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Comparison of PAPR Reduction by Coding in OFDM System

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Abstract* - Orthogonal Frequency Division Multiplexing (OFDM) is an efficient modulation technique in both broadband wired and wireless communication. It has many advantages such as eliminating Inter symbol Interference (ISI), efficient use of spectrum and dividing the channel into narrowband flat fading sub channels and many more. One of the major disadvantages of OFDM is that the time domain OFDM signal which is a sum of several sinusoids leads to high peak to average power ratio (PAPR). This paper investigates three methods for PAPR reduction such as combined Rotated Hadamard transform, combined Riemann transform and Zadoff-Chu Matrix Transform to achieve more novel PAPR reduction. The performance of these PAPR reduction techniques are analyzed by plotting the complementary cumulative distribution function (CCDF) plot and calculating BER and a comparison is made between all the above methods. The simulation results shows that the Zadoff-Chu Matrix Transform reduces PAPR much better than the Rotated Hadamard transform, Riemann matrix transform and Conventional OFDM.

Keywords- OFDM, PAPR, Hadamard transform, Riemann, ZCT, companding, CCDF, BER,

I. INTRODUCTION

In today's world, wireless communications has become an essential part of everyday life. Orthogonal frequency division multiplexing (OFDM) has become popular in wireless applications. OFDM provides greater immunity to multi-path fading and also reduces the complexity of equalizers [1]. OFDM is widely adopted in various communication standards like Digital Audio Broadcasting (DAB), Digital video Broadcasting (DVB), Wireless Local Area Networks (WLAN), Wireless Metropolitan Area Networks (WMAN), IEEE 802.11 and IEEE 802.16 wireless broadband access systems, etc.

Large PAPR could cause poor power efficiency or serious performance degradation to transmit power amplifier [2]. Therefore, nonlinearities may get overloaded by high signal peaks, causing inter modulation among subcarriers and, more critical, undesired out-of-band radiation. If RF power amplifiers are operated without large power back-offs, it is impossible to keep the out-of-band power below specified limits. This leads to very inefficient amplification and expensive transmitters so that it is highly desirable to reduce the PAPR [3].

Several schemes have been proposed to reduce the PAPR. These techniques can mainly be categorized into three types namely Signal scrambling techniques, Signal distortion techniques and block coding. Signal scrambling techniques are all variations on how to scramble the codes to decrease the PAPR. Coding techniques can be used for signal scrambling [4]. Comparatively signal distortion techniques are straight forward techniques. Signal distortion methods distort the high peak valued portion of OFDM signals using different techniques for PAPR reduction. The precoding based techniques are simple linear techniques to implement without the need of any side information. This paper compares the three precoding methods for PAPR reduction such as combined Rotated Hadamard transform (RHT), combined Riemann matrix Transform (RMT) and Zadoff-Chu Matrix Transform (ZCT) to achieve more novel PAPR reduction.

The rest of the paper is organized as follows: section II describes the PAPR problem in OFDM system. Section III compares the different transforms. Section IV reports steps involved in the scheme. Section V reports the simulation results and conclusion is presented in section VI.

II. SYSTEM MODEL

This section illustrates the basic OFDM system and the PAPR definition. Figure 1 gives the block diagram of an OFDM system consisting of N subcarriers. Baseband modulated symbols are passed through serial to parallel converter which generates complex vector of size N. We can write the complex vector of size N as $X = [X_0\, X_1 \ldots\ldots.. X_{N-1}]^T$. X is then passed through the IFFT block. The complex baseband OFDM signal with N subcarriers can be written as

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\Pi nk/N} \qquad n=0,\ 1,\ 2 \ldots\ N\text{-}1 \qquad (1)$$

The PAPR is the ratio of peak power to the average power of OFDM signal. For discrete time domain OFDM signal, it can be calculated as.

$$PAPR(dB) = 10\log_{10}\left[\frac{\max\{|x[n]^2|\}}{E\{|x[n]^2|\}}\right] \qquad (2)$$

