Comparison of Controllers for Active Vibration Control of A Plate with Piezo-Patches as Sensors/Actuators

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***Abstract:*-- This work is aimed at the comparative study of classical controllers such as PID controllers and new controllers (LQG and H∞ controllers) which are employed for the active vibration control of a cantilever plate with piezoelectric patches as sensors and actuators. These controllers are working for the purpose of vibration suppression over the plate. The cantilever plate is embedded with piezoelectric patches which are acting as actuators and sensors. The position of patches may be collocated or non-collocated depending upon the controller type. A comparison among the PID, LQG and H∞ controllers for the same pupose has been carried out.**

***Keywords:* PID, LQG, H∞, cantilever plate, piezoelectric patches, active vibration control.**

Nomenclature:

a = half length of the finite element in *x* direction, m
A = relative to surface area
b = half length of the finite element in *y* direction, m
C = elastic constant, N/m^2
C_s = piezoelectric sensor capacitance, F
D = electric displacement vector, C/m^2
e = piezoeletric stress coefficient, C/m^2
E = Young's modulus, N/m^2
f = force, N
h = thickness, m
K = stiffness matrix
M = mass matrix
q = displacement field vector
q_i = nodal displacement field, m
k = kinetic energy, J
u = displacement field in *x* direction, m
U = potential energy, J
v = displacement field in *y* direction, m
V = volume, m^3
w = displacement field in *z* direction, m
W = work, *J*

Greek Symbols

ε = strain field
σ = stress, N/m
ν = Poisson ratio
θ_u = rotation about *u*-axis
ξ = dielectric tensor
ζ = nodal displacement vector
Φ = electric potential, Volts
ω = frequency, rad/s
ρ = material density, kg/m^3

Subscripts

a = refers to the actuator
b = relative to the body
p = relative to the plate structure
s = relative to the sensor
sa = relative to the sensed voltage in the actuator
x = relative to *x* direction
y = relative to *y* direction
qq = relative to the stiffness
*q*Φ = relative to the piezoelectric stiffness
ΦΦ = relative to the dielectric stiffness

Superscripts

e = relative to the element
S = relative to constant strain
T = matrix transpose

I. INTRODUCTION

Active vibration control is the technique in which equal and opposite force is applied to suppress the external vibrations. Industrial processes which are precise cannot occur due to vibrations. Control of such vibrations is always being a field of curiosity for researchers. Today, there are different controllers which are employed for the vibration suppuration such as Proportional integrative differentiate, Linear quadratic Gaussian, Linear quadratic regulator and H∞ controllers etc.

