, the subcarrier vector A_{μ} is partitioned into V pair wise disjoint subblocks $A_{\mu}^{(\nu)}, 1 \le \upsilon \le V$. All subcarrier positions in $A_{\mu}^{(\upsilon)}$, which are already represented in another subblock are set to zero, so that $A_{\mu} = \sum_{i=1}^{V} A_{\mu}^{(\upsilon)}$



Fig. 2 PAPR reduction in PTS-OFDM [2]

We introduce complex – valued rotation factors $b_{\mu}^{(\upsilon)} = e^{+j\varphi_{\mu}(\upsilon)}, \varphi_{\mu}^{(\upsilon)} \in [0, 2\pi), 1 \le \upsilon \le V, \forall_{\mu}$ enabling a modified subcarrier vector $\tilde{A}_{\mu} = \sum_{\nu=1}^{V} b_{\mu}^{(\upsilon)} \cdot A_{\mu}^{(\upsilon)}$, which represents the same information as A_{μ} , if the set $\left\{ b_{\mu}^{(\upsilon)}, 1 \le \upsilon \le V \right\}$ (as side information) is known for each μ . Clearly, simply a joint rotation of all subcarriers in sub block υ by the same angle $\varphi_{\mu}^{(\upsilon)} = \arg\left(b_{\mu}^{(\upsilon)}\right)$ is performed. To calculate $\tilde{a}_{\mu} = IDFT\left\{\tilde{A}_{\mu}\right\}$, the linearity of the IDFT is exploited. Accordingly, the subblocks are transformed by V

separate and parallel D-point IDFTs, yielding

$$\tilde{a}_{\mu} = \sum_{\nu=1}^{V} b_{\mu}^{(\nu)} IDFT \left\{ A_{\mu}^{(\nu)} \right\} = \sum_{\nu=1}^{V} b_{\mu}^{(\nu)} \cdot a_{\mu}^{(\nu)}$$
(8)

Where the V so-called partial transmit sequences $a_{\mu}^{(\nu)} = IDFT \left\{ A_{\mu}^{(\nu)} \right\}$ have been introduced. The PTS-OFDM transmitter is shown in below Fig. 2 with the hint, that one PTS can always be left unrotated [2]. Based on them a peak value optimization is performed by suitably choosing the free parameters $b_{\mu}^{(\nu)}$ such that the PAPR is minimized for $\tilde{b}_{\mu}^{(\nu)}$. The $b_{\mu}^{(\nu)}$ may be chosen with continuous-valued phase angle, but more appropriate in practical systems is a restriction on a finite set of W (e.g. 4) allowed phase angles. The optimum transmit sequence then is,

$$\tilde{a}_{\mu} = \sum_{\nu=1}^{V} \tilde{b}_{\mu}^{(\nu)} \cdot a_{\mu}^{(\nu)} \,. \tag{9}$$

Both scheme require, that the receiver has knowledge about the generation of the transmitted OFDM signal in symbol period μ . Thus, in PTS the set with all rotation factors $\tilde{b}_{\mu}^{(v)}$ and in SLM the number \tilde{u}_{μ} of the selected $P^{(\tilde{u}_{\mu})}$ has to be transmitted to the receiver unambiguously so that this one can denote the subcarriers appropriately. The number of bits required for canonical representation of this side information is the redundancy R_{ap} introduced by the PAPR reduction scheme with PTS & SLM. As this side information is of highest importance to recover the data, it should be carefully protected by channel coding. In PTS the number of admitted combinations of rotation angles $\left\{ b_{\mu}^{(\upsilon)} \right\}$ should not be excessively high, to keep the explicitly transmitted side information within a reasonable limit. If in PTS each $b_{\mu}^{(\nu)}$ is exclusively chosen from a set of W admitted angles, then R_{ap} = $(V-1) \log_2 W$ bits per OFDM symbol are needed for this purpose. In SLM Rap = log_2U bits are required for side information. In PTS the choice $b_{\mu}^{(v)} \in \{\pm 1, \pm j\}$ (W = 4) is very interesting for an efficient implementation, as actually no multiplication must be performed, when rotating and combining the PTSs $a_{u}^{(v)}$ to the peak-optimized transmit sequence $ilde{a}_{\mu}$ in Eq.9. For SLM, choosing $P_{\upsilon}^{(u)}$ from the latter set has the same advantage, when generating the alternative subcarrier vectors by applying Eq.7 [2][4].

V. DESIGN AND SIMULATION RESULT

The OFDM system is implemented using MATLAB to allow various parameters of the system to be varied and tested. The following OFDM system parameters are considered.

Mapping: 16- QAM

Number of data sub-carriers: 52

Number of FFT points: 64

U alternative sub carrier vectors: 16 V sub block partitions: 8 Channel Mode: AWGN Data rate: 54 Mbps Total no. of symbols to TX. : 10000

The 64 point IFFT mapping is shown in Fig. 3. For generation of real output, IFFT mapping is done by taking its conjugate.



Fig.3 IFFT mapping for Complex time output OFDM signal

The results are shown in Fig. 4, 5 and 6. In Fig. 4 with N = 64 & U=16 PAPR reduction is 4.6 dB. In Fig. 5 with V=8 PAPR reduction is 5 dB. Fig. 6 shows comparison of both methods. It follows from this figure that PTS with W = 4 rotations and V = 8 IFFTs achieves a slightly better performance than SLM with U = 8 IFFTs. With PTS PAPR reduction achieves 5 dB and redundancy Rap= (V-1)log₂W = 14 bits / OFDM symbol ,whereas with SLM PAPR reduction achieves 4 dB and redundancy Rap = log₂U = 3 bits / OFDM symbol. From Fig.7 and Fig.8, 5 dB PAPR reduction can be achieved with SLM using U=32 alternative sequences (no. of IFFTs) and 5 bits of redundancy whereas with PTS using V= 8 sub-block partitioning (no. of IFFTs) and 14 bits of redundancy.



Fig. 4 Plot of SLM technique for different values of U



Fig. 5 Plot of PTS technique for different values of V





Fig. 7 PAPR reduction performance vs. redundancy for different values of U and V $% \left(V_{1}^{2}\right) =0$



Fig. 8 PAPR reduction performance vs. additional system complexity for different values of U and V

VI. CONCLUSION

SLM & PTS both schemes utilize several IFFTs instead of one and choose one signal from a multiplicity of transmit sequences. SLM-OFDM and PTS-OFDM can work with arbitrary numbers of subcarriers and any symbol mapping scheme. SLM outperforms PTS in terms of PAPR reduction vs. redundancy, but PTS is considerably better with respect to PAPR reduction vs. additional system complexity

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