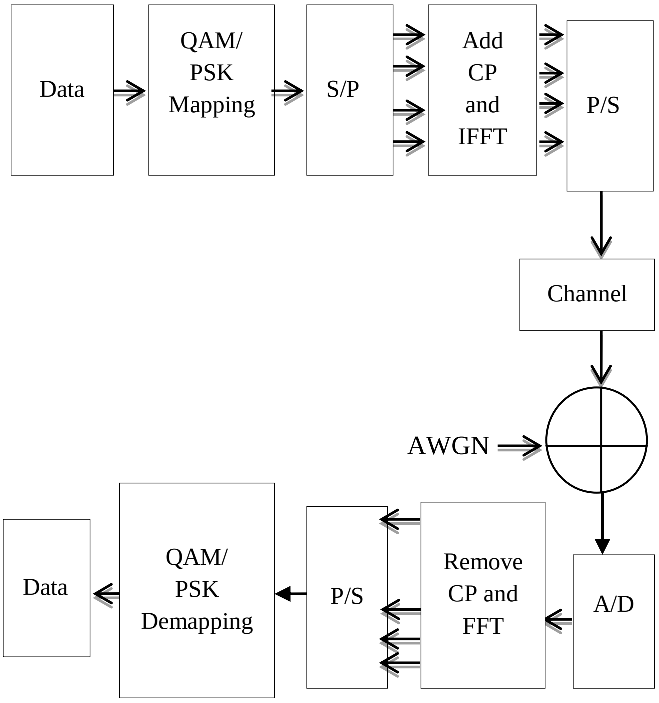

Fig 1. Block diagram of OFDM system

Where E [.] denotes expectation and the CCDF for an OFDM signal can be written as.

$$P\left(PAPR > PAPR_0\right) = \left(1 - \exp\left(-PAPR_0\right)\right)^N \qquad (3)$$

Where $PAPR_0$ is the clipping level

Equation (2) predicts that the PAPR can be defined by the amplitude of output signal. So when output signal exceed a certain value then obviously PAPR also take a higher value.

III. COMPARISION OF THE DIFFERENT TRANSFORMS

A. Rotated Hadamard Transform

The rotated Hadamard matrix [5] is a Hadamard matrix with the rotation described in Equation 4 below.

$$U = \frac{1}{\sqrt{N}} H_{M\times M}\, diag\left(exp\left(\frac{j\pi m}{C}\right)\right) \qquad (4)$$

Where C is the rotation value from which the modulation is rotated back on to itself. H is the Hadamard matrix and M is the size of the matrix. The Hadamard matrix of order 2 is given by

$$H_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad (5)$$

The fig 2 shows the system block. The modulation data is multiplied by U and the rotation takes place producing a higher modulation scheme. The rotated Hadamard is capable of achieving 16QAM. So this rotated Hadamard produces a higher order scheme than the traditional Hadamard. This is directly translated into a better BER performance in OFDM system of rotated Hadamard over Hadamard.

B. Riemann matrix transform

The Riemann matrix [6] is defined by

A = B (2: n+1, 2: n+1)

Where,

$$B(i,j) = \begin{cases} i-1 & \text{if i divides j} \\ -1 & \text{otherwise} \end{cases} \qquad (6)$$

The Riemann matrix has these properties:

- Each Eigen value e (i) satisfies $\left|e(i)\right| \le \frac{m-1}{m}$, where m = n+1.
- $i \le e(i) \le i+1$ With at most $m-\sqrt{m}$ exceptions.
- All integers in the interval [m/3, m/2] are Eigen values.

Using Equation (6), Riemann Matrix (A) of order 4 can be written as:

$$A = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 2 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix} \qquad (7)$$

C. Companding transforms

A companding transform [7] [8] performs compression at the transmit end after the IFFT process and expansion at the receiver end prior to FFT process. After companding, the signal now becomes.

$$s(n) = C\left\{x(n)\right\} = \frac{vx(n)}{\ln(1+u)\left|x(n)\right|}\ln\left(1+\frac{u}{v}\left|x(n)\right|\right) \qquad (8)$$

Where v is the average amplitude of signal and u is the companding parameter. This transform reduces the PAPR of OFDM signal by amplifying the small signal and shortening the big signal. In receiver end, the receiver signal is to be expanded by the inverse companding transform before it is sent to the FFT process unit. The expanding equation is

$$y(n) = C^{-1}\left\{r(n)\right\}\frac{vr(n)}{u\left|r(n)\right|}\left\{exp\left[\frac{r\left|n\right|\ln(1+u)}{v}\right]-1\right. \qquad (9)$$

D. Zadoff-Chu Matrix Transform

Zadoff-Chu sequences [9] are class of poly phase sequences having optimum correlation properties. Zadoff-Chu sequences have an ideal periodic autocorrelation and constant magnitude. The Zadoff-Chu sequences of length L can be defined as:

$$z(k)=\begin{cases} e^{\frac{j2\pi r}{L}\left(\frac{k^2}{2}+qk\right)} & for\ L\ even \\ e^{\frac{j2\pi r}{L}\left(\frac{k(k+1)}{2}+qk\right)} & for\ L\ odd \end{cases} \tag{10}$$

Where k = 0, 1, 2… L-1, q is any integer, r is any integer relatively prime to L and j= $\sqrt{-1}$.