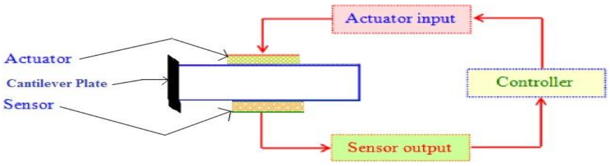


Fig.1. Schematic diagram for Active Vibration Control

Caruso G. et al.[1] studied the vibration control of an elastic cantilever plate, clamped on one side and excited by impulsive force acting on the free side. A modal model obtained by employing a suitable finite-element formulation together with a modal reduction, was used in the controller design. Qiu Z. C. et al.[2] used piezoelectric ceramics patches as sensors and actuators to control the vibration of the smart flexible clamped plate. A method for optimal placement of piezoelectric actuators and sensors on a cantilever plate was developed. Experimental set-up was build for smart plate and results were found on it. C.M.A. Vasques[3] presented a comparative study between the classical control strategies, amplitude velocity feedback and constant gain, and optimal control strategies, linear quadratic regulator (LQR) and linear quadratic Gaussian (LQG) controller, is performed in order to investigate their effectiveness to diminish vibrations in beams with piezoelectric patches acting as sensors or actuators. Vasudevan et. al.[4] showed the optimal control strategy based on the full state dynamic observer to control vibrations of a beam under limited magnetic field intensity. PID and LQR based on full state observer can reduce settling time and tip deflection response of free vibration oscillations. Shimon P. et al.[5] presented an efficient controller for vibration control in a fully clamped plate and an experiment between control methodologies and actuators was done. Theoretical and experimental studies were undertaken with verifying results. Chhabra et. al.[6][7] presented the active vibration control of beam like structures with laminated piezoelectric sensor and actuator layers bonded on top and bottom surfaces of the beam. The contribution of the piezoelectric sensor and actuator layers on the mass and stiffness of the beam has been considered with modeling of the structure in a state space form. The designing of state/output feedback control by Pole placement technique and LQR optimal control approach are exercised to achieve the desired control. Feedback Gain was find out by using Hybrid Multiobjective Genetic Algorithm-Artificial Neural Network. Pradeep et.al.[8][9] presented the work deals with the mathematical formulation and the computational model for vibration control of a beam with piezoelectric smart structures. A successful scheme of analyzing and designing piezoelectric smart structures with control laws is developed. Mukherjee A. et al.[10] studied the active vibration control of stiffened plates. The stiffened plate was formulated with finite element and piezoelectric effects. A velocity feedback algorithm was employed. Numerical examples for vibration control of isotropic and orthotropic stiffened plates were presented. Chhabra et. al.[11] showed that active vibration control of beam like structures with distributed piezoelectric actuator and sensor layers bonded on top and bottom surfaces of the beam. The control effect can be improved by locating the patches optimally. The piezoelectric patches are placed on the free end, middle end and fixed end. The study is done through simulation in MATLAB for various controllers like POF, PID and Pole Placement technique. Varun et. al.[12] studied the active vibration control in the cantilever beam with collocated sensors/actuators by using fuzzy controller. Amit et. al.[13] studied the basic techniques for analysis of active vibration control using piezoelectric sensor and actuator. A smart cantilever plate with the Neural Network and LQG controller is developed. Neeraj et. al.[14] presented various optimization techniques which for the optimal placement of piezoelectric sensors/actuators on a smart structure for active vibration control. And also how these optimization techniques can be implemented is studied. Meta-heuristic approaches such as Genetic algorithms, swarm intelligence, simulated annealing, tabu search, and other recent approaches are explained.

In this paper, different controllers used in active vibration control are stated. Classical controller like PID and new controllers like LQG and H∞ controllers with their formulations are explained. The terms like proportional, integrate and derivative are taken as parameters in PID controller, while noise reduction and collocation of patches is another requirement for LQG and H controller respectively. By using MATLAB, a comparison among these three controllers has been done by plotting graphs for different optimal positions of piezoelectric patches for respective controllers have been drawn. And a comparison by these plots is discussed.

II. PID CONTROLLER:

A. *A proportional-integral-derivative controller (PID controller):*

A proportional-integral-derivative controller (PID controller) is a control loop feedback mechanism (controller) widely used in vibration control techniques. A PID controller calculates an error value as the difference between a measured process variable and a desired set point value. Manipulated variable is used to minimize the error by adjusting the process. The PID controller algorithm involves three separate constant parameters: the proportional, the integral and derivative values, denoted P, I, and D. Simply put, these values can be interpreted in terms of time: P depends on the present error, I on the accumulation of past errors, and D is a prediction of future errors, based on current rate of change. The response of the controller can be explained in terms of the responsiveness of the controller to an error, the degree to which the controller overshoots the set point, and the degree of system oscillation.(Neeraj et. al.[16])

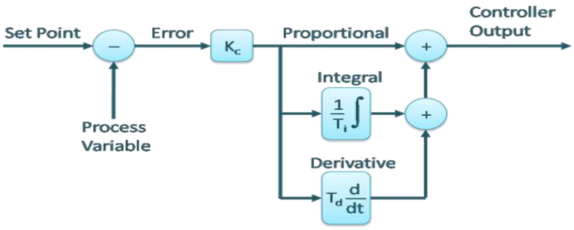


Fig.2: Block Diagram of PID controller

B. *PID Controller Theory:*

The PID controller is named after its three correcting terms, whose sum constitutes the manipulated variable (MV). The proportional, integral, and derivative terms are summed to calculate the output of the PID controller. Defining $u(t)$ as the controller output, the final form of the PID algorithm is:

$$u(t) = MV(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{d}{dt} e(t)$$

where

K_p: Proportional gain, a tuning parameter

K_i: Integral gain, a tuning parameter

K_d: Derivative gain, a tuning parameter

e: Error = $SP - PV$

t: Time or instantaneous time (the present) τ: Variable of integration; takes on values from time 0 to the present t.

III. LINEAR QUADRIC GAUSSIAN CONTROLLER

More general problem is LQG problem that deals with optimization of a quadratic performance measure for stochastic system (i.e. noise problem is also taken in the LQG problems).

LQG problem statement:

Consider the stochastic system

$$\{\dot{X}_m\} = [A_m]\{X_m\} + [B_{u_m}]\{u\} + [\Gamma_m]\{w_m\} \quad (1)$$

Where $([A_m]\{X_m\})$ are the number of states with n x 1 matrix, $([B_{u_m}]\{u\})$ are number of inputs with m x 1 matrix and $([\Gamma_m]\{w_m\})$ is noise input with r x 1 matrix.

In the case of optimal control, the following Lyapunov quadratic functional, to be minimized, is defined:

$$J_m = \left(\frac{1}{2}\right)\int_0^\infty (\{X_m\}^T[Q_m]\{X_m\} + \{u^T\}[R_m]\{u\})dt \quad (2)$$

Then, the following equations hold:

$$\{X_m\} = [\Phi]^{-1}\{X\}$$

$$\begin{Bmatrix}\{\eta\}\\ \{\dot{\eta}\}\end{Bmatrix} = \begin{bmatrix}[\Phi]^{-1} & 0\\ 0 & [\Phi]^{-1}\end{bmatrix}\begin{Bmatrix}\{q\}\\ \{\dot{q}\}\end{Bmatrix}, \quad (3)$$

Modal weighting matrices $[Q_m]$and$[R_m]$ are related to the well known traditional weighting matrices [Q] and [R], respectively, by

$$[Q_m] = [[\Phi]^{-T}[Q][\Phi]^{-1}] \text{ and } [R_m] = [[\Phi]^{-T}[R]\,[\Phi]^{-1}] \quad (4)$$

The input forces are defined by the relation:

$$\{u\} = -[K_m]\{X_m\} = -[K_m]\begin{Bmatrix}\{\eta\}\\ \{\dot{\eta}\}\end{Bmatrix}, \quad (5)$$

Where $[K_m]$, the modal gain matrix, is given by $[K_m] = \left[[R_m]^{-1}[B_{u_m}]^T[S_m]\right]$; and obtained solving the following Ricatti equation in the modal state space:

$$[S_m][A_m] + [A_m]^T[S_m] - [S_m][B_{u_m}][R_m]^{-1}[B_{u_m}]^T[S_m] + [Q_m] = [0] \quad (6)$$

$\hat{X}_m$ is the estimated modal state obtained from Kalman Filter (KF)

[Minimizes the Performance Index]

$$J_e = E\,[(\{X_m\} - \{\hat{X}_m\})^T.(\{X_m\} - \{\hat{X}_m\})]$$
$$= tr\,[E[(\{X_m\} - \{\hat{X}_m\})(\{X_m\} - \{\hat{X}_m\})^T]]$$

Where tr is trace value of performance index.

$$\{\dot{\hat{X}}_m\} = [A_m]\{\hat{X}_m\} + [B_{u_m}]\{u\} + [L](\{Y_m\} - [C_m]\{\hat{X}_m\}) \quad (7)$$

$$= ([A_m] - [L][C_m])\{\hat{X}_m\} + [B_{u_m}]\{u\} + [L]\{Y_m\} \quad (8)$$

$[L]$ is computed as follows:

As $[K_m] = \left[[R_m]^{-1}[B_{u_m}]^T[S_m]\right]$

And replacing values as;

$[A_m] \to [A_m]^T, [B_{u_m}] \to [C_m]^T, [Q_m] \to [W_m], [R_m] \to [V_m], [L] \to [K_m]^T$

So, $[L] = [S_{m_e}][C_m]^T[V_m]^{-1}$

$[S_{m_e}]$ is computed by solving Continuous Filter Algebraic Riccati Equation(CFARE).

$$[A_m][S_{m_e}] + [S_{m_e}][A_m]^T - [S_{m_e}][C_m]^T[V_m]^{-1}[C_m][S_{m_e}] + [\Gamma][W_m][\Gamma]^T = 0 \quad (9)$$

Block Diagram of LQG controller:

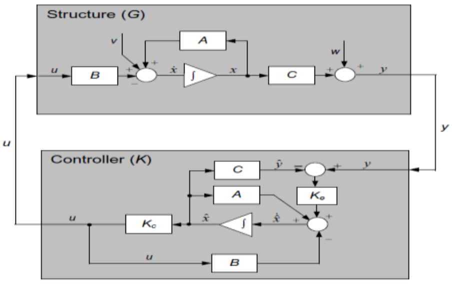


Fig.3: The inner structure of the LQG closed-loop system.

The LQG separation properly:

From equation (1),

$$\{\dot{\hat{X}}_m\} = ([A_m] - [L][C_m] - [B_{u_m}][K_m])\,\{X_m\} + [L]\{Y_m\} \quad (10)$$

From (2) and (10) with $[\Gamma] = [I]_{rxr}$, we get

$$\begin{bmatrix}\{\dot{X}_m\}\\ \{\dot{\hat{X}}_m\}\end{bmatrix} = \begin{bmatrix}[A_m] & -[B_{u_m}][K_m]\\ [L][C_m] & [A_m] - [B_{u_m}][K_m] - [L][C_m]\end{bmatrix}\begin{bmatrix}\{X_m\}\\ \{\hat{X}_m\}\end{bmatrix} + \begin{bmatrix}[I] & [0]\\ [0] & [L]\end{bmatrix}\begin{bmatrix}[W_m]\\ [V_m]\end{bmatrix} \quad (11)$$

Error becomes $\{e_m\} = \{X_m\} - \{\hat{X}_m\}$,

$$\begin{bmatrix}\{\dot{X}_m\}\\ \{e_m\}\end{bmatrix} = \begin{bmatrix}[A_m] - [B_{u_m}][K_m] & [B_{u_m}][K_m]\\ [0] & [A_m] - [L][C_m]\end{bmatrix}\begin{bmatrix}\{X_m\}\\ \{e_m\}\end{bmatrix} + \begin{bmatrix}[I] & [0]\\ [I] & -[L]\end{bmatrix}\begin{bmatrix}[W_m]\\ [V_m]\end{bmatrix} \quad (12)$$

$([A_m]-[B_{u_m}][K_m]), [K_m]([A_m]-[L][C_m])$ matrices are stable.

Eigen values of $([A_m]-[L][C_m])$ should be 5 to 10 times$([A_m]-[B_{u_m}][K_m])$, to get better performance.(Wodek K. Gawronski[15])