In this technique the baseband modulated data is passed through serial to parallel converter which generates a complex vector of size N that can be written as $X=\left[X_0 X_1 \ldots\ldots X_{N-1}\right]^T$. Later Zadoff-Chu Transform (ZCT) is applied to this complex vector which transforms this complex vector into new vector of length N that can be written as Y=RX= $\left[Y_0 Y_1 \ldots\ldots Y_{N-1}\right]^T$

Where R is a ZCT based row-wise precoding matrix of size L = N×N. With the use of reordering as given in equation (11)

$$k = mN + l \tag{11}$$

Matrix R with row wise reshaping can be written as

$$R=\begin{bmatrix} r_{00} & r_{01} & \cdots & r_{(N-1)0} \\ r_{10} & r_{11} & \cdots & r_{(N-1)1} \\ \vdots & \vdots & \ddots & \vdots \\ r_{(N-1)0} & r_{(N-1)1} & \cdots & r_{(N-1)(N-1)} \end{bmatrix} \tag{12}$$

By letting q = 1 and r = 1 the ZCT for Even L can be written as rk= exp [(j*pi*k2) / L2]. Accordingly, precoding X gives rise to Y as follows:

$$Y = RX \tag{13}$$

$$Y_m=\sum_{l=0}^{N-1} r_{m,l} X_l \qquad m=0,1,\ldots\ldots N-1 \tag{14}$$

Here $r_{m,l}$ means mth row and lth column of precoder matrix. The complex baseband OFDM signal with N subcarriers without precoding is given by

$$x_n=\frac{1}{\sqrt{N}}\sum_{m=0}^{N-1} X_m e^{j2\Pi nm/N} \qquad n=0,1,2\ldots,N-1 \tag{15}$$

However, expanding (15) while using q = 1 and r = 1 in (10), gives complex baseband ZCT precoding based OFDM signal with N subcarriers as

$$\hat{x}_n=\frac{1}{\sqrt{N}}\sum_{m=0}^{N-1}\left\{e^{j2\Pi nm/N}\left[e^{j\pi m^2}\sum_{l=0}^{L-1}\left(Y_l e^{\frac{j\pi l^2}{L^2}}\right)e^{j\frac{2\pi ml}{L}}\right]\right\} \tag{16}$$

The expression in (18) suggests that x_n are IFFT of constellation data X_1 pre multiplied with quadratic phase and IFFT precoded, and then alternated with ±1. The PAPR of ZCT-OFDM signal in (16) can be written as

$$PAPR(dB)=\left[\frac{\max\{|x[n]^2|\}}{E\{|x[n]^2|\}}\right] \tag{17}$$

IV. STEPS INVOLVED

The PAPR of OFDM signal is reduced by using the above techniques with companding transform. The input data is transformed by Rotated Hadamard Transform or Riemann Matrix transform or by using ZCT, The transformed data stream is given as input to IFFT signal processing unit. The block diagram of the system is as shown in fig 2.

The signal processing step is as below:

Step 1: The sequence X is transformed by RHT or RMT or ZCT i.e. Y=KX.

Step 2: y=IFFT(Y), where y=[y (1) y (2)… y (N)] ^{T}

Step 3: Do companding transform to y, i.e. s (n) =C{y (n)}.

Step 4: Transmit the signal through an AWGN channel

Step 5: Do inverse companding transform to the received signal r (n) i.e. $\hat{y}(n)=C^{-1}\{r(n)\}$

Step 6: Do FFT transform to signal $\hat{y}(n)$ i.e. $\hat{Y}=FFT(\hat{y})$

Step 7: Do inverse RHT or RMT or ZCT to the signal $\hat{Y}$, i.e. $\hat{X}=K^T\hat{Y}$. Then the signal $\hat{X}$ is de-mapped to bit stream.

V. SIMULATION RESULTS

The simulation of the above OFDM system for PAPR reduction is performed using MATLAB. The channel is modelled as AWGN channel. In simulation, an OFDM system with subcarrier N=64 is considered and for mapping M-QAM (Where M=16, 64) modulations are used. The simulation results show the comparison between RHT, RMT, ZCT and conventional OFDM.