IV. H∞ CONTROLLER:

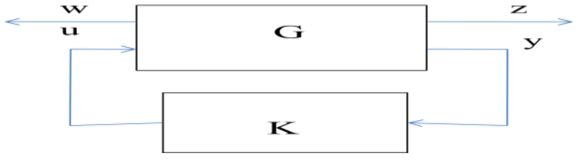
In the LQG controller design we assumed that the control inputs were collocated with disturbances, and that the control outputs were collocated with the performance. This assumption imposes significant limits on the LQG controller possibilities and applications. The locations of control inputs do not always coincide with the disturbance locations, and the locations of controlled outputs are not necessarily collocated with the location where the system performance is evaluated. This was discussed earlier, when the generalized structure was introduced. The H2

and H∞ controllers address the controller design problem in its general configuration of non-collocated disturbance and control inputs, and non-collocated performance and control outputs. The H∞ method addresses a wide range of the control problems, combining the frequency- and time-domain approaches. The design is an optimal one in the sense of minimization of the H norm of the closed-loop transfer function. The H∞ model includes colored measurement and process noise. It also addresses the issues of robustness due to model uncertainties, and is applicable to the single-input–single-output systems as well as to the multiple-input–multiple output systems.(Monu et. al. [17])

In this chapter we present the H∞ controller design for flexible structures. We chose the modal approach to H∞ controller design, which allows for the determination of a stable reduced-order H∞ controller with performance close to the full-order controller.

A. *Definition and Gains*

The closed-loop system architecture is shown in figure. In this figure *G* is the transfer function of a plant (or structure), *K* is the transfer function of a controller, *w* is the exogenous input (such as commands, disturbances), *u* is the actuator input, *z* is the regulated output (at which performance is evaluated), and *y* is the sensed (or controlled) output. This system is different from the LQG control system besides the actuator input and controlled output it has disturbance input and the regulated output. Needless to say, it represents a broader class of systems than the LQG control system.



The H∞ closed-loop system configuration: *G*—plant, *K*—controller, *u*— actuator input, *w*—exogenous input, *y*—sensed output, and *z*—regulated output.

For a closed-loop system as in Fig.1 the plant transfer function *G*(*s*) and the controller transfer function *K*(*s*) a

$$\begin{pmatrix} z(s) \\ y(s) \end{pmatrix} = G(s)\begin{pmatrix} w(s) \\ u(s) \end{pmatrix}$$

$$u(s) = K(s)y(s)$$

where *u*, *w* are control and exogenous inputs and *y*, *z* are measured and controlled outputs, respectively. The related state-space equations of a structure are as follows:

$$\dot{x} = Ax + B_1w + B_2u,$$

$$z = C_1x + D_{12}u,$$

$$y = C_2x + D_{21}w.$$

Hence, the state-space representation in the H∞ controller description consists of the quintuple (*A*,*B*1,*B*2 ,*C*1,*C*2). For this representation (*A*,*B*2) is stabilizable and (*A*,*C*2) is detectable, and the conditions H∞ and H2 controllers

$$D_{12}^T[C_1 \quad D_2] = [0 \quad I],$$

$$D_{21}[B_1^T \quad D_{21}^T] = [0 \quad I]$$

are satisfied. When the latter conditions are satisfied the H∞ controller is called the central H∞ controller. These are quite common assumptions, and in the H2 control they are interpreted as the absence of cross terms in the cost function $(D_{12}^TC_1 = 0)$ and the process noise and measurement noise are uncorrelated$(B_1D_{21}^T = 0)$.

The H∞ control problem consists of determining controller *K* such that the H□ norm of the closed-loop transfer function G_{wz} from *w* to *z* is minimized over all realizable controllers *K*, that is, one needs to find a realizable *K* such that

$$\|G_{wz}(K)\|_\infty$$

is minimal. Note that the LQG control system depends on *y* and *u* rather than on *w* and *z*, as above.

The solution says that there exists an admissible controller such that$\|G_{wz}\|_\infty < \rho$, where ρ is the smallest number such that the following four conditions hold :

1) $S_{\infty c} > 0$ solve the following central H∞ controller algebraic Riccati equation (HCARE),

$$S_{\infty c}A + A^TS_{\infty c} + C_1^TC_1 - S_{\infty c}(B_2B_2^T - \rho^{-2}B_1B_1^T)S_{\infty c} = 0$$

2) $S_{\infty e} > 0$ solves the following central H∞ filter (or estimator) algebraic Riccati equation (HFARE),

$$S_{\infty e}A^T + AS_{\infty e} + B_1B_1^T - S_{\infty c}(C_2^TC_2 - \rho^{-2}C_1^TC_1)S_{\infty c} = 0$$

3) $\lambda_{max}(S_{\infty c}\,S_{\infty e}.) < \rho^2$

where λ_{max} (*X*) is the largest eigen value of *X*.

4) The Hamiltonian matrices

$$\begin{bmatrix} A & \rho^{-2}B_1B_1^T - B_2B_2^T \\ -C_1^TC_1 & -A^T \end{bmatrix},$$

$$\begin{bmatrix} A^T & \rho^{-2}C_1C_1^T - C_2C_2^T \\ -B_1B_1^T & -A \end{bmatrix},$$

do not have eigen values on the $j\omega$-axis.

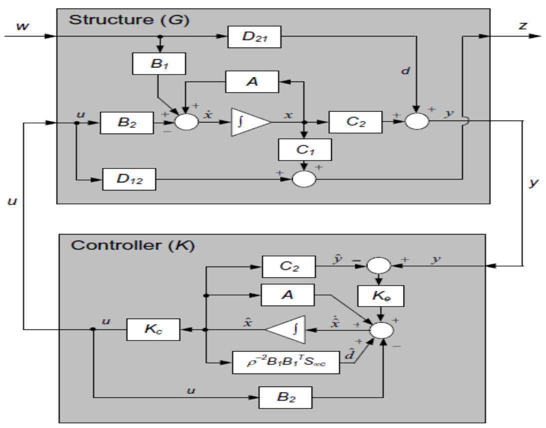


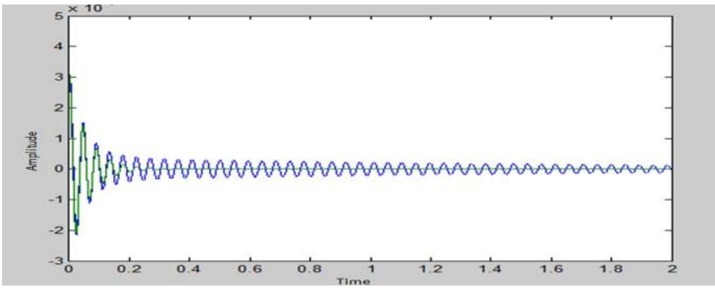
Fig.4. Block diagram for H∞ Controller

V. COMPARISON IN PID, LQG, H∞ CONTROLLERS:

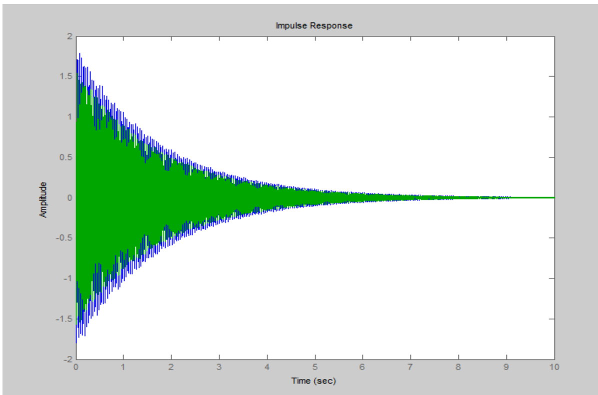
As we are using three types of Controllers namely PID, LQG and H∞, so it is necessary to find out that which controller would give better performance in controlling the vibrations in the cantilever plate. For this purpose, we plot the graphs between the parameters showing properties of the mentioned controllers and hence see the comparison among these three. All graphs are drawn in Matlab.