Fig 3 and Fig 4 shows the CCDF comparisons of ZCT with RHT, RMT and OFDM conventional without and with companding (u=1) respectively for N=64. At clip rate of 10^{-1}, the PAPR gain of 5.3dB, 4.8dB and 4.4dB is achieved without companding and it is 5.7dB, 3dB and 2.7dB with companding.

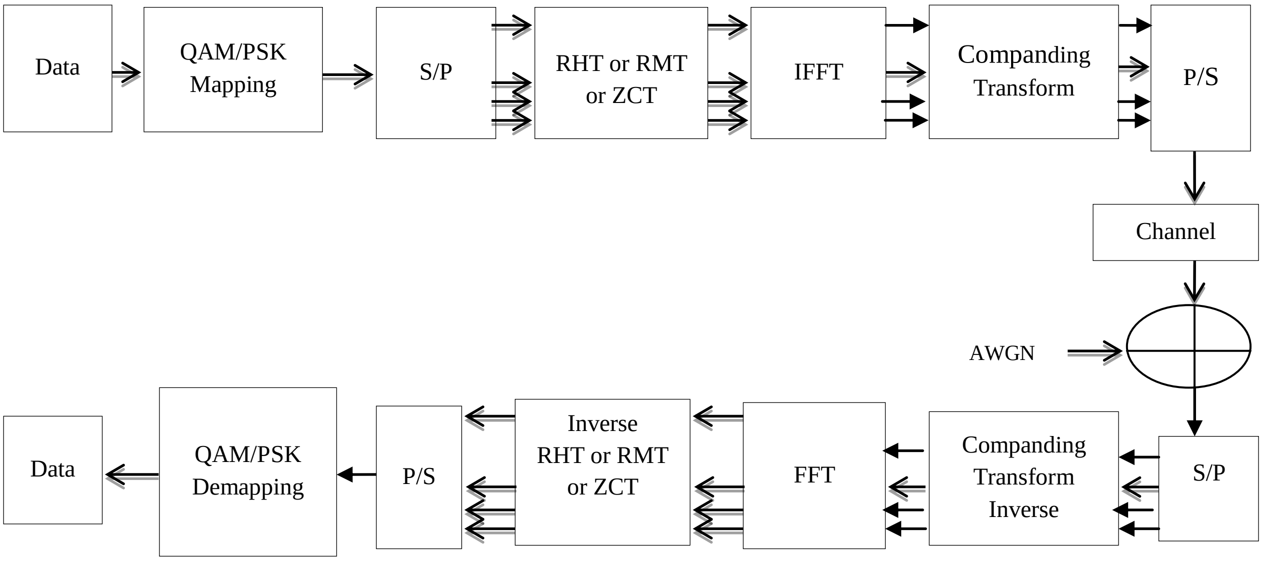

Fig 2 . General block diagram for the above Techniques

When ZCT OFDM system is compared with conventional OFDM, RMT and RHT for 16-QAM modulation