As compared to LQG and H∞ controllers, the PID controller is much simple and easy. LQG and H∞ control approaches are well suited for the requirements of damping out the effect of disturbances as quickly as possible and maintaining stability robustness, whereas in PID control approach it is not possible. In LQG controller, the design is based on the independent mode space control techniques to suppress the modes vibration of the system, but in PID controller the design is dependent inputs given, whereas in H∞ controller multi- inputs which are dependent and independent mode space control can be given. PID algorithm for control does not guarantee optimal control of the system or system stability. The main limitation of PID controller in that it is a feedback system, with constant parameters, and no direct knowledge of the process and thus overall performance is reactive and a compromise. There is also noise problem in the derivative term of PID controller, but due to Kalman's filter in LQG controller, there is noise elimination up to a good extent. Also in H∞ controller the reduction of noise can be done up to a certain limit. In H∞ controller collocated as well as non-collocated positions of piezoelectric patches can be used but in PID and LQG controllers, it is not possible. LQG controller concerns with uncertain linear systems disturbed by additive white Gaussian noise, having incomplete state information. Also, LQG controller applies to both linear time-invariant systems as well as linear time-varying systems, whereas in the PID controller it is not possible. PID and LQG controllers are single input- single output controllers whereas H∞ controller is a multi input- multi output controller.

VI. RESULTS AND DISCUSSION:

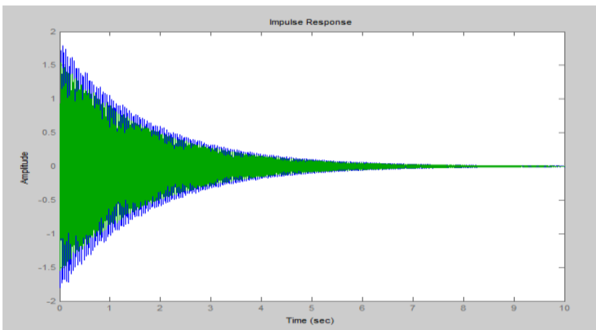


First, taking PID controller,In graph 1, the plot is drawn between the Amplitude (on ordinate) and Time also with Impulse response (on abscissa). In the following graph the value for the proportional input is taken as 98, whereas the values for integrative and derivative inputs are taken as zero. The green lines show the system (i.e. input noise/vibrations) and blue lines show the model system (i.e. controllers output). As the blue lines are exceeding the green lines, so we can conclude that our vibrations are under control.



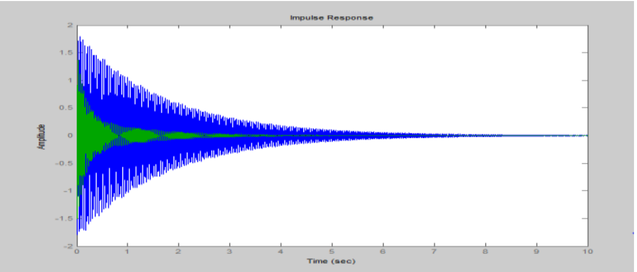
Graph 1: PID controller with P=98, I=0, D=0

In graph 2, the plot is drawn between the Amplitude (on ordinate) and Time also with Impulse response (on abscissa). Values are P= 105, I= 0, D= 0.



Graph 2: PID controller with P=105, I=0, D=0

In graph 3, the plot is drawn between the Amplitude (on ordinate) and Time also with Impulse response (on abscissa). Values are P= 110, I= -0.1, D= 0.1.



Graph 3: PID controller with P=110, I= -0.1, D=0.1

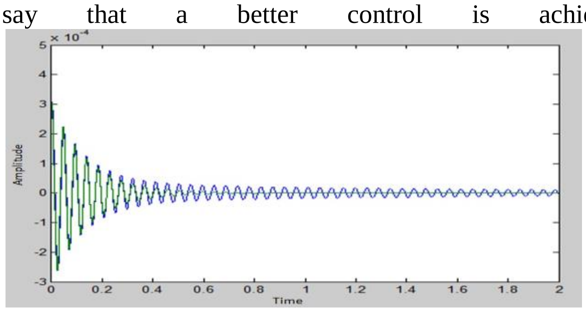
Now, taking Linear Quadratic Gaussian (LQG) controller, similar to PID controller, we can plot different type of graphs between amplitude/tip displacement and time. By changing the values of different parameters, we can find out various results. Also, for LQG controller position of the piezoelectric patches matters. The value of output varies with the position of PZT patches. The open and control loop graphs can also be drawn.

First, drawing a graph between the amplitude/tip displacement (ordinate) and time (abscissa) for LQG controller in closed loop condition.

Graph 4: LQG controller (closed loop case)

In close loop case for LQG controller, the graph can be plotted as shown above. The green sinusoidal wave shows the vibrations of the system, and the blue sinusoidal wave shows the controller's output to control the same vibrations. The amplitude for the closed loop LQG controller varies from 3 to -2 units and decreases continuously. For some time say, 0.15 sec. controller's wave traces the initial vibrations wave but after it, the controller controls the vibrations very significantly.

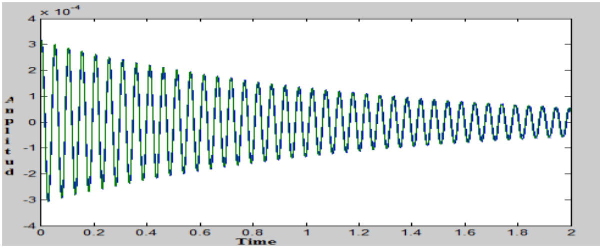
We can draw the LQG controller graphs for the different positions of piezoelectric patches which are acting as the actuators/sensors. LQG controller graph for the PZT position 16, can be shown as below. In which, the amplitude varies from the value 3 to -2.5 units and the settling time is 0.7 sec. The green wave shows the initial vibrations and blue waves shows the controller's output. As the green and blue waves are tracing each other, so we can say that a better control is achieved.



Graph 5: LQG controller at PZT position 16.

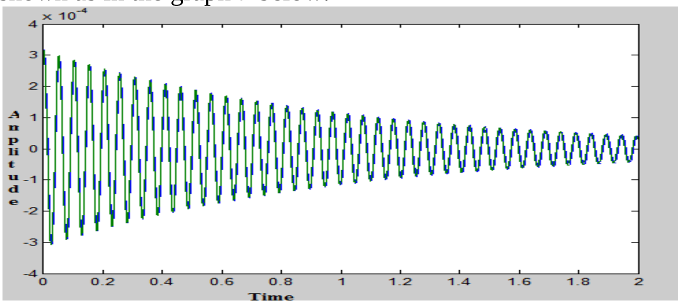
Similarly, we can draw the H∞ controller graphs for the different positions of piezoelectric patches which are acting as the actuators/sensors.

H∞ controller graph for the PZT position 46, can be shown as below. In which, the amplitude varies from the value 3.2 to -3.2 units and the settling time is more than LQG controller. The green wave shows the initial vibrations and blue waves shows the controller's output. As the green and blue waves are tracing each other, so we can say that a better control is achieved.



Graph 6: H∞ controller at PZT position 46

Likewise, H∞ controller plot at PZT position 52 can be shown as in the graph 7 below:



Graph 7: H∞ controller at PZT position 52

VII. CONCLUSION:

This work shows the basic technique of analysis of cantilever plate for Active Vibration Control using piezoelectric sensors and actuators. The optimal location and size of sensor actuator pair for cantilever plate and control effectiveness of PID, LQG and H∞ controller is obtained. Results concluded that the sensor actuator pair is optimally located based on the lower settling time criteria. It is noted that the control effectiveness of PID controller is insignificant when compares to the LQG and H∞ controller's. Study also revealed that LQG controller offers optimal effectiveness with lower peaks in settling time as compared to other classical control like PID strategies. Whereas H∞ controller is better than both PID and LQG controllers in the sense of input- output mode and collocated phenomenon.

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