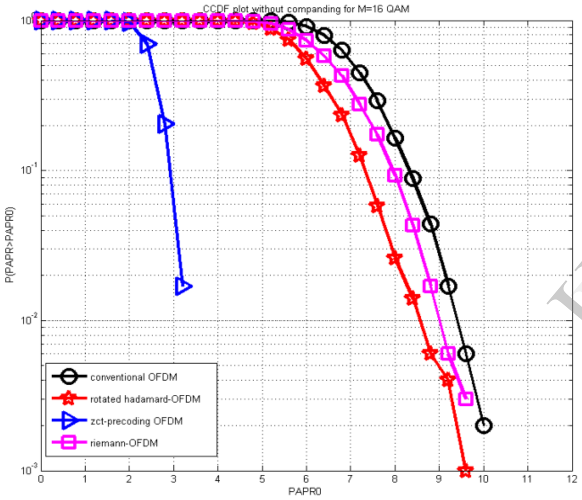

Fig 3 . CCDF plot without companding for N=64 and M=16 QAM

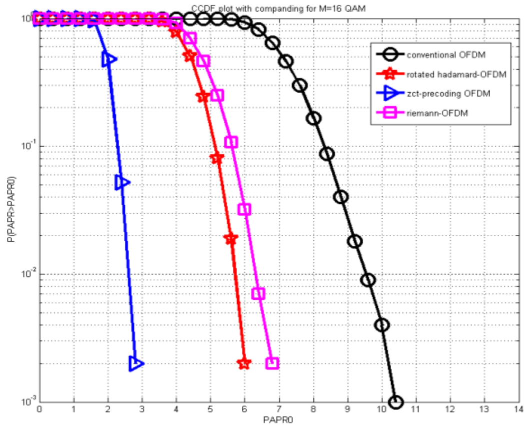

Fig 4 CCDF plot with companding for N=64 and M=16 QAM

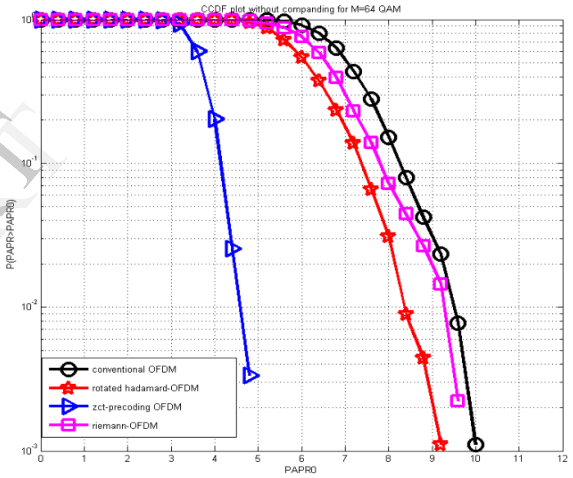

Fig 5 CCDF plot with companding for N=64 and M=16 QAM

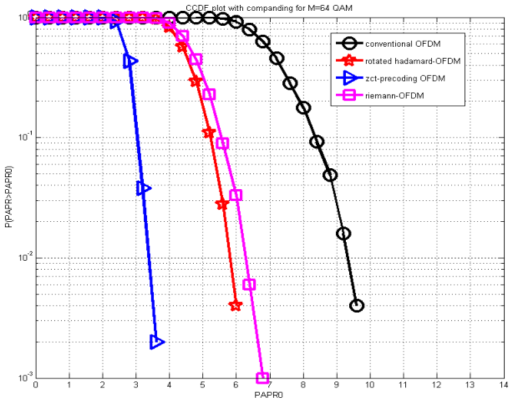

Fig 6 . CCDF plot without companding for N=64 and M=64 QAM

Similarly Fig.5 and Fig.6 shows that at clip rate of 10^{-1} the PAPR gain of 4.3dB, 3.8dB and 3.3dB is achieved without companding and it is 5.5dB, 2.6dB and 2.2dB with companding when ZCT OFDM system is compared with conventional OFDM, RMT and RHT for 64-QAM modulation.

Fig 7 and fig 8 shows the BER performance of above mentioned techniques in AWGN channel for 16 QAM modulations without and with companding. Similarly fig 9 and fig 10 shows the BER performance of above techniques in AWGN channel for 64 QAM modulations without and with companding.

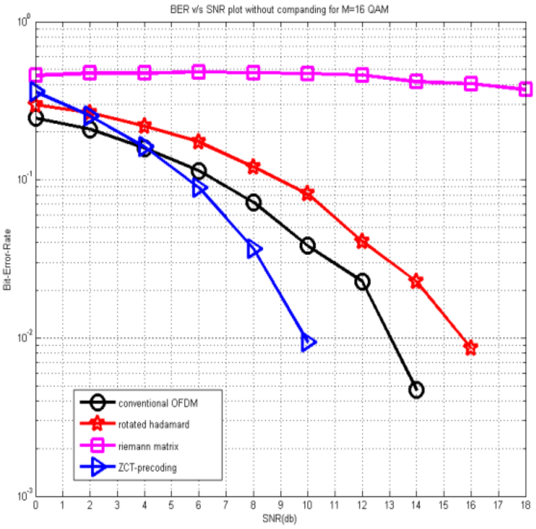

Fig 7 . BER plot without companding for N=64 and M=16 QAM

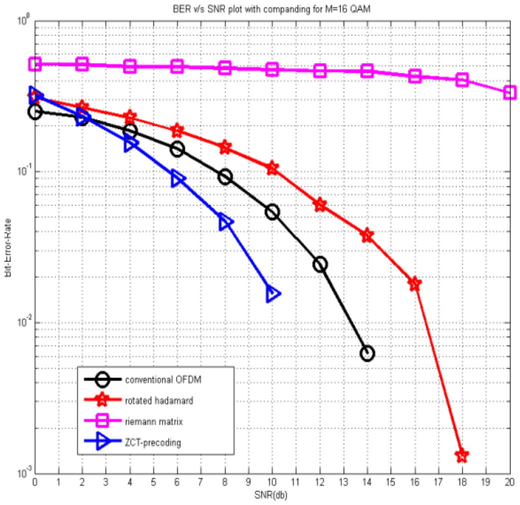

Fig 8 . BER plot with companding for N=64 and M=16 QAM

From fig 7 it is observed that to achieve BER of 10^{-1} the ZCT OFDM requires 5.5 dB SNR whereas conventional OFDM requires 6.8dB and RHT requires 9dB SNR hence ZCT based OFDM system shows better BER reduction compared to all the other transforms.

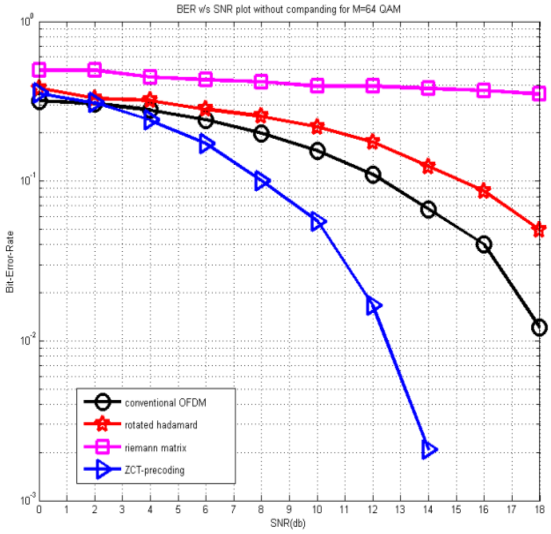

Fig 9 . BER plot without companding for N=64 and M=64 QAM

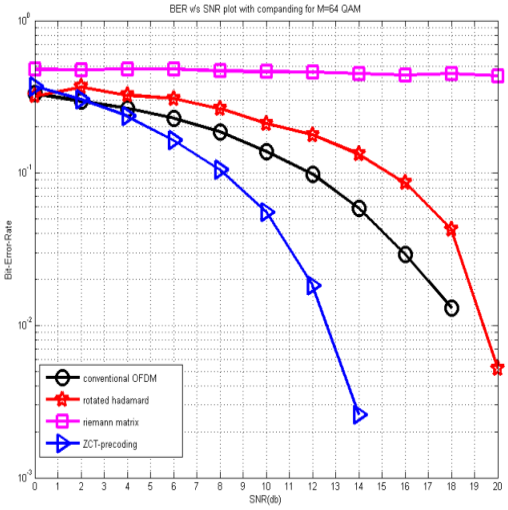

Fig 10 . BER plot with companding for N=64 and M=64 QAM

Table 1 and table 2 summarize the PAPR comparison for the above mentioned schemes without and with companding technique. It is observed that the companding technique greatly reduces PAPR in RMT and RHT compared to ZCT and conventional OFDM for both 16-QAM and 64-QAM modulation.

TABLE I. PAPR COMPARISON WITHOUT COMPANDING

WITHOUT COMPANDING (CLIP RATE 10^{-1})				
Type of Modulation (M-QAM)	***PAPR of OFDM (dB)***	***PAPR of RMT (dB)***	***PAPR of RHT (dB)***	***PAPR of ZCT (dB)***
16	8.3	7.8	7.4	3
64	8.5	8	7.5	4.2

TABLE II. PAPR COMPARISON WITH COMPANDING

WITH COMPANDING (CLIP RATE 10-1)				
Type of Modulation (M-QAM)	***PAPR of OFDM (dB)***	***PAPR of RMT (dB)***	***PAPR of RHT (dB)***	***PAPR of ZCT (dB)***
16	8.3	5.4	5.1	2.4
64	8.5	5.6	5.2	3

VI. CONCLUSION

In this paper Comparison of three different transforms for PAPR reduction in companding technique for OFDM system is performed. The BER and CCDF plots for these three transforms are obtained for 16-QAM and 64-QAM modulation techniques.

It is clear from tables 1 and table 2 that ZCT based OFDM system reduces PAPR much better than the Rotated Hadamard matrix transform, Riemann matrix transform and Conventional OFDM and it also reduces BER much better than all the other mentioned schemes. In addition ZCT OFDM systems don’t require any side information to be sent for the receiver. Thus it is concluded that ZCT based OFDM system are more favourable than Rotated Hadamard Transform, Riemann matrix transform and Conventional OFDM.

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